The Area Of A Rectangle Is 42ft^2, And The Length Of The Rectangle Is 11 Less Than Three Times The Width.

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Understanding the dimensions of a rectangle is fundamental in geometry, especially when working with real-world problems involving area, length, and width. In this article, we will explore a specific problem: given that the area of a rectangle is 42 square feet, and that its length is 11 feet less than three times its width, how can we determine its dimensions? We will break down the problem, set up the appropriate equations, and solve step-by-step to find the rectangle's length and width. This approach not only helps in solving this particular problem but also illustrates how to approach similar geometric problems involving algebra.

Understanding the Problem: Area and Dimensions of a Rectangle

Before diving into calculations, it's important to understand the key components of the problem:

- The area of the rectangle is 42 square feet (ft^2).
- The length of the rectangle is described in terms of its width: it is 11 feet less than three times the width.

Let's define our variables:
- **w** = width of the rectangle (in feet)
- **l** = length of the rectangle (in feet)

Given these definitions, our goal is to find the numerical values of **w** and **l** that satisfy the conditions.

Setting Up the Equations

To solve for the dimensions, we need to translate the problem's description into algebraic equations.

1. Expressing Length in Terms of Width

Based on the problem statement:

- The length (l) is 11 less than three times the width (w).

This translates to:

$l = 3w - 11$

2. Using the Area Formula for a Rectangle

The area (A) of a rectangle is given by:

$A = length \times width$

Substituting the known area and the expression for length:

$42 = (3w - 11) \times w$

This is the key equation we will solve to find the width.

Solving for the Width

Let's proceed to solve the quadratic equation derived from the area formula.

Step 1: Write the equation

$42 = (3w - 11)w$

Expanding the right side:

$42 = 3w^2 - 11w$

Step 2: Rearrange into standard quadratic form

Bring all terms to one side:

$3w^2 - 11w - 42 = 0$

Step 3: Solve the quadratic equation

Using the quadratic formula:

$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Where:
- a = 3
- b = -11
- c = -42

Calculate the discriminant:

$\Delta = b^2 - 4ac = (-11)^2 - 4 \times 3 \times (-42) = 121 + 504 = 625$

Since the discriminant is positive, there are two real solutions:

$w = \frac{11 \pm \sqrt{625}}{2 \times 3} = \frac{11 \pm 25}{6}$

Calculate both solutions:

1. $w = \frac{11 + 25}{6} = \frac{36}{6} = 6$
2. $w = \frac{11 - 25}{6} = \frac{-14}{6} = -\frac{7}{3}$

Since width cannot be negative in this context, we discard the negative solution.

Therefore, the width is w = 6 feet.

Finding the Length of the Rectangle

Using the expression for length:

$l = 3w - 11$

Substitute w = 6:

$l = 3 \times 6 - 11 = 18 - 11 = 7$

Hence, the length of the rectangle is 7 feet.

Verifying the Solution

Always verify your solutions to ensure they satisfy all conditions.

- Check the area:

$\text{Area} = \text{length} \times \text{width} = 7 \text{ ft} \times 6 \text{ ft} = 42 \text{ ft}^2$

- Confirm the length condition:

Is length 11 less than three times the width?

$3 \times 6 = 18$

$18 - 11 = 7$

Yes, the length is 7 feet, matching our calculation.

The solution is consistent and correct.

Practical Applications of the Problem

Understanding how to find the dimensions of a rectangle with given area and relationships between length and width has many real-world applications:

- Designing rectangular plots of land with specific area requirements.
- Creating rectangular containers or boxes where dimensions are constrained by volume or surface area considerations.
- Planning room layouts where certain proportional relationships between length and width are desired.

Knowing how to set up and solve such problems empowers students, professionals, and hobbyists to approach various practical tasks confidently.

Additional Tips for Solving Similar Problems

When faced with similar geometric problems, keep these tips in mind:

- Clearly define your variables and translate word problems into algebraic equations.
- Identify the key relationships described, such as “less than,” “more than,” or proportional relationships.
- Always verify your solutions by substituting back into original conditions.
- Be mindful of the context—discard any solutions that don’t make sense physically or practically (e.g., negative dimensions).
- Practice solving quadratic equations regularly, as they are common in geometry problems involving areas and dimensions.

Conclusion

In summary, given that the area of a rectangle is 42 square feet and that its length is 11 feet less than three times its width, we determined the dimensions by setting up and solving a quadratic equation. The width is 6 feet, and the length is 7 feet. This problem exemplifies how algebra and geometry work together to solve real-world measurement challenges. Mastering such problems enhances your problem-solving skills and deepens your understanding of geometric relationships, which are essential in many fields, from architecture to engineering.

Whether you're a student tackling homework or a professional designing a space, understanding how to approach and solve these types of problems is invaluable. Keep practicing, and you'll find that many geometric challenges become manageable and even enjoyable to solve!

Frequently Asked Questions

How do you find the width of a rectangle if the area is 42 ft^2 and the length is described as 11 less than three times the width?

Let the width be w. The length is 3w - 11. Since Area = length × width, set up the equation: w(3w - 11) = 42. Solve for w to find the width.

What is the mathematical equation representing the problem where the area of the rectangle is 42 ft^2 and the length is 11 less than three times the width?

The equation is w(3w - 11) = 42, where w is the width of the rectangle.

How can I solve for the width of the rectangle given the equation w(3w - 11) = 42?

Expand the equation to $3w^2$ - 11w = 42, then move 42 to the left side to get $3w^2$ - 11w - 42 = 0. Use the quadratic formula to find w.

What are the steps to find the length of the rectangle once the width is known?

First, solve for the width w using the quadratic equation. Then, substitute the value of w into the length expression: 3w - 11.

Is the width of the rectangle always positive in this problem, and why?

Yes, because width represents a physical measurement, which must be positive. When solving the quadratic, only positive solutions are valid for the width.

What are the approximate dimensions of the rectangle if the width is found to be 4 ft?

If the width w = 4 ft, then the length is 3(4) - 11 = 12 - 11 = 1 ft. So, the rectangle's dimensions are approximately 4 ft by 1 ft.

Additional Resources

The Area Of A Rectangle Is $42ft^2$, And The Length Of The Rectangle Is 11 Less Than Three Times The Width — this statement presents a classic algebraic problem that combines geometry with algebraic reasoning. Whether you're a student trying to understand the relationship between length, width, and area, or a professional seeking to clarify a mathematical concept, this guide aims to provide a comprehensive breakdown of how to interpret and solve this problem step-by-step. By the end of this article, you'll have a clear understanding of how to formulate and solve algebraic expressions related to rectangle dimensions.

Understanding the Problem Statement

Before jumping into calculations, it's essential to interpret what the problem is asking and identify the key pieces of information provided:

- The area of the rectangle is 42 square feet.
- The length of the rectangle is 11 less than three times the width.

This statement contains two critical pieces of information:

1. Area: The area formula for a rectangle is length × width.
2. Relationship between length and width: The length is expressed as a function of the width.

Breaking Down the Key Variables

To solve this problem systematically, assign variables:

- Let w represent the width of the rectangle (in feet).
- Let l represent the length of the rectangle (in feet).

Based on the problem statement, we can formulate an algebraic expression for the length:

$l = 3w - 11$

This means the length is "11 less than three times the width."

Formulating the Equation

The core of the problem revolves around the relationship between the area, length, and width. The area A of a rectangle is calculated as:

$$A = l \times w$$

Given that:

$$A = 42 \text{ ft}^2$$

and

$$l = 3w - 11$$

we substitute the expression for l into the area formula:

$$
42 = (3w - 11) \times w
$$

which simplifies to:

$$
42 = 3w^2 - 11w
$$

This is a quadratic equation in standard form:

$$
3w^2 - 11w - 42 = 0
$$

Solving the Quadratic Equation

To find the width w, we need to solve the quadratic:

$$
3w^2 - 11w - 42 = 0
$$

Method options:

- Factoring (if possible)
- Quadratic formula
- Completing the square

In this case, factoring may be cumbersome, so we'll use the quadratic formula:

$$
w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

where:

- $a = 3$
- $b = -11$
- $c = -42$

Calculating the discriminant:

$$
D = b^2 - 4ac = (-11)^2 - 4 \times 3 \times (-42) = 121 + 504 = 625
$$

Since $D = 625$, and $\sqrt{625} = 25$, the solutions are:

$$
w = \frac{11 \pm 25}{2 \times 3} = \frac{11 \pm 25}{6}
$$

Now, examine both solutions:

1. Using the positive root:

$$
w = \frac{11 + 25}{6} = \frac{36}{6} = 6
$$

2. Using the negative root:

$$
w = \frac{11 - 25}{6} = \frac{-14}{6} = -\frac{7}{3} \approx -2.33
$$

Since a width cannot be negative in a real-world context, we discard the negative solution. Therefore:

$$
w = 6 \text{ ft}
$$

Finding the Length

With the width known, calculate the length:

$$
l = 3w - 11 = 3 \times 6 - 11 = 18 - 11 = 7 \text{ ft}
$$

Summary:

- Width = 6 ft
- Length = 7 ft

Verifying the Solution

Always verify the dimensions satisfy the original problem:

Calculate the area with these dimensions:

$$
\text{Area} = l \times w = 7 \times 6 = 42 \text{ ft}^2
$$

which matches the given area, confirming the solution's correctness.

Additional Considerations and Real-World Applications

While this problem involves a straightforward algebraic approach, understanding the relationship between dimensions and area has broader applications:

- Design and Construction: Determining dimensions based on area constraints.
- Manufacturing: Calculating material requirements for rectangular products.
- Landscaping: Planning plots with specific area and dimension relationships.

Key points to remember:

- Always interpret the problem carefully.
- Assign variables to unknown quantities.
- Translate word statements into algebraic expressions.
- Formulate and solve the resulting equations systematically.
- Verify your solutions in the context of the problem.

Summary and Final Insights

To summarize:

- The problem involves a rectangle with an area of 42 ft^2.
- The length is expressed as $3w - 11$, where w is the width.
- Formulating the quadratic equation and solving using the quadratic formula yields $w = 6$ ft.
- The corresponding length is 7 ft.
- The calculated dimensions satisfy the original area condition.

This example illustrates how algebra and geometry intertwine to solve real-world problems, emphasizing the importance of variable assignment, equation formulation, and verification.

Practice Problems for Reinforcement

To solidify your understanding, try solving these similar problems:

1. The area of a rectangle is 56 ft^2, and its length is 2 less than four times its width. Find the dimensions of the rectangle.
2. A rectangle's length is 5 ft more than its width, and its area is 84 ft^2. Determine the width and length.
3. The perimeter of a rectangle is 36 ft, and its length is twice its width minus 4 ft. Find the dimensions.

By mastering these steps, you'll enhance your problem-solving skills and deepen your understanding of how algebra applies to geometry.

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