Sketch The Region Bounded By The Curve 5xe X, The X-axis, And X=4, And Find The Area Of The Region. Round

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Understanding the geometric and calculus concepts involved in sketching regions bounded by curves and calculating their areas is fundamental in many fields such as mathematics, engineering, and physics. This article aims to provide a detailed, step-by-step guide to sketching the region bounded by the given curve $y = 5xe^x$, the x-axis, and the vertical line $x=4$. Additionally, we will demonstrate how to compute the exact area of this region and offer insights into rounding the result appropriately for practical use.

Understanding the Given Curve and Boundaries

Examining the Curve $y = 5xe^x$

The function $y = 5xe^x$ is a product of a linear term $5x$ and an exponential term e^x. This combination results in a curve that exhibits exponential growth but is modulated by the linear factor $5x$.

Key features of this curve include:

- Intercept at $x=0$: When $x=0$,
$$
y = 5 \times 0 \times e^{0} = 0
$$
so the curve passes through the origin.

- Behavior for $x > 0$: Since both x and e^x are positive for $x > 0$, the function $y = 5xe^x$ is positive, and the curve rises rapidly as x increases.

- Behavior for $x < 0$: For negative x, e^x is positive but decreases rapidly, and x is negative, so y becomes negative. This indicates the curve dips below the x-axis for some negative x.

Sketching the Region

Step 1: Plotting Key Points

To sketch the region, begin by calculating some key points:

- At $x=0$:
$$
y=0
$$
point: (0, 0).

- At $x=1$:
$$
y=5 \times 1 \times e^{1} \approx 5 \times 2.718 \approx 13.59
$$
point: (1, 13.59).

- At $x=2$:
$$
y=5 \times 2 \times e^{2} \approx 10 \times 7.389 \approx 73.89
$$
point: (2, 73.89).

- At $x=4$:
$$
y=5 \times 4 \times e^{4} \approx 20 \times 54.598 \approx 1091.96
$$
point: (4, 1091.96).

- For negative x, say $x=-1$:
$$
y=5 \times -1 \times e^{-1} \approx -5 \times 0.368 \approx -1.84
$$
point: (-1, -1.84).

Plotting these points helps visualize the curve's shape, especially noting that it crosses the x-axis at $x=0$.

Step 2: Determining the Region's Boundaries

The boundaries are:

- The curve $y = 5xe^x$,

- The x-axis $y=0$,
- The vertical line $x=4$.

Since $y = 5xe^x$ is positive for $x>0$, the region of interest is from $x=0$ to $x=4$, where the curve remains above the x-axis.

However, for $x<0$, the curve dips below the x-axis, but the boundary is defined by the x-axis itself, which acts as a boundary from below where the curve is negative.

Important: The region bounded by the curve, x-axis, and $x=4$ is primarily from $x=0$ to $x=4$, since the curve is above the x-axis in this interval.

Calculating the Area of the Region

Step 1: Setting Up the Integral

The area enclosed by the curve $y=5xe^x$, the x-axis, and $x=4$ can be computed by integrating the absolute value of y over the relevant interval.

Because $y=5xe^x$ is positive for $x \ge 0$, the area A from $x=0$ to $x=4$ is:

$$
A = \int_{0}^{4} 5xe^x \, dx
$$

This integral gives the exact area under the curve from 0 to 4.

Step 2: Computing the Integral $\int 5xe^x dx$

To evaluate:

$$
A = 5 \int_{0}^{4} x e^x \, dx
$$

We factor out the constant 5:

$$
A = 5 \times \int_{0}^{4} x e^{x} \, dx
$$

Method: Use integration by parts.

Step 3: Applying Integration by Parts

Recall the integration by parts formula:

$$
\int u \, dv = uv - \int v \, du
$$

Let:

- $u = x \Rightarrow du = dx$,
- $dv = e^x dx \Rightarrow v = e^x$.

Applying integration by parts:

$$
\int x e^x dx = x e^x - \int e^x dx = x e^x - e^x + C
$$

Therefore:

$$
\int x e^x dx = e^x (x - 1) + C
$$

Multiplying by 5:

$$
\int 5 x e^x dx = 5 e^x (x - 1) + C
$$

Step 4: Evaluating the Definite Integral

Now, compute:

$$
A = 5 [e^x (x - 1)]_{0}^{4}

\]

Calculate at the upper limit $x=4$:

$$
e^4 (4 - 1) = e^4 \times 3
$$

Calculate at the lower limit $x=0$:

$$
e^0 (0 - 1) = 1 \times (-1) = -1
$$

Putting it all together:

$$
A = 5 [e^4 \times 3 - (-1)] = 5 [3 e^4 + 1]
$$

Recall that $e^4 \approx 54.598$. Therefore:

$$
A \approx 5 [3 \times 54.598 + 1] = 5 [163.794 + 1] = 5 \times 164.794 = 823.97
$$

Final Result and Rounding

The approximate area of the region bounded by the curve $y=5xe^x$, the x-axis, and $x=4$ is roughly 823.97 square units.

Depending on the context, you might round this to:

- Two decimal places: 823.97
- Nearest whole number: 824

Summary:

- The area is approximately 823.97 square units.
- The calculation involved setting up the integral, applying integration by parts, and evaluating the definite integral.
- The key insight was recognizing that the curve is positive from $x=0$ to $x=4$, allowing straightforward integration without considering absolute value.

Additional Considerations

Handling the Region for $x < 0$

If you wish to consider the entire bounded region, including where the curve dips below the x-axis (for $x<0$), you must:

- Calculate the area where the curve is below the x-axis by integrating $-y$ over that interval.
- Find the points where $y=0$, which occurs at $x=0$, so the negative region extends from some negative x to 0.

However, since the problem specifies the region bounded by the curve, the x-axis, and $x=4$, and the curve is above the x-axis for $x \in [0, 4]$, the calculation above is sufficient for the area in that domain.

Conclusion

Sketching the region bounded by the curve $y=5xe^x$, the x-axis, and the vertical line $x=4$ involves understanding the behavior of the function, plotting key points, and identifying the relevant interval where the region exists. Calculating the area requires integrating y over this interval, which, through the use of integration by parts, yields an exact expression that approximates

Frequently Asked Questions

What is the first step to sketch the region bounded by the curve 5xe^x, the x-axis, and x=4?

The first step is to evaluate the curve 5xe^x at key points, especially at x=4, and determine where the curve intersects the x-axis to understand the region's boundaries.

How do you find the x-intercepts of the curve 5xe^x?

Set the function equal to zero: 5xe^x = 0. Since e^x is never zero, the x-intercept occurs at x=0.

What is the significance of the x=4 line in sketching the region?

The line x=4 serves as one of the vertical boundaries of the region, limiting the area to the interval from the x-intercept at x=0 up to x=4.

How do you set up the integral to find the area of the region bounded by the curve and x-axis?

Since the curve is above the x-axis between x=0 and x=4, the area is given by the integral from 0 to 4 of 5xe^x dx.

What method should be used to evaluate the integral of 5xe^x dx?

Integration by parts is appropriate here, choosing u= x and dv= e^x dx, to evaluate ∫ x e^x dx, then multiply the result by 5.

How do you round the final area value, and what is its approximate value?

After computing the integral, round the result to the desired decimal place. The approximate value of the area is around 27.36 square units (depending on rounding).

Why is it important to verify the points where the curve intersects the x-axis when sketching the region?

Because these points determine the limits of the region and ensure the sketch accurately represents the bounded area between the curve and the x-axis.

Can the area be negative? Why or why not?

No, the area cannot be negative. When calculating the area, the integral's absolute value or proper bounds are used to ensure the result is positive, representing the actual size of the region.

Additional Resources

Sketching the Region Bounded by the Curve 5xe^x, the X-axis, and X=4: An In-Depth Analytical Review

Understanding the geometric and analytical properties of functions is fundamental in calculus, especially when it comes to visualizing regions and calculating their areas. This article offers a comprehensive exploration of the problem: sketching the region bounded by the curve $y = 5xe^{x}$, the x-axis, and the vertical line $x=4$. We will delve into the detailed process of graphing, interpreting the bounded region, and computing its exact area—culminating in a rounded numerical result. The discussion is structured to guide readers through each step, providing clarity and insight into the mathematical reasoning involved.

Introduction to the Problem and Its Significance

The task of sketching a region bounded by a specific curve, the x-axis, and a vertical boundary is a classic problem in integral calculus, especially in the context of finding areas under curves. Understanding these regions is not only a theoretical exercise; it has practical applications in physics (e.g., work done by a variable force), economics (e.g., consumer surplus), and engineering (e.g., material stress analysis).

Specifically, the curve $y = 5xe^{x}$ presents an interesting case due to its exponential component intertwined with a linear term. Its shape is influenced by both polynomial and exponential behaviors, leading to a region with potentially complex features. The challenge is to accurately sketch this region and determine its area, which requires a clear understanding of the function's behavior, the points of intersection with the axes, and the boundaries defined by the vertical line $x=4$.

Understanding the Function $y=5xe^{x}$

Behavior and Characteristics of the Curve

The function $y=5xe^{x}$ combines a linear factor $5x$ with an exponential factor e^{x}. To analyze its graph:

- Domain: All real numbers ($-\infty < x < \infty$)
- Range: Since $e^{x} > 0$ for all real x, the sign of y depends on x:

- For $x > 0$, $y > 0$
- For $x < 0$, $y < 0$
- At $x=0$, $y=0$

- Key features:

- Zeros: The function crosses the x-axis at $x=0$.
- Behavior as $x \to -\infty$: $e^{x} \to 0$, so $y \to 0^-$ (approaching zero from below).
- Behavior as $x \to \infty$: Both x and e^{x} grow large, so $y \to \infty$.

Critical Points and Concavity

Finding the critical points helps identify local minima or maxima, which influence the shape of the graph:

- First derivative:

$$
\frac{dy}{dx} = 5 \left(e^{x} + x e^{x} \right) = 5 e^{x} (1 + x)
$$

- Critical points occur where:

$$
1 + x = 0 \Rightarrow x = -1
$$

- Second derivative:

$$
\frac{d^{2}y}{dx^{2}} = 5 e^{x} (1 + x) + 5 e^{x} = 5 e^{x} (2 + x)
$$

- At $x=-1$:

$$
\frac{d^{2} y}{dx^{2}} = 5 e^{-1} (2 - 1) = 5 e^{-1} (1) > 0
$$

This indicates $x = -1$ is a local minimum.

Sketching the Region: Methodical Approach

Step 1: Plotting the Key Features

To sketch the region bounded by the curve, x-axis, and $x=4$, start by plotting:

- The x-axis (line $y=0$)
- The vertical line $x=4$
- The curve $y=5xe^{x}$

Key points to plot include:

- At $x=0$: $y=0$
- At $x=1$: $y=5 \times 1 \times e^{1} \approx 5 \times 1 \times 2.718 \approx 13.59$
- At $x=-1$: $y=5 \times (-1) \times e^{-1} \approx -5 \times 0.368 \approx -1.84$
- At $x=2$: $y=5 \times 2 \times e^{2} \approx 10 \times 7.389 \approx 73.89$
- At $x=4$: $y=5 \times 4 \times e^{4} \approx 20 \times 54.598 \approx 1091.96$

Plotting these points indicates the curve crosses the x-axis at $x=0$ and rapidly rises for $x > 0$. For $x<0$, the curve dips below the x-axis, reaching a minimum at $x=-1$.

Step 2: Identifying the Boundaries of the Region

The region of interest is bounded by:

- The curve $y=5xe^{x}$ from $x=0$ to $x=4$
- The x-axis (from $y=0$) between $x=0$ and $x=4$
- The vertical line $x=4$

Note that for $x<0$, the curve lies below the x-axis, so the bounded region is only relevant for $x \geq 0$, where the curve is above the x-axis.

Thus, the region is:

- Starting at the point $(0, 0)$
- Under the curve $y=5xe^{x}$ from $x=0$ to $x=4$
- Bounded below by the x-axis
- Bounded on the right by $x=4$

Calculating the Area of the Region

Step 1: Setting Up the Integral

The area A of the region bounded by the curve and the x-axis from $x=0$ to $x=4$ is given by:

$$
A = \int_{0}^{4} y\, dx = \int_{0}^{4} 5x e^{x} \, dx
$$

Since this is a definite integral, the goal is to evaluate:

$$
A = 5 \int_{0}^{4} x e^{x} \, dx
$$

Step 2: Computing the Integral $\int x e^{x} dx$

To evaluate $\int x e^{x} dx$, we employ integration by parts:

- Let:

$$
u = x \quad \Rightarrow \quad du = dx
$$
$$
dv = e^{x} dx \quad \Rightarrow \quad v= e^{x}
$$

- Applying integration by parts:

$$
\int x e^{x} dx = x e^{x} - \int e^{x} dx = x e^{x} - e^{x} + C
$$

- Therefore,

$$
\int x e^{x} dx = e^{x}(x - 1) + C
$$

Step 3: Incorporating the Multiplier 5 and Limits

Now, the area becomes:

$$
A = 5 \times \left[e^{x}(x - 1) \right]_0^{4}
$$

Calculate at the bounds:

- At $x=4$:

$$
e^{4}(4 - 1) = e^{4} \times 3 \approx 54.598 \times 3 \approx 163.794
$$

- At $x=0$:

$$
e^{0}(0 - 1) = 1 \times (-1) = -1
$$

Thus,

$$
A = 5 \times (163.794 - (-1)) = 5 \times (164.794) \approx 5 \times 164.794 \approx 823.97
$$

Final Result and Rounding

The area of the region bounded by the curve $y=5xe^{x}$, the x-axis, and the line $x=4$ is approximately 823.97 square units when rounded to two decimal places.

Interpretation and Practical Implications

This computed area provides a quantitative measure of the region under the curve $y=5xe^{x}$

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