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Understanding the Vector Field F(7) = 2r in the Plane

When exploring vector fields, one fundamental example in multivariable calculus involves understanding how vectors change across a plane. Specifically, the vector field F(7) = 2r—where r = (x, y)—serves as an essential case study for visualizing vector directions and magnitudes. This article aims to provide a comprehensive guide to sketching this vector field, interpreting its characteristics, and understanding the implications of the given mathematical form.

What Is the Vector Field F(7) = 2r?

Definition and Components

The vector field F(7) = 2r is a linear, radially symmetric field defined in the two-dimensional plane. Here:

- r = (x, y) is the position vector pointing from the origin to the point (x, y).
- The field assigns to each point (x, y) the vector F(7) = 2r = (2x, 2y).

This means that at every point in the plane, the vector points directly away from the origin, with a magnitude proportional to the distance from the origin, scaled by a factor of 2.

Mathematical Interpretation

- The field is radially outward, meaning vectors emanate outward from the origin.
- The magnitude of the vector at point (x, y) is:

$$
|F(7)| = \sqrt{(2x)^2 + (2y)^2} = 2 \sqrt{x^2 + y^2} = 2r
$$

- The direction of the vectors is directly away from the origin, aligned with the position vector r.

Visualizing the Vector Field: Sketching Techniques

Step 1: Understanding the Direction of Vectors

Since F(7) = 2r, the vectors at any point (x, y) point radially outward from the origin. The direction is the same as the position vector r.

Step 2: Magnitude of Vectors at Various Points

- Vectors farther from the origin are longer.
- For example:
- At (1, 0), the vector is (2, 0) with magnitude 2.
- At (0, 3), the vector is (0, 6) with magnitude 6.
- At (−2, −2), the vector is (−4, −4) with magnitude approximately 5.66.

Step 3: Drawing the Vectors

To sketch F(7):

- Select key points in the plane, such as (1, 0), (0, 1), (−1, 0), and (0, −1).
- Draw vectors at these points pointing outward, with lengths proportional to their distances from the origin, scaled by 2.
- Ensure the vectors are aligned with the position vectors, pointing away from the origin.

Step 4: Adding Additional Vectors for Completeness

- To accurately visualize the field, include vectors at various radii and angles.
- Use consistent scaling to depict the proportionality of magnitudes.

Characteristics of the Vector Field F(7) = 2r

Radial Symmetry

- The field exhibits radial symmetry; vectors point directly away from (or toward) the origin, depending on the sign.
- In this case, vectors point outward because the coefficient is positive.

Magnitude Variation

- Magnitude increases linearly with the distance from the origin.
- Closer to the origin, vectors are shorter; farther away, they are longer.

Divergence and Curl

- The divergence of F is positive everywhere except at the origin, indicating a source-like behavior.
- The curl of this vector field is zero, indicating it is irrotational.

Practical Applications and Significance

Physics Context: Radial Force Fields

- The vector field F(7) = 2r models phenomena like radial force fields emanating from a point source, such as electric fields around a positive charge or gravitational fields around a mass.

Engineering and Fluid Dynamics

- Such vector fields are used to model flow patterns where fluid moves outward uniformly from a source point.

Mathematical Analysis

- The field's simple form makes it an ideal example for studying divergence, gradient fields, and vector calculus operators.

Select All That Apply: Analyzing the Given Options

Option A: All The Vectors

- Since the vectors radiate outward from the origin with magnitudes proportional to the distance, all vectors in the field point away from the origin.
- The field is radially outward, with vector length increasing linearly as you move away from the origin.

Additional Considerations

- Are all vectors in the same direction? Yes, they are aligned with the position vectors, pointing outward.
- Are vectors of equal length? No, their lengths depend on their position, increasing with distance from the

origin.
- Are vectors at the same distance from the origin equal in magnitude? Yes, vectors at the same radius have the same magnitude.

Visual Sketching Tips for Accurate Representation

- Use a consistent scaling factor to relate vector length to position.
- Plot vectors at multiple points forming concentric circles to visualize the radial symmetry.
- Remember that the vectors at the origin are zero-length (theoretically), so they can be omitted or represented as a point.

Summary: Key Takeaways

- The vector field F(7) = 2r is a classic example of a radial vector field.
- Vectors point away from the origin, with magnitude increasing linearly with distance.
- The field is divergent, with a positive divergence indicating a source at the origin.
- Visualizing the field involves plotting vectors with lengths proportional to their distance from the origin and pointing outward.

Concluding Remarks

Mastering the sketching and interpretation of the vector field F(7) = 2r is fundamental in multivariable calculus and physics. It provides insight into how vector fields behave, especially those that are radially symmetric and exhibit linear magnitude growth. By understanding these characteristics, students and professionals can better analyze complex systems involving fields, flows, and forces in two-dimensional space.

FAQs

Q1: Why does the vector field increase in magnitude with distance?

A: Because F(7) = 2r, the magnitude depends on the length of the position vector r = (x, y), scaled by 2. As the distance from the origin increases, the magnitude of the vectors increases proportionally.

Q2: How do I determine the direction of vectors at various points?

A: The vectors point in the same direction as the position vector r at each point. For example, at (x, y), the vector points directly away from the origin toward (x, y).

Q3: What is the significance of the divergence being positive?

A: A positive divergence indicates a source at the origin, meaning the field behaves like something emanating outward from a point source.

Final Notes

Understanding and sketching vector fields like F(7) = 2r is crucial in many scientific disciplines. Recognizing the properties of radial symmetry, magnitude variation, and divergence lays the foundation for more advanced topics in vector calculus and physics. Practice plotting vectors at various points and analyzing their behavior to develop intuition for these fields.

Frequently Asked Questions

What is the general form of the vector field F(r) = 2r in the plane?

The vector field F(r) = 2r assigns to each point r = (x, y) a vector pointing radially outward from the origin, scaled by a factor of 2, i.e., F(x, y) = (2x, 2y).

How do the vectors in the field F(7) = 2r behave at various points in the plane?

At any point (x, y), the vector points directly away from the origin, with magnitude proportional to the distance from the origin, specifically 2 times the distance.

What does the phrase 'Select All That Apply' suggest about analyzing the vector field?

It indicates multiple properties or characteristics of the vector field may be correct, and one should select all options that accurately describe the field.

Are all vectors in the field F(7) = 2r of the same length?

No, the vectors' lengths vary proportionally with their distance from the origin; vectors farther from the origin are longer.

Does the vector field F(7) = 2r have any particular symmetry?

Yes, it exhibits radial symmetry about the origin, with vectors pointing directly outward in all directions.

Could the vector field F(7) = 2r be considered a gradient field?

Yes, since it can be derived from the scalar potential function $\varphi(x, y) = x^2 + y^2$, with $F = \nabla\varphi$.

Is the vector field F(7) = 2r divergence-free?

No, the divergence of F is constant and equal to 4 everywhere in the plane, indicating it is not divergence-free.

What is the divergence of the vector field F(7) = 2r?

The divergence is 4, calculated as $\partial(2x)/\partial x + \partial(2y)/\partial y = 2 + 2 = 4$.

Does the vector field F(7) = 2r have any rotational component?

No, it is a purely radial (conservative) field with no rotational component, meaning its curl is zero.

In the context of the given vector field, what does 'All The Vectors' refer to?

It suggests considering all vectors at every point in the plane, emphasizing the uniform radial pattern of the entire field.

Additional Resources

Vector Field Sketching: An In-Depth Examination of F(7) = 2r in the Plane

When exploring vector calculus and field visualization, understanding how to interpret and sketch vector fields is fundamental. Among the many vector fields encountered, the field defined as F(r) = 2r in the plane offers a compelling case study, combining simplicity with rich geometric insight. This article delves into the nuances of this vector field, guiding you through the conceptual framework, the properties, and the methods for sketching it effectively—particularly considering the multiple-choice question: Select All That Apply: A. All the Vectors.

Understanding the Vector Field $F(r) = 2r$

Defining the Vector Field

The vector field $F(r) = 2r$ is a linear transformation in the two-dimensional plane, where r represents a position vector $r = (x, y)$. Specifically:

$$
F(x, y) = 2(x, y) = (2x, 2y)
$$

This indicates that at every point (x, y) in the plane, the vector $F(x, y)$ points in the same direction as the position vector r, but scaled by a factor of 2.

Key Points:

- The field is radial, meaning vectors emanate outward from the origin.
- The magnitude of each vector doubles that of its position vector.
- The field exhibits linear behavior, with the vectors increasing proportionally as you move away from the origin.

Physical and Geometric Intuition

Imagine a rubber sheet stretched in the plane, with vectors representing forces or velocities at various points. The $F(r) = 2r$ field suggests that:

- Near the origin, vectors are small.
- As you move farther from the origin, vectors grow proportionally in length.
- All vectors point directly away from (or toward, depending on the sign) the origin, making the field radially symmetric.

This is analogous to a uniformly expanding or shrinking field, depending on the interpretation, with the origin acting as a center of expansion.

Mathematical Properties and Characteristics

1. Radial Nature and Directionality

Since F(r) = 2r, the vectors are always scalar multiples of the position vector r. Therefore:

- The direction of F(r) at any point (x, y) is the same as r.
- The vectors are radial, pointing directly away from or toward the origin.

Implication: When sketching, vectors should be aligned along the lines passing through the origin and their respective points.

2. Magnitude of Vectors

The magnitude at any point (x, y) is:

$$
|F(r)| = |2r| = 2|r| = 2 \sqrt{x^2 + y^2}
$$

This indicates that:

- The magnitude increases linearly with the distance from the origin.
- At the origin (0,0), the vector magnitude is zero.
- The further away from the origin, the longer the vectors.

Visual Tip: Use concentric circles around the origin to help gauge the length of vectors at different distances.

3. Divergence and Curl

- Divergence: $\nabla \cdot F = \frac{\partial}{\partial x}(2x) + \frac{\partial}{\partial y}(2y) = 2 + 2 = 4$. The positive divergence indicates a source-like behavior, with vectors emanating outward.
- Curl: $\nabla \times F = \frac{\partial}{\partial x}(2y) - \frac{\partial}{\partial y}(2x) = 0 - 0 = 0$. The zero curl confirms the field is irrotational, with no tendency to circulate.

Sketching the Vector Field F(7) = 2r

To accurately sketch $F(r) = 2r$, consider these systematic approaches:

Step 1: Establish a Coordinate Grid

Plot a representative grid with points at various distances from the origin, such as:

- (0,0)
- (1,0), (0,1), (-1,0), (0,-1)
- (1,1), (-1,1), (-1,-1), (1,-1)

This ensures a comprehensive view of the field's behavior.

Step 2: Calculate and Draw Vectors at Sample Points

For each point, compute $F(r)$:

- At (1,0): $F(1,0) = (21, 20) = (2, 0)$
- At (0,1): $F(0,1) = (0, 2)$
- At (-1,0): $F(-1,0) = (-2, 0)$
- At (0,-1): $F(0,-1) = (0, -2)$
- At (1,1): $F(1,1) = (2, 2)$
- At (-1,-1): $F(-1,-1) = (-2, -2)$

The vectors point outward along the line from the origin through each point, with lengths proportional to their distance from the origin.

Tip: Use a consistent scale for vector length to maintain proportion, e.g., 1 unit length for a vector of magnitude 1.

Step 3: Draw Vectors with Correct Magnitude and Direction

- Ensure each vector originates at the sample point.
- Draw the vector in the same direction as $F(r)$.
- Lengthen or shorten the arrow proportionally.

Step 4: Use Symmetry and Radial Lines

- The vectors should be symmetric about the origin.
- Since the vectors are proportional to r, they will always be aligned with the radius lines emanating from the origin.

Step 5: Annotate and Confirm

- Label the vectors with their magnitudes if necessary.
- Confirm vectors at symmetric points have the same length but opposite directions if on opposite sides of the origin.

Interpreting the Multiple-Choice Options: "Select All That Apply: A. All The Vectors"

The question appears to inquire whether the vectors at all points are represented or whether the vectors cover all possible directions and magnitudes consistent with F(r) = 2r.

Key considerations:

- A. All the Vectors:
- The field assigns a vector to every point in the plane except possibly the origin (where the vector is zero).
- Since F(r) = 2r is defined everywhere in \(\mathbb{R}^2\), all vectors at all points can be considered.
- The vectors are not uniform in magnitude; their length depends on the point's distance from the origin.

Conclusion:
- If the question is asking whether all vectors are included in the sketch, the answer is Yes, provided the sketch covers the entire plane or a sufficiently large region.
- If it asks whether all vectors of this form are represented, then the answer is Yes—the field includes vectors pointing radially outward, scaled by the distance from the origin.

Summary and Final Remarks

The vector field F(r) = 2r exemplifies a classic radial expansion field with elegant properties:

- Radial symmetry: vectors point directly away from the origin.
- Linearly increasing magnitude: vectors grow in length proportionally with their distance from the origin.
- Zero at the origin: the vector is null at (0,0).
- Divergence is positive: indicating a source-like behavior.
- Curl is zero: confirming irrotationality.

When sketching this field:

- Use a grid of points at various distances.
- Calculate the vectors based on the position.
- Draw arrows aligned with the radius vector, scaled appropriately.
- Maintain consistency in scale and directionality.

In conclusion, the field F(r) = 2r is a straightforward yet illustrative example of a radial vector field, showcasing how geometric and algebraic insights converge to produce an accurate and meaningful visualization. Recognizing that all vectors in such a field are emanating outward, with magnitudes proportional to the distance from the origin, is key to mastering the art of vector field sketching—making the option A. All The Vectors a fitting choice when considering the comprehensive coverage of the field in a given region.

Final Note: Whether you are a student beginning to explore vector calculus or an educator aiming to clarify the concepts, mastering the sketching of such fields enhances both intuition and analytical skills—fundamental tools for advanced mathematical and physical applications.

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