0 0 - 1 3. Show That A And B Are Not Similar Matrices. 11 2 2 1 1 A = 0 1 -1 , B = 0 1 12 0 1 4. Determine

0 0 - 1 3. Show That A And B Are Not Similar Matrices. 11 2 2 1 1 A = 0 1 -1 , B = 0 1 12 0 1 4. Determine

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Introduction

In linear algebra, the concept of matrix similarity plays a crucial role in understanding how matrices relate to each other, especially in terms of their fundamental properties such as eigenvalues and diagonalizability. When analyzing whether two matrices are similar, mathematicians often rely on invariants like eigenvalues, eigenvectors, and canonical forms. This article aims to explore a specific problem: demonstrating that two given matrices, A and B, are not similar by examining their properties and invariants.

The matrices in question are:

$$
A = \begin{bmatrix}
0 & 1 & -1 \\
0 & 1 & -1 \\
0 & 1 & -1
\end{bmatrix}
\quad \text{and} \quad
B = \begin{bmatrix}
0 & 1 & 12 \\
0 & 1 & 4 \\
0 & 1 & 4
\end{bmatrix}
$$

The goal is to analyze these matrices comprehensively, understand their properties, compute their invariants, and ultimately show that they are not similar matrices.

Understanding Matrix Similarity

What Does It Mean for Matrices to Be Similar?

Two $n \times n$ matrices A and B are similar if there exists an invertible matrix P such that:

$$
B = P^{-1} A P

\]

Matrix similarity indicates that \(A \) and \(B \) represent the same linear transformation under different bases. Similar matrices share many properties:

- Same eigenvalues (including algebraic multiplicities)
- Same characteristic polynomial
- Same minimal polynomial
- Same Jordan canonical form

Thus, to show that two matrices are not similar, it suffices to find a property that they do not share.

Step 1: Computing Eigenvalues

Eigenvalues of Matrix A

Given:

\[
A = \begin{bmatrix}
0 & 1 & -1 \\
0 & 1 & -1 \\
0 & 1 & -1
\end{bmatrix}
\]

Since the matrix is upper triangular (all entries below the main diagonal are zero), its eigenvalues are the entries on the main diagonal:

\[
\boxed{
\text{Eigenvalues of } A: \lambda = 0, 1, 1
}
\]

Eigenvalues of Matrix B

Given:

\[
B = \begin{bmatrix}
0 & 1 & 12 \\
0 & 1 & 4 \\
0 & 1 & 4
\end{bmatrix}
\]

Similarly, B is upper triangular, so eigenvalues are the diagonal entries:

\[
\boxed{
\text{Eigenvalues of } B: \lambda = 0, 1, 4
}
\]

Step 2: Comparing Eigenvalues

Since matrices \(A \) and \(B \) have different eigenvalues:

- \(A \) has eigenvalues \(0, 1, 1 \)
- \(B \) has eigenvalues \(0, 1, 4 \)

They do not share the same eigenvalues, which is a key invariant under similarity transformations.

Implication

Because similar matrices must have identical eigenvalues (with the same algebraic multiplicities), the difference in eigenvalues directly implies:

\[
\boxed{
A \text{ and } B \text{ are not similar}
}
\]

This is the primary and most straightforward argument in our proof.

Step 3: Additional Invariants and Confirmation

While eigenvalues are sufficient for the conclusion, further analysis can reinforce the argument, especially if the eigenvalues coincided.

Jordan Canonical Form

The Jordan form is a canonical form representing a matrix up to similarity. Differences in the sizes of Jordan blocks corresponding to each eigenvalue can also demonstrate non-similarity.

For Matrix A:

- Eigenvalues: \(0, 1, 1 \)
- Eigenvalue \(1 \) has algebraic multiplicity 2, but the matrix's structure suggests the possibility of a Jordan block of size 2 or 1.

Calculating the geometric multiplicity (dimension of eigenspaces):

\[

\text{Eigenvectors for } \lambda=1:
\]

\[
A - I = \begin{bmatrix}
-1 & 1 & -1 \\
0 & 0 & -1 \\
0 & 0 & -1
\end{bmatrix}
\]

Solving:

\[
(A - I) \mathbf{x} = 0
\]

leads to the eigenspace dimension (geometric multiplicity). The calculations show that the eigenspace for \(\lambda=1 \) has dimension 1, indicating a Jordan block of size 2 for eigenvalue 1.

For Matrix B:

- Eigenvalues: \(0, 1, 4 \)
- For eigenvalue \(1 \):

\[
B - I = \begin{bmatrix}
-1 & 1 & 12 \\
0 & 0 & 4 \\
0 & 1 & 3
\end{bmatrix}
\]

Similar calculations yield the eigenspace dimension, confirming the structural differences.

Since the eigenvalues differ and the Jordan block structures are incompatible, matrices \(A \) and \(B \) are not similar.

Conclusion

Summary of Findings

- Eigenvalues differ: \(A \) has eigenvalues \(0, 1, 1 \), while \(B \) has \(0, 1, 4 \). The difference in eigenvalues immediately indicates that the matrices are not similar.
- Eigenvalue multiplicities and Jordan forms further confirm the non-similarity, as the structure of the eigenvalues and associated Jordan blocks cannot be matched through similarity transformations.

Final Statement

Given the fundamental invariance of eigenvalues under similarity, the fact that matrices \(A \) and \(B \) do not share the same eigenvalues (or Jordan block structures) conclusively demonstrates that A and B are not similar matrices.

Additional Considerations: Determining Other Properties

While the primary goal was to show non-similarity, the second part of the problem hints at calculating or determining further features of these matrices. For instance, if the problem involves finding their minimal polynomials, ranks, or canonical forms, similar techniques apply:

- Minimal polynomial: Compute the least degree monic polynomial annihilating the matrix.
- Rank and nullity: Use row reduction or rank calculations.
- Canonical forms: Use eigenvalues and Jordan block sizes to explicitly construct Jordan forms.

However, since the eigenvalues differ, these further calculations primarily serve to reinforce the initial conclusion.

Final Remarks

Understanding the distinctions between matrices via invariants like eigenvalues and Jordan forms is central in linear algebra. The analysis of matrices \(A \) and \(B \) exemplifies how these invariants provide powerful tools for classifying matrices up to similarity. In this particular case, the discrepancy in eigenvalues suffices to establish their non-similarity, emphasizing the importance of eigenvalue analysis in matrix theory.

References

- Lay, David C. Linear Algebra and Its Applications. 5th Edition. Pearson, 2015.
- Hoffman, Kenneth, and Ray Kunze. Linear Algebra. 2nd Edition. Prentice-Hall, 1971.
- Strang, Gilbert. Introduction to Linear Algebra. 5th Edition. Wellesley-Cambridge Press, 2016.

This article provides a comprehensive approach to show that two matrices are not similar, emphasizing the significance of eigenvalues and canonical forms in matrix analysis.

Frequently Asked Questions

What does it mean for two matrices A and B to be similar?

Two matrices A and B are similar if there exists an invertible matrix P such that $B = P^{-1}AP$. Similar matrices represent the same linear transformation under different bases.

Given matrices A = [[0, 0, -1], [3, 11, 2], [2, 1, 1]] and B = [[0, 1, -1], [0, 1, 12], [0, 1, 4]], how can we show they are not similar?

One way is to compare their eigenvalues and minimal polynomials. If they differ, the matrices are not similar. In this case, their eigenvalues are different, indicating they are not similar.

How do eigenvalues help determine if two matrices are similar?

Eigenvalues are invariants under similarity transformations. If two matrices have different sets of eigenvalues, they cannot be similar.

What are the eigenvalues of matrix A?

Calculating the eigenvalues of A involves solving the characteristic polynomial $\det(A - \lambda I) = 0$. For the given A, the eigenvalues are $\lambda = 0, 2, 1$.

What are the eigenvalues of matrix B?

Similarly, for B, the eigenvalues are $\lambda = 0, 1, 12$.

Why do differing eigenvalues imply matrices are not similar?

Because eigenvalues are invariant under similarity transformations, different eigenvalues mean the matrices cannot be similar.

Can matrices with the same eigenvalues be non-similar?

Yes, matrices can share the same eigenvalues but differ in their Jordan canonical forms, so they may not be similar.

What is the significance of minimal polynomials in matrix similarity?

The minimal polynomial is also invariant under similarity. If two matrices have different minimal polynomials, they are not similar.

How does the Jordan form help in determining whether two matrices are similar?

Two matrices are similar if and only if they have the same Jordan canonical form (up to permutation). Different Jordan forms mean they are not similar.

What conclusions can be drawn from the difference in eigenvalues of matrices A and B?

Since the eigenvalues are different, matrices A and B are not similar.

Additional Resources

Matrix Similarity Analysis: Unveiling the Distinction Between Matrices A and B

Matrices are fundamental constructs in linear algebra, serving as the backbone for numerous applications across science, engineering, and mathematics. When analyzing matrices, one key question often arises: are two matrices similar? Understanding whether matrices are similar not only enriches our theoretical comprehension but also has practical implications—particularly in simplifying complex matrix operations, diagonalization, and understanding matrix transformations.

In this detailed exploration, we examine the matrices:

$$A = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 2 & 2 \\ 1 & 1 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 1 & 12 \\ 0 & 1 & 4 \end{bmatrix}$$

and demonstrate why they are not similar matrices. The analysis will include eigenvalues, eigenvectors, characteristic polynomials, minimal polynomials, and other invariants essential in the theory of matrix similarity.

Understanding Matrix Similarity

What Does It Mean for Matrices to Be Similar?

Two square matrices A and B of size $n \times n$ are said to be similar if there exists an invertible matrix P such that:

$$B = P^{-1} A P$$

This relation indicates that A and B represent the same linear transformation under different bases. Similar matrices share many fundamental properties, particularly:

- Eigenvalues (including algebraic multiplicities)
- Characteristic polynomial
- Minimal polynomial
- Jordan canonical form

However, they may differ in their specific eigenvectors or basis representations.

Step 1: Analyzing the Matrices' Dimensions and Basic Properties

Before delving into the eigenvalues and other invariants, it's crucial to recognize the dimensions:

- Matrix A is a 3×3 matrix.
- Matrix B is a 2×3 matrix.

Important note: For matrices to be similar, they must be of the same size. Since A is 3×3 and B is 2×3, they are not even the same size, and thus cannot be similar matrices.

Step 2: Clarifying the Matrices’ Dimensions and Context

Given the matrices:

$$
A = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 2 & 2 \\ 1 & 1 & 0 \end{bmatrix}
$$

$$
B = \begin{bmatrix} 1 & 1 & 12 \\ 0 & 1 & 4 \end{bmatrix}
$$

we observe that:

- A is a 3x3 matrix.
- B is a 2x3 matrix.

In standard linear algebra, similarity is defined only between square matrices of the same size. Therefore, matrices A and B are of different sizes, which immediately indicates they are not similar.

Step 3: Interpreting the Problem Context

However, often in advanced contexts, matrices can be compared under different equivalence relations, such as:

- Row or column equivalence
- Rank equivalence
- Similarity after embedding or via transformations

But, in the classical sense, similarity applies strictly to square matrices of the same dimension. The question perhaps intends to compare their properties or to analyze whether the matrices are similar in some other sense.

Step 4: Eigenvalues and Invariants of \(A\)

Assuming the focus is on matrix \(A\), as it's square, analyze its eigenvalues, characteristic polynomial, and minimal polynomial.

4.1. Characteristic Polynomial of \(A\)

Calculate:

\[
\det(A - \lambda I) = 0
\]

where \(I\) is the \(3 \times 3\) identity matrix.

\[
A - \lambda I = \begin{bmatrix}
-\lambda & 0 & -1 \\
1 & 2 - \lambda & 2 \\
1 & 1 & -\lambda
\end{bmatrix}
\]

The determinant:

\[
\det(A - \lambda I) =
\begin{vmatrix}
-\lambda & 0 & -1 \\
1 & 2 - \lambda & 2 \\
1 & 1 & -\lambda
\end{vmatrix}
\]

Expanding along the first row:

$$
= (-\lambda) \cdot \begin{vmatrix} 2 - \lambda & 2 \\ 1 & -\lambda \end{vmatrix} - 0 \cdot (\text{minor}) + (-1) \cdot \begin{vmatrix} 1 & 2 - \lambda \\ 1 & 1 \end{vmatrix}
$$

Calculating each minor:

- First minor:

$$
\begin{vmatrix} 2 - \lambda & 2 \\ 1 & -\lambda \end{vmatrix} = (2 - \lambda)(-\lambda) - 2 \cdot 1 = -\lambda (2 - \lambda) - 2
$$

- Second minor:

$$
\begin{vmatrix} 1 & 2 - \lambda \\ 1 & 1 \end{vmatrix} = 1 \cdot 1 - (2 - \lambda) \cdot 1 = 1 - (2 - \lambda) = 1 - 2 + \lambda = \lambda - 1
$$

Putting it all together:

$$
\det(A - \lambda I) = (-\lambda) \left(-\lambda (2 - \lambda) - 2 \right) - (-1)(\lambda - 1)
$$

Simplify:

$$
= (-\lambda) \left(-\lambda (2 - \lambda) - 2 \right) + (\lambda - 1)
$$

Calculate the inner term:

$$
-\lambda (2 - \lambda) - 2 = -\lambda (2 - \lambda) - 2 = -\lambda (2 - \lambda) - 2
$$

Expand:

$$
-\lambda (2 - \lambda) = -2 \lambda + \lambda^2
$$

Thus:

$$
-\lambda (2 - \lambda) - 2 = (-2 \lambda + \lambda^2) - 2
$$

$$
$$

Now multiply by $-\lambda$:

$$
(-\lambda) \left(-2 \lambda + \lambda^2 - 2 \right) = -\lambda (-2 \lambda + \lambda^2 - 2)
$$

Distribute:

$$
= -\lambda (-2 \lambda) + (-\lambda)(\lambda^2) + (-\lambda)(-2)
$$

$$
= 2 \lambda^2 - \lambda^3 + 2 \lambda
$$

Finally, add the remaining term $\lambda - 1$:

$$
\det(A - \lambda I) = 2 \lambda^2 - \lambda^3 + 2 \lambda + \lambda - 1 = -\lambda^3 + 2 \lambda^2 + 3 \lambda - 1
$$

Rearranged:

$$
\boxed{
\det(A - \lambda I) = -\lambda^3 + 2 \lambda^2 + 3 \lambda - 1
}
$$

or equivalently:

$$
\lambda^3 - 2 \lambda^2 - 3 \lambda + 1 = 0
$$

Eigenvalues of A

To find eigenvalues, solve:

$$
\lambda^3 - 2 \lambda^2 - 3 \lambda + 1 = 0
$$

Try rational root theorem: possible roots are factors of constant term (±1):

- $\lambda = 1$:

$$
1 - 2 - 3 + 1 = -3 \neq 0
$$

- $\lambda = -1$:

$$
-1 - 2 + 3 + 1 = 1 \neq 0
$$

No rational roots; we need to solve the cubic explicitly or approximate roots.

Step 5: Eigenvalues of B and Their Implications

The matrix B is not square, so eigenvalues are not defined in the traditional sense. If the problem's goal is to compare A and B, perhaps the focus is on their row spaces, rank, or other invariants.

Step 6: Determining the Incompatibility of A and B Being Similar

6.1. Size Difference

As already established, matrices of different sizes cannot be similar because similarity is an equivalence relation for square matrices of the same dimension.

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