Solve 8 Cos(2x) = 5 For The Smallest Positive Solution. Give Your Answer Accurate To At Least Two Decimal

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When tackling trigonometric equations such as 8 Cos(2x) = 5, it is crucial to understand the methods involved in solving for the variable x, especially when seeking the smallest positive solution. This type of problem combines algebraic manipulation with the properties of the cosine function, which is periodic and oscillates between -1 and 1. By carefully isolating the cosine term and applying inverse cosine functions, you can find the solutions with precision. In this comprehensive guide, we will explore step-by-step how to solve the equation, interpret the solutions, and verify the smallest positive value of x, accurate to at least two decimal places. This article aims to enhance your understanding of solving trig equations efficiently and correctly, beneficial for students, educators, and math enthusiasts alike.

Understanding the Equation and Its Components

Before diving into the solution process, it's helpful to analyze the structure and key features of the equation:

Equation: 8 Cos(2x) = 5

Key points:

- The equation involves a cosine function with a doubled angle, 2x.
- The coefficient 8 indicates the amplitude of the cosine term before being equated to a constant.
- The right side, 5, is a constant within the range of the cosine function multiplied by 8.

Important considerations:

- The cosine function, Cos(θ), has a range of [-1, 1].
- Since the equation involves 8 Cos(2x), the maximum value of 8 Cos(2x) is 8 (when Cos(2x) = 1), and the minimum is -8 (when Cos(2x) = -1).
- Therefore, the equation 8 Cos(2x) = 5 is solvable because 5 lies within the range [-8, 8].

Step-by-Step Solution Process

Step 1: Isolate the Cosine Term

Start by dividing both sides of the equation by 8 to isolate Cos(2x):

$$
\cos(2x) = \frac{5}{8}
$$

This simplifies to:

$$
\cos(2x) = 0.625
$$

Step 2: Find the General Solutions for 2x

Next, find the angles for which cosine equals 0.625. Since cosine is positive in the first and fourth quadrants, the solutions for 2x are:

$$
2x = \arccos(0.625) + 2\pi n \quad \text{or} \quad 2x = -\arccos(0.625) + 2\pi n
$$

where n is any integer representing the periodicity of the cosine function.

Calculating the principal value:

$$
\arccos(0.625) \approx 0.895 \text{ radians}
$$

Step 3: Write the General Solution for x

Divide both sides by 2 to solve for x:

$$
x = \frac{1}{2} \arccos(0.625) + \pi n
$$

and

$$
x = -\frac{1}{2} \arccos(0.625) + \pi n
$$

Using the principal value:

$$
x = \frac{0.895}{2} + \pi n \quad \Rightarrow \quad x \approx 0.4475 + \pi n
$$

and

$$
x = -\frac{0.895}{2} + \pi n \quad \Rightarrow \quad x \approx -0.4475 + \pi n
$$

Finding the Smallest Positive Solution

To identify the smallest positive x, examine the solutions for different integer values of n:

- For $n = 0$:

- $x \approx 0.4475$ (positive)
- $x \approx -0.4475$ (negative, discard)

- For $n = 1$:

- $x \approx 0.4475 + \pi \approx 0.4475 + 3.1416 \approx 3.5891$
- $x \approx -0.4475 + \pi \approx -0.4475 + 3.1416 \approx 2.6941$

- For $n = -1$:

- $x \approx 0.4475 - \pi \approx 0.4475 - 3.1416 \approx -2.6941$ (negative)
- $x \approx -0.4475 - \pi \approx -0.4475 - 3.1416 \approx -3.5891$ (negative)

From these calculations, the smallest positive solution occurs at $n=0$:

$$
x \approx 0.4475
$$

which is approximately 0.45 when rounded to two decimal places.

Final Answer and Verification

The smallest positive solution to the equation 8 Cos(2x) = 5, accurate to at least two decimal places, is approximately:

$$
\boxed{x \approx 0.45}
$$

Additional Tips for Solving Trigonometric Equations

Key points to remember:

- Always verify whether the value of the cosine is within its range [-1, 1] before proceeding.
- When solving for angles, consider the periodicity of the cosine function, which repeats every 2π.
- Use inverse cosine (arccos) carefully, noting that it returns an angle in the range $[0, \pi]$.
- Remember to account for all solutions within the interval of interest, especially when seeking the smallest positive solution.
- Round your final answer to the desired decimal places for accuracy.

Summary

This comprehensive guide has walked you through the process of solving the equation 8 Cos(2x) = 5, highlighting key steps such as isolating the cosine, calculating inverse cosine, and considering periodic solutions. The critical takeaway is that the smallest positive solution is approximately 0.45 radians, which provides a precise answer suitable for academic and practical applications. Understanding this method enhances your problem-solving skills in trigonometry and prepares you for more complex equations involving multiple angles and functions.

Frequently Asked Questions (FAQs)

Q1: Why is it necessary to consider multiple solutions?
Because cosine is a periodic function, solutions repeat every 2π. To find all solutions within a specific interval, you must account for this periodicity.

Q2: How do I convert radians to degrees if needed?

Multiply the radian measure by \(\frac{180}{\pi} \). For example, \(0.45 \text{ radians} \times \frac{180}{\pi} \approx 25.77^\circ \).

Q3: Can I use a calculator to find the solutions?
Yes, but ensure your calculator is in radian mode when working with radians. Always verify the mode before calculation.

Q4: What if the right side of the equation was outside the range [-8,8]?
The equation would have no solutions because the cosine function cannot produce values outside its amplitude range.

By understanding these steps and concepts, you are now equipped to solve similar trigonometric equations accurately and efficiently.

Frequently Asked Questions

What is the first step to solve the equation 8 cos(2x) = 5 for the smallest positive solution?

Divide both sides by 8 to isolate cos(2x): $\cos(2x) = 5/8$.

What is the value of cos(2x) after isolating it in the equation 8 cos(2x) = 5?

$\cos(2x) = 0.625$.

How do you find the general solutions for 2x when cos(2x) = 0.625?

Use the inverse cosine: $2x = \arccos(0.625) + 2\pi k$ or $2x = -\arccos(0.625) + 2\pi k$, where k is any integer.

What is the value of arccos(0.625) approximately?

$\arccos(0.625) \approx 0.895$ radians.

How do you find the smallest positive value of x from 2x = 0.895 radians?

Divide both sides by 2: $x \approx 0.895 / 2 \approx 0.4475$ radians.

Convert the solution $x \approx 0.4475$ radians into degrees and provide the approximate value.

$x \approx 0.4475 \times (180/\pi) \approx 25.65°$.

What is the approximate smallest positive solution for x in radians, accurate to two decimal places?

$x \approx 0.45$ radians.

What is the final answer for the smallest positive solution to 8 cos(2x) = 5, accurate to two decimal places?

$x \approx 0.45$ radians.

Additional Resources

Mathematical Problem-Solving: Solving 8 Cos(2x) = 5 for the Smallest Positive Solution

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Introduction

Mathematics often presents intriguing problems that challenge our understanding of functions, equations, and their solutions. One such problem involves solving a trigonometric equation: 8 Cos(2x) = 5, with the goal of finding the smallest positive solution accurate to at least two decimal places. This type of problem combines fundamental trigonometric identities and algebraic manipulation, offering an excellent opportunity to deepen one's understanding of cosine functions, their properties, and the methods used to solve such equations.

In this comprehensive review, we will explore every detail involved in solving the equation, including the underlying concepts, step-by-step solution procedures, and the importance of precision in mathematical answers. Whether you're a student preparing for exams or an enthusiast wanting to sharpen your problem-solving skills, this discussion aims to clarify each aspect of the problem thoroughly.

Understanding the Problem Statement

The problem is:

$$ 8 \cos(2x) = 5 $$

Our task is to find the smallest positive value of x (in radians or degrees, based on the context—here, we will work in radians unless otherwise specified) that satisfies this equation, with accuracy to at least two decimal places.

Step 1: Isolate the Trigonometric Function

The initial step in solving the equation involves isolating $\cos(2x)$:

$$
8 \cos(2x) = 5
$$

Divide both sides by 8:

$$
\cos(2x) = \frac{5}{8}
$$

The simplified form is:

$$
\cos(2x) = 0.625
$$

This simplification is crucial because it sets the stage for applying inverse cosine functions and understanding the periodic nature of cosine.

Step 2: Understanding the Cosine Function and Its Range

Before proceeding to find solutions, it’s important to recall some key properties of the cosine function:

- Range: $\cos(\theta)$ always lies between -1 and 1, inclusive.

- Since 0.625 is within $[-1, 1]$, the equation has real solutions.

- Period: Cosine has a fundamental period of 2π, meaning:

$$
\cos(\theta) = \cos(\theta + 2\pi n), \quad n \in \mathbb{Z}
$$

- Symmetry: $\cos(\theta) = \cos(-\theta)$ and $\cos(\theta) = \cos(2\pi - \theta)$.

This periodicity and symmetry imply multiple solutions for $2x$, and subsequently for x.

Step 3: Find the General Solutions for $2x$

To find x, first solve for $2x$:

\[
2x = \arccos(0.625) + 2\pi n \quad \text{or} \quad 2x = -\arccos(0.625) + 2\pi n
\]

where \(n\) is any integer, representing the periodic nature of cosine.

Calculating \(\arccos(0.625)\) is the next step.

Step 4: Calculating \(\arccos(0.625)\)

Using a calculator or computational tool:

\[
\arccos(0.625) \approx 0.8956 \text{ radians}
\]

This value is approximate and will be used to derive the solutions.

Step 5: Expressing the Solutions for \(2x\)

Based on the properties of cosine, the solutions for \(2x\) are:

\[
2x = 0.8956 + 2\pi n
\]
and
\[
2x = -0.8956 + 2\pi n
\]

for all integers \(n\).

Step 6: Solving for \(x\)

Divide both sides of each equation by 2 to find \(x\):

\[
x = \frac{0.8956 + 2\pi n}{2}
\]
and
\[
x = \frac{-0.8956 + 2\pi n}{2}
\]

Expressed more cleanly:

$$
x = 0.4478 + \pi n
$$
and
$$
x = -0.4478 + \pi n
$$

This set of solutions encompasses all possible x values satisfying the original equation.

Step 7: Finding the Smallest Positive Solution

The goal is to identify the smallest positive solution $x > 0$.

Let’s analyze each set:

Set 1: $x = 0.4478 + \pi n$

- For $n=0$:

$$
x = 0.4478
$$

which is positive.

- For $n=-1$:

$$
x = 0.4478 - \pi \approx 0.4478 - 3.1416 = -2.6938
$$

which is negative.

- For $n=1$:

$$
x = 0.4478 + 3.1416 = 3.5894
$$

larger than the previous positive solution.

The smallest positive solution from this set is $x \approx 0.4478$ radians.

Set 2: $x = -0.4478 + \pi n$

- For $n=1$:

$$
x = -0.4478 + 3.1416 = 2.6938
$$

positive, but larger than 0.4478.

- For $n=0$:

$$
x = -0.4478
$$

negative, so discard.

- For $n=-1$:

$$
x = -0.4478 - 3.1416 = -3.5894
$$

negative, discard.

Thus, the smallest positive solution from this set is approximately $x \approx 2.6938$ radians, which is larger than 0.4478.

Conclusion:

The smallest positive solution to the original equation 8 Cos(2x) = 5 is approximately:

$$
x \approx 0.4478 \text{ radians}
$$

Final answer (accurate to at least two decimal places):

$$
\boxed{
x \approx \mathbf{0.45}
}
$$

Additional Considerations and Context

Converting to Degrees

If required to express the answer in degrees:

$$
x \approx 0.4478 \times \frac{180}{\pi} \approx 25.66^\circ
$$

\]

which, rounded to two decimal places, is:

\[
x \approx \boxed{25.66^\circ}
\]

Verifying the Solution

To verify, substitute \(x \approx 0.4478\) into the original equation:

\[
8 \cos(2 \times 0.4478) \approx 8 \cos(0.8956) \approx 8 \times 0.625 = 5
\]

which confirms the solution is valid.

Significance and Practical Applications

Understanding how to solve equations like \(8 \cos(2x) = 5\) is fundamental in many fields, including physics, engineering, and computer science. For instance:

- In wave mechanics, phase shifts often involve solving similar trigonometric equations.
- In signal processing, understanding the periodicity of functions is crucial for Fourier analysis.
- In engineering design, ensuring the smallest positive solution aligns with physical constraints or operational limits.

The ability to manipulate and interpret these solutions accurately impacts the design, analysis, and problem-solving in various technical disciplines.

Summary of Key Takeaways

- Isolate the trigonometric function before applying inverse functions.
- Remember cosine's periodicity and symmetry when deriving general solutions.
- Always consider the domain and range of inverse cosine.
- Find the smallest positive solution by evaluating solutions at different integer values of \(n\).
- Ensure your answer is accurate to the required decimal places, verifying by substitution.

Final Remarks

This detailed exploration underscores the importance of systematic problem-solving in trigonometry. The key steps—simplifying, understanding periodicity, calculating inverse functions, and verifying solutions—are critical skills. Mastery of these techniques ensures confidence when tackling a broad array of mathematical problems, from straightforward exercises to complex real-world applications.

Whether in academic assessments or practical scenarios, being precise and thorough, as demonstrated here, is essential for producing accurate and reliable solutions.

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