

The Coordinate Of A Figure Is (1,5). What Will Be The New Coordinate After A Dilation With A Scale Factor

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Understanding how geometric transformations affect figures on the coordinate plane is fundamental in mathematics, especially in the study of geometry and coordinate geometry. One such transformation is dilation, which involves resizing a figure either larger or smaller relative to a fixed point, known as the center of dilation. When working with points on the coordinate plane, it's essential to grasp how their coordinates change under dilation to accurately analyze and manipulate geometric figures.

In this article, we will explore the concept of dilation in coordinate geometry, focusing on how a point's coordinates change after dilation about the origin or another fixed point. Using the initial coordinate (1, 5) as an example, we will demonstrate how to determine the new coordinate after applying a dilation with a given scale factor. Whether you're a student preparing for exams, a teacher designing lessons, or an enthusiast seeking to deepen your understanding of geometry, this comprehensive guide will clarify the process with detailed explanations, formulas, and illustrative examples.

Understanding Dilation in Coordinate Geometry

What Is Dilation?

Dilation is a type of similarity transformation that enlarges or reduces a figure while preserving its shape. Unlike translation or rotation, dilation changes the size of the figure but maintains the proportional relationships between points. The key characteristics of dilation include:

- Center of Dilation: The fixed point about which the figure is expanded or contracted.
- Scale Factor (k): A positive number indicating how much the figure is scaled. If $k > 1$, the figure enlarges; if $0 < k < 1$, it shrinks.

Mathematically, dilation involves multiplying the distances from the center of dilation to each point by the scale factor.

Coordinate Geometry and Dilation

In coordinate geometry, dilation can be applied to points on the coordinate plane. When dilating a point (x, y) about a center point (x_0, y_0) with scale factor k , the new point (x', y') is calculated as:

$$\begin{aligned} x' &= x_0 + k(x - x_0) \\ y' &= y_0 + k(y - y_0) \end{aligned}$$

This formula indicates that the new coordinate is obtained by moving from the center of dilation towards (or away from) the original point by a factor of k times the distance.

Applying Dilation to the Point (1, 5)

Case 1: Dilation About the Origin (0, 0)

When the dilation center is at the origin, the transformation simplifies to multiplying the coordinates by the scale factor:

$$\begin{aligned} x' &= k \times x \\ y' &= k \times y \end{aligned}$$

Suppose the scale factor is $k = 3$. Then, the new coordinate (x', y') for the point $(1, 5)$ would be:

$$\begin{aligned} x' &= 3 \times 1 = 3 \\ y' &= 3 \times 5 = 15 \end{aligned}$$

Result: The dilated point is $(3, 15)$.

Similarly, if the scale factor is $k = 0.5$:

$$\begin{aligned} x' &= 0.5 \times 1 = 0.5 \\ y' &= 0.5 \times 5 = 2.5 \end{aligned}$$

\]

Result: The new coordinate is (0.5, 2.5).

Summary for dilation about the origin:

Scale Factor (k)	New Coordinate (x', y')	Description
3	(3, 15)	Enlarged figure by factor 3
0.5	(0.5, 2.5)	Reduced figure by half

Case 2: Dilation About a Different Center Point

When dilating about a point other than the origin, you must apply the general formula:

$$\begin{aligned}x' &= x_0 + k(x - x_0) \\ y' &= y_0 + k(y - y_0)\end{aligned}$$

Example: Dilation of (1, 5) about the point (2, 3) with a scale factor of 2

Calculations:

$$\begin{aligned}x' &= 2 + 2(1 - 2) = 2 + 2(-1) = 2 - 2 = 0 \\ y' &= 3 + 2(5 - 3) = 3 + 2(2) = 3 + 4 = 7\end{aligned}$$

Result: The new coordinate is (0, 7).

If the scale factor is 0.5:

$$\begin{aligned}x' &= 2 + 0.5(1 - 2) = 2 + 0.5(-1) = 2 - 0.5 = 1.5 \\ y' &= 3 + 0.5(5 - 3) = 3 + 0.5(2) = 3 + 1 = 4\end{aligned}$$

Result: The new coordinate is (1.5, 4).

Step-by-Step Guide to Find the Dilated Coordinate

To systematically determine the new coordinate after dilation, follow these steps:

1. Identify the Center of Dilation (x_0, y_0): This could be the origin or any other fixed point.
2. Determine the Scale Factor (k): Based on the problem, decide whether the figure enlarges or shrinks.
3. Apply the Dilation Formula:
 - If dilating about the origin:
$$(x', y') = (k \times x, k \times y)$$
 - If dilating about a point other than the origin:
$$(x', y') = (x_0 + k(x - x_0), y_0 + k(y - y_0))$$
4. Calculate the New Coordinates: Plug in the known values and solve.

Real-World Applications of Coordinate Dilation

Understanding dilation in coordinate geometry isn't just an academic exercise; it has practical applications across multiple fields:

- Computer Graphics: Scaling images and objects in digital environments.
- Engineering and Design: Creating scaled models of structures or components.
- Geography and Mapping: Adjusting maps for different scales.
- Robotics: Calculating movements and transformations in coordinate space.
- Art and Architecture: Designing scaled drawings and blueprints.

These applications rely on precise calculations of how points move under dilation, making the understanding of scale factors and transformation formulas essential.

Common Mistakes and Tips for Mastery

- Always identify the correct center of dilation: Mistaking the origin for another point can lead to incorrect results.
- Pay attention to the scale factor: Remember that $k > 1$ enlarges, $0 < k < 1$ shrinks, and negative k reflects points across the center.
- Double-check calculations: Small arithmetic errors can lead to incorrect coordinates.
- Visualize the transformation: Drawing the original and dilated points can help verify the correctness of your calculations.

- Practice with different centers: Dilation about various points enhances understanding and flexibility.

Conclusion

Dilation is a powerful geometric transformation that allows for resizing figures while maintaining their shape and proportions. When working with points on the coordinate plane, understanding how to apply the dilation formula is crucial for accurate calculations. Starting from a simple example like the point (1, 5), we've demonstrated how the new coordinate depends on the scale factor and the center of dilation.

Whether dilating about the origin or another point, the key is to identify the correct formula and apply it carefully. Mastery of this concept enables you to analyze and manipulate geometric figures effectively, with applications spanning various scientific and artistic fields. Keep practicing with different points, centers, and scale factors to develop confidence and proficiency in coordinate dilation.

Keywords: coordinate geometry, dilation, scale factor, transformation, coordinate (1, 5), dilated point, geometric transformation, scale, origin, center of dilation, coordinate plane, math tips, geometry formulas

Frequently Asked Questions

What is the original coordinate of the figure before dilation?

The original coordinate of the figure is (1, 5).

How do you determine the new coordinate after dilation with a scale factor?

Multiply both the x and y coordinates by the scale factor to find the new coordinate.

What is the new coordinate if the scale factor is 2?

The new coordinate will be (2, 10) because $1 \times 2 = 2$ and $5 \times 2 = 10$.

What is the effect of a scale factor less than 1 on the figure's coordinate?

A scale factor less than 1 will produce a new coordinate closer to the origin, shrinking the figure.

If the scale factor is 0.5, what will be the new coordinate after dilation?

The new coordinate will be (0.5, 2.5) because $1 \times 0.5 = 0.5$ and $5 \times 0.5 = 2.5$.

Can the scale factor be negative? If so, what does it imply for the coordinate?

Yes, a negative scale factor reflects the figure across the origin and scales it; the coordinates will have opposite signs accordingly.

Additional Resources

The coordinate of a figure is (1, 5). What will be the new coordinate after a dilation with a scale factor? This question delves into one of the fundamental concepts in coordinate geometry—dilation. Understanding how a figure transforms under dilation, especially how individual points move, is essential for students, educators, and anyone interested in geometric transformations. In this comprehensive guide, we will explore the process of calculating the new coordinate after a dilation, the role of the scale factor, and practical examples to solidify your understanding.

Understanding Dilation in Coordinate Geometry

Before diving into calculations, it's crucial to grasp what dilation entails in the context of coordinate geometry.

What Is Dilation?

Dilation, also known as resizing or scaling, is a transformation that enlarges or reduces a figure relative to a fixed point called the center of dilation. The key characteristic of dilation is that it preserves the shape of the figure but alters its size.

Fixed Point: The Center of Dilation

- The center of dilation is the point about which all other points are scaled.
- It can be any point in the plane, but often, the origin (0,0) is used for simplicity.
- The position of the center significantly influences how points move during dilation.

Scale Factor

- The scale factor determines how much the figure enlarges or shrinks.
- If the scale factor $k > 1$, the figure enlarges.
- If $0 < k < 1$, the figure shrinks.
- If $k = 1$, the figure remains unchanged.
- Negative scale factors, though less common, reflect the figure across the center as well as scale it.

The Core Concept: Calculating New Coordinates After Dilation

When a point $P(x, y)$ is dilated with a scale factor k about a center point $C(h, k)$, its new position P' can be calculated using a simple formula.

General Formula

If the center of dilation is at $C(h, k)$, then the new coordinate $P'(x', y')$ is given by:

$$- x' = h + k(x - h)$$

$$- y' = k + k(y - k)$$

This formula essentially moves the point along the line connecting the center to the original point, scaling the distance from the center by k .

Step-by-Step Guide: Calculating the New Coordinate

Given the original coordinate, the scale factor, and the center of dilation, follow these steps:

1. Identify the Given Data

- Original point: (x, y)
- Center of dilation: (h, k)
- Scale factor: k

2. Find the Difference from the Center

Calculate:

$$- x - h$$

$$- y - k$$

These represent the horizontal and vertical distances from the center to the original point.

3. Apply the Scale Factor

Multiply each difference by the scale factor:

$$- k(x - h)$$

$$- k(y - k)$$

4. Calculate the New Coordinates

Add these scaled differences back to the center coordinates:

$$- x' = h + k(x - h)$$

$$- y' = k + k(y - k)$$

Practical Example: Dilation of the Point (1, 5)

Let's apply this process to the specific scenario:

- Original point: (1, 5)
- Assume the center of dilation is at the origin (0, 0) for simplicity.
- The scale factor: k

Example 1: Scale factor of 2

Step 1: Identify data

- $(x, y) = (1, 5)$
- $(h, k) = (0, 0)$
- $k = 2$

Step 2: Calculate differences

- $x - h = 1 - 0 = 1$
- $y - k = 5 - 0 = 5$

Step 3: Scale differences

- $2 \cdot 1 = 2$
- $2 \cdot 5 = 10$

Step 4: Find new coordinates

- $x' = 0 + 2 = 2$
- $y' = 0 + 10 = 10$

Result: The new coordinate after dilation is (2, 10).

Example 2: Scale factor of 0.5

Using the same point and center:

- $k = 0.5$

Step 1: Differences

- $(x - h) = 1$
- $(y - k) = 5$

Step 2: Scale differences

- $0.5 \cdot 1 = 0.5$
- $0.5 \cdot 5 = 2.5$

Step 3: New coordinates

$$- x' = 0 + 0.5 = 0.5$$

$$- y' = 0 + 2.5 = 2.5$$

Result: The dilated point is (0.5, 2.5).

Handling Different Centers of Dilation

If the center of dilation is not at the origin but at an arbitrary point (h, k), the process remains the same, but you substitute the specific center coordinates.

Example: Center at (2, 3), scale factor 3

Original point: (1, 5)

Step 1: Differences

$$- (x - h) = 1 - 2 = -1$$

$$- (y - k) = 5 - 3 = 2$$

Step 2: Scale differences

$$- 3 (-1) = -3$$

$$- 3 2 = 6$$

Step 3: New coordinates

$$- x' = 2 + (-3) = -1$$

$$- y' = 3 + 6 = 9$$

Result: The new coordinate is (-1, 9).

Visualizing the Transformation

Understanding the calculation is essential, but visualizing the transformation helps solidify the concept.

- Draw the original point and the center of dilation.
- Mark the line connecting them.
- The scaled point P' will lie on this line, at a distance from the center proportional to the scale factor.
- When $k > 1$, the point moves away from the center, enlarging the figure.
- When $0 < k < 1$, the point moves closer to the center, shrinking the figure.
- When $k < 0$, the point reflects across the center and scales accordingly.

Summary and Key Takeaways

- Dilation in coordinate geometry involves resizing a figure relative to a fixed point called the center of dilation.
- The scale factor determines the degree of resizing—greater than 1 for enlargement, between 0 and 1 for reduction.
- The new coordinate after dilation can be calculated using the formula:

$$x' = h + k(x - h)$$

$$y' = k + k(y - k)$$

- The process involves calculating the difference between the point and the center, scaling that difference by the scale factor, then adding it back to the center.
- Visualizing the transformation helps in understanding how the figure enlarges or shrinks relative to the center.

Final Thoughts

Understanding how to find the new coordinate after a dilation is a fundamental skill in coordinate geometry, applicable in various fields such as computer graphics, engineering, and architecture. Whether working with simple points or complex figures, mastering this process provides a solid foundation for analyzing and performing geometric transformations efficiently. Remember to always identify the center of dilation and the scale factor before applying the formula, and visualize the transformation for a clearer understanding.

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