Sketch Two Periods Of The Graph Of The Function H(x)=4sec(4(x+3)).Identify The Stretching Factor, Period,

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Understanding how to graph trigonometric functions like the secant function is essential for students and professionals working in mathematics, engineering, and physics. The function H(x) = 4sec(4(x+3)) presents an interesting case due to its transformations, including shifts, stretches, and periodicity. In this comprehensive guide, we will delve into how to sketch two periods of this function, identify the stretching factor, and determine its period, providing clear, step-by-step instructions to enhance your understanding.

Understanding the Basic Secant Function

Before analyzing the transformed function H(x) = 4sec(4(x+3)), it's crucial to recall the basic properties of the secant function.

What Is the Secant Function?

- The secant function, denoted as sec(x), is the reciprocal of the cosine function:

sec(x) = 1 / cos(x)

- Its domain excludes points where cos(x) = 0, which occur at x = (π/2) + nπ, where n is an integer.
- The graph of sec(x) consists of a series of branches opening upward and downward, with asymptotes at the zeros of cosine.

Basic Characteristics of sec(x)

- Period: 2π
- Range: (−∞, −1] ∪ [1, ∞)

- Asymptotes: $x = (\pi/2) + n\pi$

Visualizing the basic secant graph helps us understand how transformations like shifts, stretches, or compressions affect the graph's shape and period.

Analyzing the Function H(x) = 4sec(4(x+3))

Within the function H(x), several transformations are applied to the basic secant function. Let's dissect each component to understand their effects.

Breaking Down the Function Components

- The coefficient 4 outside the secant, i.e., 4sec(...), indicates a vertical stretch.
- The argument of the secant, 4(x+3), involves horizontal compression and a horizontal shift.
- The (x + 3) inside the argument shifts the graph horizontally.

Step-by-Step Transformation Analysis

1. Horizontal Shift:
- The term (x + 3) shifts the graph 3 units to the left because adding inside the function shifts leftward.

2. Horizontal Compression:
- The coefficient 4 inside the argument compresses the graph horizontally by a factor of 1/4.
- This affects the period, making the graph "squeeze" along the x-axis.

3. Vertical Stretch:
- The coefficient 4 outside the secant stretches the graph vertically by a factor of 4, making the branches steeper and extending the range.

Determining the Period of H(x) = 4sec(4(x+3))

The period of a basic secant function, sec(x), is 2π. When transformations are applied, particularly horizontal compression, the period changes accordingly.

Calculating the Period

- The general form of a transformed secant function is:

$\sec(b(x - c))$

- The period of this function is given by:

$\text{Period} = (2\pi) / |b|$

- In our case, the argument is $4(x + 3)$, which can be written as $4(x - (-3))$.
- Here, $b = 4$.

- Therefore, the period of $H(x)$ is:

$\text{Period} = 2\pi / 4 = \pi / 2$

Conclusion:
The function $H(x)$ repeats every $\pi/2$ units along the x-axis.

Sketching Two Periods of H(x)

To accurately sketch two periods, follow these steps:

1. Identify the Domain Range for Two Periods:
- Since one period is $\pi/2$, two periods span from an initial x-value to initial + π.

2. Locate Asymptotes:
- Asymptotes occur where $\cos(4(x+3)) = 0$.
- Solve for x:

$\cos(4(x+3)) = 0$
$4(x + 3) = (\pi/2) + n\pi, n \in \mathbb{Z}$

$x + 3 = [(\pi/2) + n\pi] / 4$
$x = [(\pi/2) + n\pi] / 4 - 3$

- For two periods, consider n = 0 and n = 1:

- For n=0:
$x = (\pi/2) / 4 - 3 = \pi/8 - 3$

- For n=1:
$x = (\pi/2 + \pi) / 4 - 3 = ((\pi/2) + \pi) / 4 - 3 = ((\pi/2) + 2\pi/2) / 4 - 3 = (3\pi/2) / 4 - 3 = (3\pi/8) - 3$

- These asymptotes partition the graph into sections, guiding the shape of the secant branches.

3. Plot Key Points:

- Maximum and Minimum Values:
Since the range is $(-\infty, -1] \cup [1, \infty)$, the branches tend toward vertical asymptotes and extend infinitely upwards or downwards.

- Calculate points where secant reaches its minimum or maximum:
- These occur where $\cos(4(x+3)) = \pm 1$.
- Solve for x:
$\cos(4(x+3)) = 1 \rightarrow 4(x+3) = 2n\pi \rightarrow x = (2n\pi)/4 - 3 = (n\pi/2) - 3$
- For $n=0$: $x=-3$
- When $\cos(4(x+3))=-1$: $4(x+3) = (2n+1)\pi \rightarrow x = [(2n+1)\pi]/4 - 3$

4. Construct the Graph:

- Draw the asymptotes at the calculated x-values.
- Plot points at $x = -3$ (max point), and other key points.
- Sketch the branches approaching asymptotes, ensuring the vertical stretch makes the branches steep.

5. Repeat for the Second Period:

- Continue plotting asymptotes and key points at intervals of $\pi/2$ to cover two periods.

Summary of Key Features of $H(x) = 4\sec(4(x+3))$

Feature	Description
Period	$\pi/2$
Horizontal Compression Factor	1/4 (since coefficient of x in the argument)
Vertical Stretch	4 (amplitude scaling)
Horizontal Shift	3 units to the left
Vertical Asymptotes	$x = (\pi/8) - 3 + n(\pi/2)$, where n is an integer

Visual Tips for Sketching

- Always start by plotting the asymptotes.
- Mark key points where $\cos(4(x+3))$ equals ± 1.
- Remember the range and asymptotic behavior of secant.
- Use the period to mark repeating sections.
- Incorporate the vertical stretch to accurately depict the steepness.

Conclusion

Sketching the graph of H(x) = 4sec(4(x+3)) requires understanding the effects of horizontal compression, shifts, and vertical stretching. By calculating the period as π/2, identifying the asymptotes, and plotting key points, you can accurately represent two periods of the graph. Recognizing these transformations not only helps in graphing but also deepens comprehension of how trigonometric functions behave under various modifications.

Mastering these concepts provides a solid foundation for tackling more complex trigonometric functions and their applications across different fields such as physics, engineering, and mathematics. Keep practicing by sketching different transformed secant functions to build confidence and improve your analytical skills.

Frequently Asked Questions

What is the general form of the function H(x) = 4sec(4(x+3))?

The function is a secant function with a vertical stretch/compression of 4 and a horizontal transformation involving a shift and a frequency factor of 4 inside the cosine function.

How do you determine the period of the function H(x) = 4sec(4(x+3))?

The period of the secant function is derived from its cosine counterpart. Since the basic secant function has a period of 2π, and the argument is 4(x+3), the period is calculated as (2π)/4 = π/2.

What is the stretching factor applied to the graph of H(x)?

The stretching factor is 4, as indicated by the coefficient in front of the secant function, which vertically stretches the graph by a factor of 4.

How do you find the two key periods of the graph of H(x) within one full cycle?

Since the period is π/2, the two key points (such as the start and end of one cycle) can be found at x = -3 and x = -3 + π/2, i.e., x = -3 and x = -3 + 1.5708.

What transformations are applied to the basic secant graph to obtain H(x)?

The graph is horizontally shifted 3 units to the left (due to +3 inside the argument), scaled horizontally by a factor of 1/4 (since the argument is multiplied by 4), and vertically stretched by a factor of 4.

How would you sketch two periods of H(x)?

Identify the starting point at $x = -3$, then mark key points at intervals of $\pi/2$ (approximately 1.57) to the right, noting where the secant function has asymptotes and key points, and apply the vertical stretch of 4 to visualize the graph accurately.

What are the asymptotes of H(x) in one period, and how are they determined?

The asymptotes occur where the cosine inside the secant is zero, i.e., when $4(x+3) = \pi/2 + k\pi$. Solving for x gives the asymptote positions at $x = (-3 + (\pi/2 + k\pi)/4)$. For one period, they occur at $x = -3 + (\pi/2)/4$ and $x = -3 + (3\pi/2)/4$.

Additional Resources

Sketch Two Periods Of The Graph Of The Function H(x)=4sec(4(x+3)))

Understanding the behavior and characteristics of the function $H(x) = 4 \sec(4(x + 3))$ is essential for accurately sketching its graph and analyzing its properties. This function involves the secant function, which is the reciprocal of the cosine function, scaled and shifted in multiple ways. To develop a comprehensive understanding, it is vital to explore its period, stretching factors, asymptotes, key points, and how these elements influence its graph. This article provides an in-depth review of the features of H(x), guides on sketching two periods of its graph, and discusses the implications of its transformations.

Understanding the Function $H(x) = 4 \sec(4(x + 3))$

The function H(x) combines the secant function with a horizontal transformation and amplitude scaling. To analyze its graph, we need to understand each component:

- Secant function, sec(x): Defined as 1 / cos(x). Its graph has vertical asymptotes where cos(x) = 0, i.e., at odd multiples of $\pi/2$, and it exhibits a series of "U"-shaped branches extending to infinity.
- Scaling factor (4): Multiplied outside the secant, affects the amplitude and the overall shape vertically.
- Horizontal compression factor (4 inside): The argument of secant is multiplied by 4, impacting its period.
- Horizontal shift (+3 inside): Translates the graph left or right.

The goal is to identify key features such as the period, amplitude, vertical asymptotes, and key points, which are essential in sketching the graph.

Identifying the Period of the Function

The period of a secant function is derived from the period of its cosine base. Recall that:

- The basic cosine function, cos(x), has a period of 2π.
- For sec(x), the period remains 2π, but when the argument is scaled, the period changes accordingly.

Calculating the period:

Given the function H(x) = 4 sec(4(x + 3)), we focus on the inner argument:

- The inner function is 4(x + 3).
- The period of sec(kx) is given by $(2\pi) / |k|$.

Here, the coefficient k = 4 (inside the secant).

Therefore:

$$\text{Period} = \frac{2\pi}{|4|} = \frac{2\pi}{4} = \frac{\pi}{2}$$

Implication:

- The secant graph repeats every $\pi/2$ units along the x-axis.
- Since the argument is shifted by +3, the entire graph shifts horizontally without affecting the period.

Note: The horizontal shift by +3 affects the location of asymptotes but does not alter the period.

Determining the Stretching Factor and Amplitude

The coefficient outside the secant function, 4, affects the graph's vertical stretch and amplitude:

- Amplitude: Since secant is unbounded, it doesn't have a maximum or minimum value, but the "height" of the branches is scaled by 4.
- Vertical Stretch: The graph is stretched vertically by a factor of 4, making the branches extend further from the x-axis compared to the basic secant function.

Summary of vertical features:

- The basic secant (or cosine) ranges between -1 and 1.
- After multiplication by 4, the range becomes $-\infty$ to -4 and 4 to ∞, with asymptotes at points where cos(4(x + 3)) = 0.

Locating Asymptotes and Key Points

Since secant is undefined where cosine equals zero, the asymptotes occur at:

$$
\cos(4(x + 3)) = 0
$$

which occurs at:

$$
4(x + 3) = \frac{\pi}{2} + n\pi, \quad n \in \mathbb{Z}
$$

Solving for x:

$$
x + 3 = \frac{\pi/2 + n\pi}{4}
$$

$$
x = -3 + \frac{\pi/2 + n\pi}{4}
$$

For the first two periods:

- When $n=0$:

\[

$$
x = -3 + \frac{\pi/2}{4} = -3 + \frac{\pi}{8}
$$

- When $n=1$:

$$
x = -3 + \frac{\pi/2 + \pi}{4} = -3 + \frac{\pi/2 + \pi}{4} = -3 + \frac{3\pi/2}{4} = -3 + \frac{3\pi}{8}
$$

- When $n=-1$:

$$
x = -3 + \frac{\pi/2 - \pi}{4} = -3 - \frac{\pi/2}{4} = -3 - \frac{\pi}{8}
$$

Thus, the asymptotes for the first two periods are located at:

- $x = -3 - \frac{\pi}{8}$
- $x = -3 + \frac{\pi}{8}$
- $x = -3 + \frac{3\pi}{8}$
- $x = -3 - \frac{3\pi}{8}$

Between these asymptotes, the secant function's branches are continuous and resemble "U" or "n" shapes, opening upwards or downwards depending on the interval.

Key points:

- At the midpoints between asymptotes, the cosine function reaches ±1, and secant reaches ±4.
- For example, at the midpoint between asymptotes:

$$
x = -3 + \frac{(2n+1)\pi}{16}
$$

- At these points:

$$
\cos(4(x+3)) = \pm 1
$$

$$
H(x) = \pm 4
$$

Sketching Two Periods of the Graph

To sketch two periods, follow these systematic steps:

1. Mark Asymptotes:
Calculate and plot the asymptotes using the formulas above. For the first two periods, identify all asymptotes within the x-range that covers two periods, i.e., from:

$$
x = -3 - \frac{\pi}{8} \quad \text{to} \quad x = -3 + \frac{3\pi}{8}
$$

2. Plot Key Points:
Determine the points where secant reaches its maximum and minimum, i.e., where cosine equals ±1.

- At these points, $H(x) = \pm 4$.

3. Draw the Branches:
Between asymptotes, sketch the characteristic "U" or "n" shapes of secant, ensuring they approach the asymptotes asymptotically and reach the maximum or minimum at the midpoints.

4. Incorporate Horizontal Shift and Vertical Stretch:
Ensure the entire graph is shifted left by 3 units and vertically stretched by a factor of 4.

Features and Observations of the Graph

- Period: The graph repeats every $\pi/2$, resulting in two periods spanning an x-range of length π.
- Asymptotes: Occur at points derived from solving $\cos(4(x+3))=0$. They partition the graph into segments where the secant branches extend infinitely.
- Amplitude: The "height" of the secant branches is scaled by 4, leading to asymptotes at ±4.
- Vertical stretch: The graph appears more "stretched" vertically compared to the basic secant function.
- Horizontal compression: The period is compressed to $\pi/2$, making the graph more "compressed" along the x-axis.

Pros and Cons of the Graph Features

Pros:

- The periodicity allows for predictable repetition, making the graph easier to analyze over multiple cycles.
- The vertical stretch highlights the points where secant reaches its maximum and minimum.
- The asymptotes clearly indicate discontinuities, helping avoid misinterpretation of the graph's behavior near undefined points.
- The horizontal compression makes the graph more compact, useful in applications requiring tight periodic behavior.

Cons:

- The multiple asymptotes can make the graph appear cluttered or complex, especially for beginners.
- The unbounded nature of secant means the graph extends infinitely, which may be challenging to depict fully in limited space.
- The horizontal shift adds an extra layer of complexity in locating asymptotes and key points, requiring careful calculations.
- For practical applications, the infinite nature of the graph means only a finite portion can be realistically visualized.

Summary and Final Thoughts

Sketching two periods of the function H(x) = 4 sec(4(x + 3)) involves understanding its fundamental transformations, period, asymptotes, and key points. The period of \(\pi/2\) results from the

Sketch Two Periods Of The Graph Of The Function H X 4sec 4 X 3 Identify The Stretching Factor Period

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