The Altitude Of A Right Circular Cylinder Is Twice The Radius Of The Base. Find The Height. If The Volume

The Altitude Of A Right Circular Cylinder Is Twice The Radius Of The Base. Find The Height. If The Volume

Understanding the properties and dimensions of geometric shapes is fundamental in mathematics, especially in the study of three-dimensional figures like cylinders. In this article, we explore a specific problem involving a right circular cylinder where the altitude (height) is twice the radius of its base. Such problems are common in geometry and help reinforce understanding of volume, surface area, and the relationships between different dimensions of cylinders. We will analyze the problem step by step, derive relevant formulas, and demonstrate how to find the height given the volume of the cylinder.

Understanding the Problem Statement

Restating the Given Data

The problem states:

- The altitude (height) of a right circular cylinder is twice the radius of its base.
- The volume of the cylinder is given (though the specific volume value is not provided in the initial statement — we will assume it is given or work through the general solution).

From this, we can extract the key variables:

- Let r be the radius of the base of the cylinder.
- Since the altitude is twice the radius, h = 2r.
- The volume V is expressed in terms of r (and h).

Our goal is to find the height h when the volume is known, or to express h in terms of volume and radius.

Formulating the Volume of a Cylinder

Basic Volume Formula

The volume V of a right circular cylinder is given by:

$$ V = \pi r^2 h $$

where:

- r = radius of the base,
- h = height of the cylinder,
- $\pi \approx 3.1416$.

Given the problem's condition that h = 2r, we can substitute this into the volume formula:

$$ V = \pi r^2 (2r) = 2 \pi r^3 $$

This simplifies the problem to relating volume directly to the radius:

$$ V = 2 \pi r^3 $$

Expressing the Height in Terms of Volume and Radius

Solving for the Radius

Suppose the volume V is known; then:

$$ V = 2 \pi r^3 $$

Rearranged to solve for r:

$$ r^3 = \frac{V}{2 \pi} $$

$$
r = \sqrt[3]{\frac{V}{2 \pi}}
$$

Once the radius is found, the height h can be computed using:

$$ h = 2r $$

Expressing the Height in Terms of Volume

Substituting for r:

$$ h = 2 \times \sqrt[3]{\frac{V}{2 \pi}} $$

This formula allows us to find the height directly once the volume V is known.

Worked Example: Calculating Height for a Given Volume

Let's consider an example where the volume of the cylinder is V = 1000 cubic units.

Step 1: Find the Radius r

$$
r = \sqrt[3]{\frac{1000}{2 \pi}} = \sqrt[3]{\frac{1000}{2 \times 3.1416}} = \sqrt[3]{\frac{1000}{6.2832}}
$$

Calculating:

$$
\frac{1000}{6.2832} \approx 159.15
$$

Then:

$$
r \approx \sqrt[3]{159.15} \approx 5.36
$$

Step 2: Find the Height h

$$
h = 2r \approx 2 \times 5.36 \approx 10.72
$$

Answer: The height of the cylinder is approximately 10.72 units when the volume is 1000 cubic units.

General Approach and Key Points

Summary of Steps to Solve Such Problems

1. Identify the given data: volume V, relationship between height and radius $h = 2r$.
2. Write the volume formula: $V = \pi r^2 h$.
3. Substitute $h = 2r$ into the volume formula to get $V = 2 \pi r^3$.
4. Solve for r: $r = \sqrt[3]{\frac{V}{2 \pi}}$.
5. Calculate r and then find $h = 2r$.

Important Considerations

- The volume must be positive; hence, $V > 0$.
- The radius r derived must be real and positive for the dimensions to make sense physically.
- If the volume is specified, the problem reduces to computing the cube root, which may involve approximation.

Additional Notes on the Geometry of the Cylinder

Surface Area of the Cylinder

While the problem focuses on volume and height, understanding surface area is also useful:

$$\text{Surface Area} = 2 \pi r^2 + 2 \pi r h$$

Given $h = 2r$:

$$\text{Surface Area} = 2 \pi r^2 + 2 \pi r (2r) = 2 \pi r^2 + 4 \pi r^2 = 6 \pi r^2$$

This reveals that for this specific relationship between height and radius, the surface area depends solely on the radius.

Implications of the Relationship $h = 2r$

- The height is directly proportional to the radius.
- Increasing the radius r scales both the volume and surface area accordingly.
- The problem exemplifies how constraints between dimensions simplify calculations.

Conclusion

In summary, when the altitude of a right circular cylinder is twice the radius of its base, the problem of finding the height given the volume becomes straightforward through algebraic substitution. The key steps involve expressing the volume formula in terms of a single variable, solving for the radius, and then deriving the height. The general formula for the height in terms of volume is:

$$h = 2 \times \sqrt[3]{\frac{V}{2 \pi}}$$

This approach not only provides a clear pathway to solving such problems but also deepens understanding of the relationships between the dimensions of cylinders. Whether in academic exercises or real-world applications like engineering and design, mastering these relationships is essential for accurate measurements and calculations.

Frequently Asked Questions

What is the relationship between the altitude and the radius in a right circular cylinder as given?

The altitude of the cylinder is twice the radius of the base.

How do you express the volume of a right circular cylinder in terms of its radius and height?

The volume $V = \pi r^2 h$, where r is the radius and h is the height of the cylinder.

Given the altitude is twice the radius, how can we express the height in terms of the radius?

The height $h = 2r$, since the altitude is twice the radius.

If the volume of the cylinder is known, how can we find the radius?

By substituting $h = 2r$ into the volume formula $V = \pi r^2 h$, then solving for r: $V = \pi r^2(2r) = 2\pi r^3$, so $r = (V / 2\pi)^{1/3}$.

What is the formula for the height of the cylinder when the volume is given?

First express the volume in terms of r: $V = 2\pi r^3$. Then, solve for r: $r = (V / 2\pi)^{1/3}$. The height $h = 2r$.

How do you find the height of the cylinder if the volume is known and the relationship between altitude and radius is given?

Calculate r from the volume using $r = (V / 2\pi)^{1/3}$, then find $h = 2r$.

Can you derive the height of the cylinder directly from the volume and the given relationship?

Yes, by substituting $h = 2r$ into $V = \pi r^2 h$, leading to $V = 2\pi r^3$, then solving for r and subsequently finding h.

What steps should be followed to find the height of a right circular cylinder when the volume and the relationship between altitude and radius are known?

1. Express $h = 2r$. 2. Substitute into volume formula: $V = \pi r^2(2r) = 2\pi r^3$. 3. Solve for r: $r = (V / 2\pi)^{1/3}$. 4. Find $h = 2r$.

Additional Resources

Cylinder Geometry Analysis: Determining the Height Based on Given Dimensions

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Introduction to the Problem

In the realm of solid geometry, understanding the relationships between the various dimensions of a three-dimensional shape is crucial for both theoretical mathematics and practical applications, such as engineering, manufacturing, and design. The problem at hand involves a right circular cylinder—a shape characterized by a circular base and a height perpendicular to that base.

The specific scenario is as follows:
- The altitude (height) of the cylinder is twice the radius of its base.
- The goal is to find the height given this relationship.
- Additionally, we are interested in the volume of the cylinder once the height is determined.

This analysis will not only derive the formula for the height but also discuss the implications, calculations, and potential applications of such a geometric configuration.

Understanding the Geometry of a Right Circular Cylinder

Basic Parameters of a Cylinder

A right circular cylinder is defined by:
- Radius (r): The radius of the circular base.
- Height (h): The perpendicular distance between the bases.
- Volume (V): The space enclosed by the cylinder.

The volume of a right circular cylinder is given by the formula:

$$
V = \pi r^2 h
$$

where:
- $\pi \approx 3.14159$,
- r is the radius,
- h is the height.

Relationship Between Height and Radius

Given in the problem:
"The altitude of the cylinder is twice the radius of the base."

Mathematically, this translates to:

$$
h = 2r
$$

This relationship simplifies the problem significantly, as it allows us to express the volume entirely in terms of a single variable, r.

Deriving the Height of the Cylinder

Step 1: Express Height in Terms of Radius

From the given relationship:

$$
h = 2r
$$

This means if we determine the radius, the height inherently follows.

Step 2: Express Volume in Terms of Radius

Substituting $h = 2r$ into the volume formula:

$$
V = \pi r^2 (2r) = 2\pi r^3
$$

Thus, the volume is directly proportional to the cube of the radius.

Step 3: Find the Height from Known or Assumed Volume

Suppose we are given a specific volume V, we can rearrange the formula to find r:

$$
V = 2\pi r^3
\Rightarrow r^3 = \frac{V}{2\pi}
\Rightarrow r = \left(\frac{V}{2\pi} \right)^{1/3}
$$

Once r is known, the height h can be computed as:

$$
h = 2r = 2 \times \left(\frac{V}{2\pi} \right)^{1/3}
$$

In the absence of a specific volume, the problem remains in this general form.

Implications and Applications

Design and Manufacturing Considerations

Understanding the relationship between height and radius is vital when designing cylindrical components such as pipes, tanks, or mechanical parts. For instance:

- If a tank must hold a specific volume and the height is constrained by spatial limitations, knowing how to adjust the radius accordingly is essential.
- Conversely, if the radius is fixed due to material constraints, the height must be adjusted, and this relationship guides those adjustments.

Mathematical and Educational Significance

From an educational standpoint, this problem exemplifies the importance of algebraic substitution and understanding proportional relationships in geometry. It reinforces how dimensional constraints affect volume calculations and how to manipulate formulas to derive unknowns.

Example Calculation: Applying a Specific Volume

Suppose a practical example:
- The volume of the cylinder is 1000 cubic units.

Using the formula:

$$
r = \left(\frac{V}{2\pi} \right)^{1/3}
$$

Plugging in the value:

$$
r = \left(\frac{1000}{2 \times 3.14159} \right)^{1/3} \approx \left(\frac{1000}{6.28318} \right)^{1/3} \approx \left(159.15 \right)^{1/3}
$$

Calculating the cube root:

$$
r \approx 5.37 \text{ units}
$$

Then, the height:

$$
h = 2r \approx 2 \times 5.37 \approx 10.74 \text{ units}
$$

Thus, for a cylinder with a volume of 1000 cubic units, the radius is approximately 5.37 units, and the height is approximately 10.74 units.

Conclusion and Summary

In summary, the relationship between the altitude and the radius of a right circular cylinder significantly simplifies the process of determining its dimensions. When the height is twice the radius, the key points are:

- The height h is directly proportional to the radius r, with $h = 2r$.
- The volume can be expressed as $V = 2\pi r^3$, highlighting the cubic relationship between radius and volume.
- Given a specific volume, the radius and consequently the height can be computed using algebraic manipulation.
- This relationship has practical applications in engineering and design, emphasizing the importance of understanding proportional geometric relationships.

By mastering these principles, professionals and students alike can confidently analyze cylindrical shapes with specified dimensional constraints, enabling precise calculations and optimized designs across various fields.

Note: For specific applications, always ensure to use accurate values for π and consider unit consistency throughout calculations.

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