The Domain For F(x) And G(x) Is The Set Of All Real Numbers.Let F(x) = X2 + 1 And G(x) = 3x.Find F G.A.x2

The Domain For F(x) And G(x) Is The Set Of All Real Numbers. Let F(x) = X2 + 1 And G(x) = 3x. Find F G. A.x2

Introduction

Mathematics, especially algebra, often involves analyzing functions and their compositions. Understanding the domain and range of functions, as well as how to evaluate compositions like $F(G(x))$, is fundamental for solving various problems in algebra, calculus, and beyond. In this article, we will explore the functions $F(x) = x^2 + 1$ and $G(x) = 3x$, analyze their domains, and work through the process of finding the composite function $F(G(x))$. We will also interpret the notation and address common questions students might have when working with these types of functions.

Understanding the Domain of F(x) and G(x)

What is the Domain?

The domain of a function is the set of all possible input values (x-values) for which the function is defined. For real-valued functions, the domain is often all real numbers unless restrictions apply due to the function's form (such as division by zero or square roots of negative numbers).

Domain of $F(x) = x^2 + 1$

- Since x^2 is defined for all real numbers, and adding 1 does not restrict the domain, the domain of $F(x)$ is all real numbers.

Domain of $G(x) = 3x$

- Similarly, $G(x) = 3x$ is a linear function, defined for every real number, so the domain of $G(x)$ is all real numbers.

Overall Domain

- Given that both functions have the domain of all real numbers, their composition will also be defined for all real numbers, assuming no restrictions are introduced during the composition process.

Function Composition: $F(G(x))$

Understanding Function Composition

- The notation $F(G(x))$ represents the composition of functions F and G. It involves applying G to x, then applying F to the result.

- In general, $F(G(x)) = F$ composed with G, meaning:

$$
F(G(x)) = F(\text{the output of } G(x))
$$

Step-by-Step Process

1. Identify $G(x)$:

$$
G(x) = 3x
$$

2. Substitute $G(x)$ into $F(x)$:

$$
F(G(x)) = F(3x)
$$

3. Apply $F(x) = x^2 + 1$ to $3x$:

$$
F(3x) = (3x)^2 + 1
$$

4. Simplify the expression:

$$
(3x)^2 + 1 = 9x^2 + 1
$$

Thus, the composite function $F(G(x))$ simplifies to:

$$
\boxed{
F(G(x)) = 9x^2 + 1
}
$$

Analyzing the Result of the Composition

The nature of $F(G(x))$

- The composite function $F(G(x)) = 9x^2 + 1$ is a quadratic function in x.

- Since the coefficient of x^2 is positive, the parabola opens upward, with its vertex at the minimum point.

Domain of $F(G(x))$

- As both F and G are defined over all real numbers, their composition $F(G(x))$ is also defined for all real numbers.

- Therefore, the domain of $F(G(x))$ is all real numbers.

Range of $F(G(x))$

- Since $9x^2 \geq 0$ for all real x, and the minimum value of $9x^2$ is 0 (at $x=0$), the smallest value of $F(G(x))$ is:

$$
9(0)^2 + 1 = 1
$$

- As $x \to \pm \infty$, $9x^2 \to \infty$, so:

$$
F(G(x)) \to \infty
$$

- Range:

$$
[1, \infty)
$$

Visual Interpretation and Graphical Insights

Graph of $F(x) = x^2 + 1$

- A parabola opening upward, vertex at $(0, 1)$.

Graph of $G(x) = 3x$

- A straight line with slope 3 passing through the origin.

Graph of $F(G(x)) = 9x^2 + 1$

- A parabola opening upward, vertex at $(0, 1)$, similar to $F(x)$ but compressed horizontally due to the factor 9.

- The graph of the composition can be viewed as F applied to the linear transformation $G(x)$.

Practical Applications and Examples

Example 1: Calculating $F(G(2))$

- First, find $G(2)$:

$$
G(2) = 3 \times 2 = 6
$$

- Then, apply F to this value:

$$
F(6) = 6^2 + 1 = 36 + 1 = 37
$$

- Thus, $F(G(2)) = 37$.

Example 2: Finding $F(G(-1))$

- Calculate $G(-1)$:

$$
G(-1) = 3 \times (-1) = -3
$$

- Apply F:

$$
F(-3) = (-3)^2 + 1 = 9 + 1 = 10
$$

Example 3: Solving for x when $F(G(x)) = 10$

- Set $9x^2 + 1 = 10$:

$$
9x^2 = 9
$$

- Solve for x:

$$
x^2 = 1
$$

$$
x = \pm 1
$$

Common Questions and Clarifications

Q1: Why is the domain of $F(x)$ and $G(x)$ all real numbers?

- Because both functions are polynomial or linear functions without restrictions such as division by zero or square roots of negative numbers, they are defined for all real inputs.

Q2: Can the composition $F(G(x))$ be undefined?

- In this case, no, because the composition simplifies to $9x^2 + 1$, which is defined for all real x.

Q3: How does the composition affect the shape of the graph?

- Since $G(x)$ is linear, applying F to $G(x)$ essentially "transforms" the linear input into a quadratic output, resulting in a parabola.

Summary

- The functions $F(x) = x^2 + 1$ and $G(x) = 3x$ both have the domain of all real numbers.
- The composition $F(G(x))$ simplifies to $9x^2 + 1$.
- The domain of $F(G(x))$ remains all real numbers, with the range starting from 1 to infinity.
- Understanding the composition of functions is crucial in algebra, enabling us to analyze complex transformations and their graphs.
- Practical examples help reinforce the process of evaluating compositions and solving for specific values.

Final Thoughts

Mastering the concepts of domain, range, and function composition is essential for progressing in mathematics. Whether working with simple functions like polynomials or more complex transformations, these foundational ideas underpin much of advanced mathematics, calculus, and applied sciences. Remember, always analyze the functions carefully, simplify step-by-step, and visualize the results for a comprehensive understanding.

Frequently Asked Questions

What is the domain for the functions $F(x) = x^2 + 1$ and $G(x) = 3x$?

The domain for both $F(x)$ and $G(x)$ is the set of all real numbers, meaning they are defined for every real value of x.

How do you find the composite function $(F \circ G)(x)$ or $F(G(x))$?

To find $F(G(x))$, substitute $G(x)$ into $F(x)$. Since $G(x) = 3x$, compute $F(3x) = (3x)^2 + 1 = 9x^2 + 1$.

What is the value of $F(G(x))$ when $x = 2$?

When $x = 2$, $G(2) = 3 2 = 6$. Then, $F(6) = 6^2 + 1 = 36 + 1 = 37$.

Is the composite function $F(G(x))$ defined for all real numbers?

Yes, since both F and G are defined for all real numbers, their composition $F(G(x))$ is also defined for all real x.

What is the general form of the composite function $F(G(x))$?

The composite function $F(G(x))$ is given by $9x^2 + 1$, derived by substituting $G(x) = 3x$ into $F(x) = x^2 + 1$.

How does the composition affect the degree of the resulting function?

Since $G(x)$ is degree 1 and $F(x)$ is degree 2, the composition $F(G(x))$ results in a degree 2 quadratic function.

What is the range of the composite function $F(G(x)) = 9x^2 + 1$?

The range is all real numbers y such that $y \geq 1$, since $9x^2 \geq 0$ for all real x, making the minimum value of y equal to 1.

Additional Resources

Understanding the Composition of Functions: F(g(x)) and Its Domain

When delving into the world of functions, one of the most intriguing and fundamental concepts is the composition of functions. This process involves creating a new function by applying one function to the result of another. In this detailed exploration, we'll examine the specific functions:

- $F(x) = x^2 + 1$
- $G(x) = 3x$

and analyze the composition $F(G(x))$. We'll discuss the domain considerations, the step-by-step evaluation, and the broader implications of function composition in mathematics.

Foundations of Function Composition

Before diving into the specific functions, it's essential to understand what function composition entails.

What Is Function Composition?

Function composition is denoted as $(F \circ G)(x)$ and represents applying the function F to the result of $G(x)$:

$$
(F \circ G)(x) = F(G(x))
$$

This process is akin to a two-step process:

1. Calculate $G(x)$.
2. Plug the result into F.

Example: If $G(x) = 3x$, and $F(x) = x^2 + 1$, then

$$
F(G(x)) = F(3x) = (3x)^2 + 1 = 9x^2 + 1
$$

Analyzing the Given Functions

Let's examine the specific functions provided:

- $F(x) = x^2 + 1$
- $G(x) = 3x$

and determine the composition $F(G(x))$.

Domain of the Functions

The problem states:

> The domain for $F(x)$ and $G(x)$ is the set of all real numbers $\mathbb{R}$.

This is crucial because it implies that both functions are defined everywhere on $\mathbb{R}$. There are no restrictions such as square roots of negative numbers or divisions by zero that might limit their domains.

Step-by-Step Evaluation of $F(G(x))$

Let's now compute $F(G(x))$ explicitly.

Substitute $G(x)$ into F

Since $G(x) = 3x$, replacing the input in F:

$$
F(G(x)) = F(3x)
$$

Applying the formula for $F(x)$:

$$
F(3x) = (3x)^2 + 1
$$

Simplify:

$$
F(3x) = 9x^2 + 1
$$

This is the composition function $F \circ G$.

Domain of the Composition $F(G(x))$

Given that:

- $G(x)$ maps $\mathbb{R}$ to $\mathbb{R}$,
- F is defined for all real numbers,

then the composition $F(G(x))$ is also defined for all real numbers, i.e., its domain is $\mathbb{R}$.

Summary:

| Function | Domain | Range |

$G(x) = 3x$	$\mathbb{R}$	$\mathbb{R}$
$F(x) = x^2 + 1$	$\mathbb{R}$	$[1, \infty)$

Since the composition involves substituting $G(x)$ into F, the domain of $F \circ G$ remains $\mathbb{R}$.

Graphical Interpretation and Behavior

Visualizing the functions enhances understanding.

Graph of $G(x) = 3x$

- Straight line passing through the origin with a slope of 3.
- Domain and range are both all real numbers.
- For any real x, $G(x)$ produces a real output.

Graph of $F(x) = x^2 + 1$

- Parabola opening upwards, vertex at $(0, 1)$.
- Range is $[1, \infty)$.

Graph of $F(G(x)) = 9x^2 + 1$

- Also a parabola opening upwards.
- Vertex at $(0, 1)$, same as F, but "stretched" vertically because of the coefficient 9 before x^2.
- Domain: all real numbers $\mathbb{R}$.

This composition transforms the linear input $3x$ into a quadratic form scaled by 9, and shifted upward by 1.

Mathematical Significance of the Composition $F \circ G$

Understanding the composite function's behavior is essential because it illustrates how transformations combine.

Effect of Composition on Shape and Range

- The original $G(x)$ is linear, which means it increases or decreases at a constant rate.
- Applying F to $G(x)$ results in a quadratic function $9x^2 + 1$, which is symmetric about the y-axis and opens upward.
- The composition effectively "stretches" the input scale due to the multiplication by 3 inside G and the square in F.

Implications for Function Behavior

- The composition inherits properties from both functions.
- Since G is linear, it preserves the input's order, but the quadratic F amplifies the input's magnitude quadratically.
- The entire composition is a parabola with a minimum point at $x = 0$, where $F(G(0)) = 1$.

Calculus Perspective: Derivatives and Critical Points

For those interested in the calculus implications:

- The derivative of $F(G(x))$:

$$
\frac{d}{dx} F(G(x)) = F'(G(x)) \times G'(x)
$$

- Compute $F'(x)$:

$$
F'(x) = 2x
$$

- Compute $G'(x)$:

$$
G'(x) = 3
$$

- Applying the chain rule:

$$
\frac{d}{dx} F(G(x)) = 2 \times G(x) \times 3 = 6 G(x) = 6 \times 3x = 18x
$$

\]

Critical point:

- Set derivative to zero:

$$
18x = 0 \Rightarrow x = 0
$$

- The minimum of $F(G(x))$ occurs at $x = 0$.

- At $x=0$:

$$
F(G(0)) = 9(0)^2 + 1 = 1
$$

which is the vertex of the parabola.

Applications and Broader Context

Understanding compositions like $F \circ G$ is fundamental in various fields:

- Mathematics Education: It helps students grasp how functions combine and transform.
- Physics: Many physical phenomena involve layered functions, such as displacement as a function of time, then velocity as a function of displacement.
- Computer Science: Function composition underpins functional programming paradigms.
- Engineering: Control systems often involve the composition of transfer functions.

Further Exploration: Inverse Functions and Composition

While not directly asked, it's worthwhile to touch on the inverse functions:

- $F(x) = x^2 + 1$ is not invertible over all $\mathbb{R}$ because it's not one-to-one (parabola).
- Restricting F to $[0, \infty)$ makes it invertible.
- $G(x) = 3x$ is invertible over $\mathbb{R}$, with inverse $G^{-1}(x) = x/3$.

Understanding these aspects can deepen the comprehension of how functions behave under composition and inversion.

Summary and Final Remarks

- The composition \(F(G(x)) \) with \(F(x) = x^2 + 1 \) and \(G(x) = 3x \) simplifies to \(9x^2 + 1 \).
- The domain of this composition is all real numbers, consistent with the individual domains.
- The graph is a parabola opening upward with vertex at \((0, 1) \).
- The derivative indicates a linear rate of change \(18x \), with a critical point at \(x=0 \).
- Understanding this composition illustrates how linear and quadratic functions combine, offering insights into function transformations.

In essence, exploring function composition provides a powerful lens through which to analyze complex behaviors, transformations, and properties in mathematics. Mastery of such concepts is foundational for advanced studies in calculus, algebra, and applied sciences.

Note: Always verify the domain and range when working with function compositions, especially when dealing with more complex functions

The Domain For F X And G X Is The Set Of All Real Numbers Let F X X2 1 And G X 3x Find F G A X2

The Domain For F X And G X Is The Set Of All Real Numbers Let F X X2 1 And G X 3x Find F G A X2

Related Articles

- In A Survey, A Group Of Students Were Asked To Name Their Favorite Elective Class. There Were 15 Students
- Kyung Works 5 Hours At A Restaurant Each Wednesday. He Also Earns $37 A Week Delivering Newspapers. Kyung
- Onsumer Holds Money To Meet Spending Needs. This Would Be An Example Of The: A. Use Of Money As A Measure

Back to Home