Suppose A Firm's Total Cost Is Given By Tc = 100 4q 2q2. Which Of The Following Statements Is (are) True?

Suppose A Firm's Total Cost Is Given By Tc = 100 + 4q + $2q^2$. Which Of The Following Statements Is (are) True?

Understanding a firm's cost structure is fundamental in economics as it influences decision-making, profit maximization, and market strategies. When given a total cost (TC) function such as $Tc = 100 + 4q + 2q^2$, analyzing its components and derivatives helps in understanding the behavior of costs in relation to output quantity (q). This article explores the implications of this specific total cost function, examines key concepts like marginal cost, average cost, and their relationships, and clarifies which statements about this cost function are true.

Breaking Down the Total Cost Function

The total cost function provided is:

$$
Tc = 100 + 4q + 2q^2
$$

where:
- 100 is the fixed cost, representing costs that do not vary with output.
- $4q$ is the variable cost component, linear in output.
- $2q^2$ is the variable cost component, quadratic in output, indicating increasing marginal costs.

Understanding each component is crucial:

Fixed Costs

- The fixed cost, 100, remains constant regardless of the output level.
- These costs are incurred even if the firm produces nothing (i.e., when $q=0$).

Variable Costs

- The terms $4q$ and $2q^2$ vary with output.
- The linear term, $4q$, suggests a base variable cost per unit.
- The quadratic term, $2q^2$, indicates that as output increases, the additional cost per unit increases at an increasing rate, reflecting increasing marginal costs.

Key Concepts Derived from the Cost Function

Analyzing the cost function involves calculating and interpreting several critical measures:

Marginal Cost (MC)

- Defined as the additional cost of producing one more unit of output.
- Mathematically, it's the first derivative of the total cost function with respect to q:

$$
MC = \frac{d(Tc)}{dq} = 4 + 4q
$$

- Interpretation:
- At $q=0$, $MC = 4$.
- As q increases, MC increases linearly due to the $4q$ term.

Average Total Cost (ATC)

- Represents the cost per unit of output.
- Calculated as:

$$
ATC = \frac{Tc}{q} = \frac{100 + 4q + 2q^2}{q} = \frac{100}{q} + 4 + 2q
$$

- Behavior:
- As q increases, $\frac{100}{q}$ decreases.
- The term $2q$ increases with q.
- The ATC curve typically has a U-shape due to the fixed cost component diminishing per unit and rising variable costs.

Average Variable Cost (AVC)

- Variable cost per unit:

$$
AVC = \frac{VC}{q} = \frac{4q + 2q^2}{q} = 4 + 2q
$$

- As q increases, AVC increases linearly.

Average Fixed Cost (AFC)

- Fixed cost per unit:

$$
AFC = \frac{Fixed\ Cost}{q} = \frac{100}{q}
$$

- Decreases continuously as output increases.

Analyzing the Statements: Which Are True?

Given the cost function, various statements can be evaluated for correctness. Let's consider commonly posed statements about such a cost structure.

Statement 1: Marginal Cost Is Increasing with Output

- Evaluation:

$$
MC = 4 + 4q
$$

- Since the derivative of MC with respect to q is:

$$
\frac{d(MC)}{dq} = 4
$$

- Which is positive and constant.

- Conclusion:
- The marginal cost increases linearly with q because of the $4q$ term. Therefore, this statement is true.

Statement 2: Average Total Cost (ATC) Is U-Shaped

- Evaluation:

$$
ATC = \frac{100}{q} + 4 + 2q
$$

- As $q \to 0$, $ATC \to \infty$.
- As $q \to \infty$, $ATC \to \infty$ because of the $2q$ term.

- The presence of a $\frac{100}{q}$ term decreasing with q, and the $2q$ term increasing with q, suggests a U-shaped curve, typical in cost functions.

- Conclusion:
- The ATC curve initially decreases, reaches a minimum, then increases, confirming the U-shape.
- This statement is true.

Statement 3: The Firm's Fixed Cost Is $100

- Evaluation:
- Fixed cost is the component of total cost unaffected by output.
- Given $Tc = 100 + 4q + 2q^2$, fixed cost is explicitly 100.

- Conclusion:
- This statement is true.

Statement 4: The Average Variable Cost (AVC) Is Increasing Linearly with Output

- Evaluation:

$$
AVC = 4 + 2q
$$

- It is a linear function of q with a positive slope.

- Conclusion:
- This statement is true.

Statement 5: The Marginal Cost (MC) Is Less Than Average Total Cost (ATC) When Output Is Small

- Evaluation:
- When q is small, $MC = 4 + 4q$ and $ATC = \frac{100}{q} + 4 + 2q$.

- At very small q, $\frac{100}{q}$ dominates, making ATC very large.
- Since MC starts at 4 when $q=0$, and ATC is very large at small q, $MC < ATC$

when q is small.

- Conclusion:
- This statement is true.

Implications for Business Decisions

Understanding the cost functions and their derivatives provides insights into optimal production levels, pricing, and profitability.

Optimal Output Level

- The point where marginal cost equals marginal revenue (or price) determines the profit-maximizing output.
- Since $MC = 4 + 4q$, as output increases, the marginal cost rises linearly.
- If the firm faces a market price P, it will produce where $P = MC$, provided $P \geq AVC$.

Cost Minimization

- The minimum of the ATC curve indicates the most cost-efficient level of production.
- To find this point, set the derivative of ATC with respect to q to zero.

$$
ATC = \frac{100}{q} + 4 + 2q
$$

$$
\frac{d(ATC)}{dq} = -\frac{100}{q^2} + 2
$$

- Setting to zero:

$$
-\frac{100}{q^2} + 2 = 0 \Rightarrow 2 = \frac{100}{q^2} \Rightarrow q^2 = \frac{100}{2} = 50
$$

$$
q = \sqrt{50} \approx 7.07
$$

- At $q \approx 7.07$, ATC is minimized, and the firm operates at most efficiency.

Conclusion: Summarizing the True Statements

Based on the analysis above, the following statements about the cost function \(Tc = 100 + 4q + 2q^2 \) are true:

- The marginal cost increases linearly with output.
- The average total cost (ATC) curve is U-shaped.
- The fixed cost of the firm is $100.
- The average variable cost (AVC) increases linearly with output.
- When output is small, the marginal cost is less than the average total cost.

These insights help firms in planning production, setting prices, and understanding cost behavior, which are essential for competitive strategy and maximizing profits.

Additional Considerations

- The quadratic nature of the cost function indicates increasing marginal costs, which can reflect limitations in production scaling, such as resource

Frequently Asked Questions

What is the total cost function of the firm?

The total cost function is given by $Tc = 100 + 4q + 2q^2$.

How can we find the marginal cost (MC) from the total cost function?

Marginal cost is the derivative of total cost with respect to quantity q, so $MC = d(Tc)/dq = 4 + 4q$.

At what level of output q does the firm reach its minimum average cost?

Minimum average cost occurs where the derivative of average cost with respect to q equals zero; it can be found by analyzing the function $AC = Tc/q$.

What is the average total cost (ATC) function for this firm?

Average total cost is $ATC = Tc / q = (100 + 4q + 2q^2) / q = 100/q + 4 + 2q$.

Is the firm's total cost function increasing at all levels of output?

No, the total cost function is increasing for all $q \geq 0$ because both the fixed cost and variable costs increase with q.

What is the fixed cost component of the total cost?

The fixed cost component is 100, which is the cost incurred even when $q = 0$.

How does the variable cost change with output q?

Variable cost is the part of total cost that varies with q, which is $4q + 2q^2$; it increases as q increases.

Can the firm achieve economies of scale based on this cost function?

Yes, since the average total cost decreases as q increases initially (due to fixed cost spreading), indicating economies of scale at certain output levels.

What is the condition for the profit-maximizing output level for this firm?

The profit-maximizing output occurs where marginal cost equals marginal revenue; without revenue data, we can only analyze cost, but typically set MC = MR to find q.

Which statement is true: Fixed costs are constant, variable costs increase with output, or both?

Both statements are true: fixed costs are constant at 100, and variable costs ($4q + 2q^2$) increase with output.

Additional Resources

Suppose A Firm's Total Cost Is Given By $Tc = 100 + 4q + 2q^2$. Which Of The Following Statements Is (are) True?

In the world of microeconomics and managerial decision-making, understanding a firm's cost structure is fundamental. The total cost (TC) function encapsulates how a firm's expenses vary with output levels, providing insights into profitability, efficiency, and strategic planning. Here, we examine a specific total cost function: $Tc = 100 + 4q + 2q^2$, where "q" represents the quantity of output produced. This quadratic form offers a rich ground to analyze the cost behavior, including fixed costs, variable costs, marginal costs, and their implications for the firm's operations.

This article aims to dissect the given cost function, interpret its components, and evaluate the validity of common statements related to such a cost structure. By doing so, readers will gain a clearer understanding of how to analyze and interpret cost functions in practical scenarios, equipping them with the tools necessary to make informed business decisions.

Understanding the Cost Function: Breakdown and Interpretation

The Structure of the Cost Function: Fixed and Variable Components

The total cost function provided is:

$Tc = 100 + 4q + 2q^2$

Breaking down this equation:

- Fixed Costs (FC): The constant term, 100, represents fixed costs. These are expenses that do not vary with the level of output. Examples include rent, salaries of permanent staff, and equipment depreciation. Fixed costs are incurred regardless of whether the firm produces nothing or a large quantity.

- Variable Costs (VC): The terms involving "q" (specifically, $4q + 2q^2$) are variable costs, which change with output level. The linear component, $4q$, suggests a constant marginal cost per unit, while the quadratic term, $2q^2$, indicates increasing marginal costs as output expands.

This structure implies that as the firm increases production, its total costs grow, initially at a steady rate (due to the 4q term) but eventually accelerating because of the quadratic component.

Implications of the Quadratic Cost Term

The presence of a quadratic term ($2q^2$) signifies that the firm's marginal cost (MC) is not constant. To analyze this, we derive the marginal cost from the total cost function:

$MC = d(Tc)/dq$ = derivative of Tc with respect to q

Calculating:

$MC = d/dq\ [100 + 4q + 2q^2] = 0 + 4 + 4q = 4 + 4q$

This linear function indicates that marginal costs increase as output increases—an essential insight for decision-making. When MC rises with q, the firm faces diminishing returns in the short run, which influences production and pricing strategies.

Analyzing Key Cost Concepts Based on the Given Function

Fixed Costs and Variable Costs

- Fixed Costs (FC): Constant at 100, regardless of output. They set a baseline for the firm's cost structure and are critical in break-even analysis.

- Variable Costs (VC): The sum of the linear and quadratic parts: $4q + 2q^2$. As output increases, variable costs become the dominant component, especially at higher levels.

Marginal Cost (MC): The Cost of Producing an Additional Unit

- Expression: $MC = 4 + 4q$

- Behavior: Since MC increases linearly with q, producing additional units becomes progressively more expensive. This has direct implications for profit maximization, optimal output levels, and pricing strategies.

Average Costs: How Cost Per Unit Changes with Output

- Average Total Cost (ATC): $ATC = Tc / q = (100 + 4q + 2q^2) / q$

Calculating:

$ATC = 100/q + 4 + 2q$

- Average Variable Cost (AVC): $AVC = VC / q = (4q + 2q^2) / q = 4 + 2q$

- Insights:

- As q increases, the fixed cost per unit ($100/q$) decreases, tending towards zero at high output levels.

- The AVC increases linearly with q due to the $2q$ component.

- The ATC initially decreases as fixed costs are spread over more units but eventually increases because of rising variable costs.

Evaluating Statements About the Cost Function

Suppose the question presents a set of statements regarding the cost function, such as:

1. The fixed cost is $100.
2. The marginal cost increases as output increases.
3. The average total cost decreases initially and then increases.
4. The total cost becomes purely variable as output becomes very large.

Let's analyze each statement based on the detailed understanding of the cost function.

Statement 1: The fixed cost is $100.

Analysis: True. The fixed component of the total cost function is the constant term, which is 100. This cost remains unchanged regardless of the level of production.

Statement 2: The marginal cost increases as output increases.

Analysis: True. The marginal cost (MC) = 4 + 4q, which is a linear function increasing in q. As q grows, MC rises, indicating increasing marginal costs due to the quadratic component in total costs.

Statement 3: The average total cost decreases initially and then increases.

Analysis: True. The ATC = 100/q + 4 + 2q.

- For small q, the fixed cost per unit, 100/q, dominates, making ATC high initially but decreasing as q increases.

- As q grows large, the fixed cost per unit diminishes, but the variable cost component, 2q, increases, causing ATC to eventually rise again.

- The ATC curve is U-shaped, a common pattern in cost analysis, with a minimum point at some output level.

Statement 4: The total cost becomes purely variable as output becomes very large.

Analysis: False. While fixed costs become negligible per unit at very high q, they do not disappear entirely. The total cost function always includes the fixed component of 100; it merely becomes less significant relative to total costs as q increases.

Practical Implications for Business Decision-Making

Understanding the cost structure helps firms determine optimal production levels, pricing strategies, and long-term planning.

Key Takeaways:

- Break-even Analysis: Knowing fixed and variable costs allows firms to calculate the minimum output needed to cover costs.

- Profit Maximization: Firms should produce where marginal cost equals marginal revenue, considering that MC increases with output, which can influence the profit-maximizing quantity.

- Scaling Production: The increasing MC suggests that scaling beyond certain points could lead to higher per-unit costs, potentially reducing profitability if prices are not adjusted.

- Cost Management: Recognizing the components of total costs guides managerial decisions to control fixed costs or find efficiencies in variable costs.

Graphical Representation:

Plotting the cost functions provides visual insights:

- The total cost curve starts at 100 on the cost axis when $q=0$.

- The marginal cost line rises linearly with q.

- The ATC curve is U-shaped, with its minimum point indicating the most efficient scale of production.

Impacts of Cost Behavior:

- Firms may prefer to operate at output levels where ATC is minimized for optimal efficiency.

- Increasing marginal costs may limit the benefits of expanding production unless market conditions justify higher costs.

Conclusion: Synthesizing the Analysis

The cost function $Tc = 100 + 4q + 2q^2$ offers a comprehensive view of how fixed and variable costs interact as output varies. It reveals that fixed costs are constant at 100, the marginal cost starts at 4 and increases linearly with output, and the average total cost exhibits a U-shaped curve—initially decreasing due to spreading fixed costs and later increasing because of rising variable costs.

Understanding these dynamics enables firms to make informed decisions, optimize production levels, and develop strategies aligned with their cost structures. The key takeaway is that the fixed costs are always present, but their relative significance diminishes at higher output levels, while variable costs—and their associated marginal costs—become dominant factors influencing operational efficiency and profitability.

In essence, the detailed analysis of this quadratic cost function underscores the importance of a nuanced understanding of cost components in microeconomic decision-making, illustrating how mathematical modeling directly informs practical business strategies.

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