The Dimensions Of A Cuboid Are In The Ratio Of 1:2:3 And Its Total Surface Area Is8 Square Meter. Findthe

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Understanding the dimensions and surface area of geometric shapes like cuboids is fundamental in various fields such as architecture, engineering, and everyday problem-solving. When given ratios and surface areas, the challenge lies in calculating the actual measurements of the shape. This article explores the problem of determining the dimensions of a cuboid when its dimensions are in the ratio of 1:2:3, and its total surface area is 8 square meters. We will walk through the problem systematically, providing clear explanations, relevant formulas, and step-by-step calculations to reach the solution.

Understanding the Problem

Before diving into calculations, it's essential to comprehend what is given and what is required.

Given Data

- The ratios of the cuboid's dimensions: length (l), breadth (b), and height (h) are in the ratio 1:2:3.
- The total surface area (TSA) of the cuboid is 8 m².

What Needs to Be Found?

- The actual dimensions (length, breadth, and height) of the cuboid.

Key Concepts and Formulas

To solve this problem, understanding the fundamental formulas related to cuboids is crucial.

1. Ratio Representation

- Since the dimensions are in ratio 1:2:3, we can express:

$l = k\ 1 = k$

$b = k\ 2 = 2k$

$h = k\ 3 = 3k$

where k is a positive real number representing the common scale factor.

2. Total Surface Area (TSA) of a Cuboid

- The formula for TSA is:

$$TSA = 2(lb + bh + hl)$$

- Substituting the expressions for l, b, h:

$$TSA = 2(k\ 2k + 2k\ 3k + 3k\ k)$$

Simplify each term:

- $lb = k \; 2k = 2k^2$
- $bh = 2k \; 3k = 6k^2$
- $hl = 3k \; k = 3k^2$

So,

$$TSA = 2(2k^2 + 6k^2 + 3k^2) = 2(11k^2) = 22k^2$$

3. Solving for the Scale Factor (k)

- Given TSA = 8 m^2:

$$22k^2 = 8$$

$$k^2 = 8 / 22 = 4 / 11$$

Therefore,

$$k = \sqrt{(4/11)} = 2 / \sqrt{11}$$

Calculating the Dimensions

Using the value of k, we can now find the actual dimensions.

1. Length (l)

- $l = k = 2 / \sqrt{11}$ meters

2. Breadth (b)

- $b = 2k = 2 (2 / \sqrt{11}) = 4 / \sqrt{11}$ meters

3. Height (h)

- $h = 3k = 3 (2 / \sqrt{11}) = 6 / \sqrt{11}$ meters

To express these more neatly, rationalize the denominator:

- $l = (2 / \sqrt{11}) (\sqrt{11} / \sqrt{11}) = (2\sqrt{11}) / 11$ meters
- $b = (4 / \sqrt{11}) (\sqrt{11} / \sqrt{11}) = (4\sqrt{11}) / 11$ meters
- $h = (6 / \sqrt{11}) (\sqrt{11} / \sqrt{11}) = (6\sqrt{11}) / 11$ meters

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Verification of the Surface Area

It's good practice to verify our calculations by plugging the dimensions back into the surface area formula.

- Calculate each product:

$lb = l\ b = (2\sqrt{11} / 11) (4\sqrt{11} / 11) = (2\ 4\ 11) / (11\ 11) = (88) / 121 = 8 / 11$

$bh = b\ h = (4\sqrt{11} / 11) (6\sqrt{11} / 11) = (4\ 6\ 11) / 121 = (264) / 121 = 264 / 121$

hl = h l = (6√11 / 11) (2√11 / 11) = (6 2 11) / 121 = (132) / 121

- Sum of the products:

lb + bh + hl = (8/11) + (264/121) + (132/121)

Convert all to denominator 121:

- 8/11 = (8 11) / 121 = 88 / 121
- 264/121 remains the same
- 132/121 remains the same

Sum = (88 + 264 + 132) / 121 = 484 / 121 = 4

- Total surface area:

TSA = 2 (sum) = 2 4 = 8 m^2

This confirms our calculations are correct.

Practical Applications of the Problem

Understanding how to determine the dimensions of a cuboid based on ratios and surface area has practical significance in various real-life scenarios:

1. Packaging Industry

- Designing boxes with specific surface area constraints to optimize material usage.

2. Construction and Architecture

- Calculating the dimensions of structural elements when given surface area data.

3. Material Science

- Determining the size of objects based on surface area to volume ratios for efficiency.

4. Educational Purposes

- Teaching students problem-solving skills related to geometry and algebra.

Summary and Key Takeaways

- When the dimensions of a cuboid are in a known ratio, express them as multiples of a common scale factor.
- Use the formulas for surface area to set up an equation in terms of the scale factor.
- Solve for the scale factor and substitute back to find the actual dimensions.
- Always verify calculations to ensure accuracy.
- Expressing dimensions in simplified and rationalized form enhances clarity and usability.

Conclusion

Determining the dimensions of a cuboid from its ratio and surface area involves a blend of understanding geometric formulas and algebraic manipulation. In this case, with the ratio 1:2:3 and a surface area of 8 m², the actual dimensions are approximately:

- Length: (2□11) / 11 meters
- Breadth: (4□11) / 11 meters
- Height: (6□11) / 11 meters

This method demonstrates how mathematical principles can be applied to solve practical problems efficiently. Whether in design, manufacturing, or education, mastering such calculations enhances problem-solving skills and deepens understanding of geometric relationships.

References:

- Geometry textbooks and resources
- Standard formulas for cuboids
- Algebraic methods for ratio and scale calculations

Frequently Asked Questions

What are the dimensions of a cuboid with a ratio of 1:2:3 if its total surface area is 8 square meters?

Let the dimensions be x, 2x, and 3x. Given the total surface area is 8 m², we find x by solving the surface area formula: 2(lb + bh + hl) = 8. Substituting, 2(x2x + 2x3x + x3x) = 8, leading to x = 0.25

meters. Therefore, the dimensions are 0.25 m, 0.5 m, and 0.75 m.

How do you determine the dimensions of a cuboid when the ratio and surface area are known?

Express the dimensions in terms of a variable, set up the surface area formula, and solve for the variable. For example, if ratios are 1:2:3, let dimensions be x, 2x, 3x. Use the surface area formula: 2(lb + bh + hl) = total area, then substitute and solve for x.

What is the formula for the total surface area of a cuboid?

The total surface area of a cuboid is given by: TSA = 2(lb + bh + hl), where l, b, and h are the length, breadth, and height respectively.

If a cuboid has dimensions in the ratio 1:2:3, what is the combined surface area formula in terms of x?

Using x as the common ratio multiplier, dimensions are x, 2x, 3x. The surface area becomes: TSA = 2(x2x + 2x3x + x3x) = $2(2x^2 + 6x^2 + 3x^2) = 2(11x^2) = 22x^2$.

How do you find the value of x given the total surface area of 8 m^2 for the cuboid?

Set $22x^2 = 8$ and solve for x: $x^2 = 8/22 = 4/11$, so x = □(4/11) □ 0.603 meters.

What are the actual dimensions of the cuboid with surface area 8 m^2 and ratio 1:2:3?

The dimensions are approximately x □ 0.603 m, 2x □ 1.206 m, and 3x □ 1.809 m.

Can the dimensions of the cuboid be expressed in simplest radical form?

Yes. Since x = √(4/11) = (2/√11) meters, the dimensions are (2/√11) m, (4/√11) m, and (6/√11) m.

What is the significance of the ratio 1:2:3 in solving such problems?

The ratio simplifies the process by allowing the dimensions to be expressed in terms of a single variable, making it easier to set up and solve the equations for surface area or volume.

Additional Resources

The Dimensions Of A Cuboid Are In The Ratio Of 1:2:3 And Its Total Surface Area Is 8 Square Meter. Find The

Understanding the dimensions and surface area of a cuboid is fundamental in geometry, architecture, engineering, and various design applications. When specific ratios and surface area constraints are given, it challenges problem-solvers to derive the precise measurements of the object in question. In this case, the problem involves a cuboid whose dimensions are in the ratio 1:2:3, with a total surface area of 8 square meters. This article aims to explore the problem comprehensively, breaking down the mathematical process involved, discussing key concepts, and examining the implications of such measurements in real-world contexts.

Introduction to Cuboids and Their Properties

A cuboid, commonly known as a rectangular prism, is a three-dimensional shape bounded by six rectangular faces. Its key dimensions are length (l), width (w), and height (h). The properties of a cuboid are foundational in understanding its surface area, volume, and other geometric features.

Key features of a cuboid:

- Opposite faces are equal rectangles.
- Edges meet at right angles (90 degrees).
- The surface area depends on the sum of the areas of all six faces.
- The volume is the product of its three dimensions.

Mathematically:

- Surface area (SA) = $2(lw + lh + wh)$
- Volume (V) = $l \times w \times h$

Understanding the Ratio of Dimensions: 1:2:3

Given that the dimensions are in the ratio 1:2:3, we can express the dimensions as:

- $l = k \times 1 = k$
- $w = k \times 2 = 2k$
- $h = k \times 3 = 3k$

where k is a positive real number scaling the ratio to actual dimensions.

This ratio helps relate the dimensions succinctly, simplifying calculations. The goal is to find the value of k using the given total surface area and then determine the actual dimensions.

Calculating the Total Surface Area

The total surface area (SA) of the cuboid is given as 8 square meters. Substituting the expressions for l, w, and h into the surface area formula:

$SA = 2(lw + lh + wh)$

$\Rightarrow 8 = 2(k \times 2k + k \times 3k + 2k \times 3k)$

Calculating each term:

- $lw = k \times 2k = 2k^2$
- $lh = k \times 3k = 3k^2$
- $wh = 2k \times 3k = 6k^2$

Adding these:

- $lw + lh + wh = 2k^2 + 3k^2 + 6k^2 = 11k^2$

Now, substituting back into the surface area equation:

$8 = 2 \times 11k^2$

$\Rightarrow 8 = 22k^2$

Solving for k:

$k^2 = 8 / 22 = 4 / 11$

$\Rightarrow k = \sqrt{}(4/11) = (2) / \sqrt{}11$

To rationalize the denominator:

$k = (2\sqrt{}11) / 11$

This value of k enables us to compute the actual dimensions.

Determining the Actual Dimensions

Using the value of k:

- Length, $l = k = (2\sqrt{11}) / 11$ meters
- Width, $w = 2k = 2 \times (2\sqrt{11}) / 11 = (4\sqrt{11}) / 11$ meters
- Height, $h = 3k = 3 \times (2\sqrt{11}) / 11 = (6\sqrt{11}) / 11$ meters

These are the precise measurements of the cuboid's dimensions, expressed in surd form. For practical interpretation, approximate decimal values can also be calculated:

- $\sqrt{11} \approx 3.3166$
- $l \approx (2 \times 3.3166) / 11 \approx 6.6332 / 11 \approx 0.603$ meters
- $w \approx (4 \times 3.3166) / 11 \approx 13.2664 / 11 \approx 1.206$ meters
- $h \approx (6 \times 3.3166) / 11 \approx 19.8996 / 11 \approx 1.809$ meters

Thus, the approximate dimensions are approximately:

- Length: 0.603 meters
- Width: 1.206 meters
- Height: 1.809 meters

Verification of Surface Area

To ensure accuracy, verify the calculation by plugging the values back into the surface area formula:

$SA = 2(lw + lh + wh)$

Calculating each:

- $lw \approx 0.603 \times 1.206 \approx 0.727\ m^2$
- $lh \approx 0.603 \times 1.809 \approx 1.089\ m^2$
- $wh \approx 1.206 \times 1.809 \approx 2.182\ m^2$

Sum:

$0.727 + 1.089 + 2.182 \approx 4.0\ m^2$

Multiply by 2:

$2 \times 4.0 \approx 8\ m^2$

This confirms the original surface area provided in the problem, validating our calculations.

Implications of the Dimensions

Knowing the precise dimensions of the cuboid allows architects, engineers, and designers to understand the spatial properties and material requirements. For example:

Pros:

- Precise measurements facilitate accurate material estimation.
- Maintaining the ratio ensures aesthetic consistency in design.
- Helps in optimizing space utilization based on specific dimensions.

Cons:

- The dimensions are relatively small, which may not suit large-scale applications without scaling.
- Strict adherence to ratios might limit design flexibility in some contexts.

Applications and Real-World Contexts

Understanding such geometric problems is not purely theoretical; they have practical significance:

- Packaging Design: Optimizing dimensions for volume and surface area to minimize material usage.
- Construction: Designing storage units or containers with specific surface area constraints.
- Manufacturing: Creating objects where surface treatments (like painting or coating) are dependent on surface area.
- Educational Purposes: Teaching problem-solving skills involving ratios, algebra, and geometry.

Summary of the Solution Process

In summary, the key steps involved in solving this problem were:

1. Expressing the dimensions in terms of a variable k based on the ratio.
2. Substituting these into the surface area formula.
3. Solving the resulting equation for k.
4. Calculating the actual dimensions using the value of k.
5. Verifying the calculations through substitution back into the surface area formula.

This systematic approach exemplifies problem-solving strategies in geometry, emphasizing algebraic manipulation and a clear understanding of geometric properties.

Conclusion

The problem of determining the dimensions of a cuboid given a ratio and total surface area is a classic exercise in geometry that combines ratio concepts, algebra, and surface area calculations. By carefully translating ratios into algebraic expressions and solving the resulting equations, one can accurately determine the dimensions needed for practical applications. This exercise also highlights the importance of precise calculations and verification, ensuring that theoretical solutions align with real-world measurements.

Whether for academic purposes, design projects, or engineering applications, mastering such problems enhances spatial reasoning and mathematical proficiency. Understanding how ratios influence dimensions and surface areas enables better planning and resource optimization across various fields.

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