1 Point) Let F(x)=|x1| |x 4|. Use Interval Notation To Indicate The Values Of X Where F Is Differentiable.

1 Point) Let $F(x)=|x_1|\ |x_4|$. Use Interval Notation To Indicate The Values Of X Where F Is Differentiable.

Understanding the differentiability of functions involving absolute value expressions is fundamental in calculus, especially when analyzing piecewise functions and their smoothness properties. In this article, we will explore the function $F(x) = |x_1| \cdot |x_4|$, where $x = (x_1, x_2, x_3, x_4)$, and determine precisely where F is differentiable using interval notation. This comprehensive analysis will include detailed explanations of absolute value functions, their derivatives, and the points of non-differentiability, providing clarity for students and practitioners alike.

Understanding the Function $F(x) = |x_1| \cdot |x_4|$

Breakdown of the Function Components

The function $F(x) = |x_1| \cdot |x_4|$ is a product of two absolute value functions:

- $|x_1|$, which depends solely on the first coordinate
- $|x_4|$, which depends solely on the fourth coordinate

The other components x_2 and x_3 do not influence the value of F(x), making the function effectively dependent on the first and fourth variables.

Properties of Absolute Value Functions

The absolute value function |x| is defined as:

|x| =
- x, if $x \geq 0$
- -x, if $x < 0$

It is continuous everywhere but not differentiable at $x = 0$, because the derivative from the left and right do not coincide at zero.

Similarly, for the function F(x), the points where $|x_1|$ or $|x_4|$ are not differentiable are precisely where $x_1 = 0$ or $x_4 = 0$.

Differentiability of $F(x) = |x_1| \cdot |x_4|$

Differentiability in Multivariable Context

In multivariable calculus, a function is differentiable at a point if all its partial derivatives exist and are continuous in a neighborhood around that point. For F(x), the partial derivatives with respect to each variable are:

- $\partial F/\partial x_1$
- $\partial F/\partial x_2$
- $\partial F/\partial x_3$
- $\partial F/\partial x_4$

Since F depends only on x_1 and x_4, the partial derivatives with respect to x_2 and x_3 are zero everywhere, and their differentiability is trivial.

The critical focus is on the partial derivatives with respect to x_1 and x_4, which involve the derivatives of absolute value functions.

Calculating Partial Derivatives

Let's compute the partial derivatives:

1. Partial derivative with respect to x_1:

$$
\frac{\partial F}{\partial x_1} = \frac{\partial}{\partial x_1} (|x_1| \cdot |x_4|) = |x_4| \cdot \frac{\partial}{\partial x_1} |x_1|
$$

Since $|x_1|$ is not differentiable at $x_1 = 0$, its derivative is:

$$
\frac{d}{dx_1} |x_1| =
\begin{cases}
1, & x_1 > 0 \\
-1, & x_1 < 0 \\
\text{undefined}, & x_1 = 0
\end{cases}
$$

Therefore,

$$
\frac{\partial F}{\partial x_1} =
\begin{cases}
$$

|x_4|, & x_1 \neq 0 \\
\text{undefined}, & x_1 = 0
\end{cases}
\]

2. Partial derivative with respect to x_4:

Similarly,

$$
\frac{\partial F}{\partial x_4} = |x_1| \cdot \frac{\partial}{\partial x_4}
|x_4| = |x_1| \times
\begin{cases}
1, & x_4 > 0 \\
-1, & x_4 < 0 \\
\text{undefined}, & x_4 = 0
\end{cases}
$$

Thus,

$$
\frac{\partial F}{\partial x_4} =
\begin{cases}
|x_1|, & x_4 \neq 0 \\
\text{undefined}, & x_4 = 0
\end{cases}
$$

Points of Non-Differentiability

The partial derivatives exist and are finite everywhere except where the derivatives of the absolute value functions are undefined, specifically at $x_1 = 0$ and $x_4 = 0$.

Therefore, the function F(x) is not differentiable at points where:

- $x_1 = 0$
- $x_4 = 0$

Since the differentiability depends on the behavior of the absolute value functions, the overall set of points where F is not differentiable is the union of the hyperplanes:

- $\{ (x_1, x_2, x_3, x_4) \mid x_1 = 0 \}$
- $\{ (x_1, x_2, x_3, x_4) \mid x_4 = 0 \}$

Conversely, F is differentiable at all points where both $x_1 \neq 0$ and $x_4 \neq 0$.

Using Interval Notation to Express Differentiability

In multivariable calculus, the set of points where a function is differentiable can be described using Cartesian product notation. For the variables x_1 and x_4:

- The set where F is differentiable with respect to these variables is:

$$
\{ (x_1, x_4) \mid x_1 \neq 0, x_4 \neq 0 \}
$$

Expressed in interval notation for each variable:

- For x_1:

$$
(-\infty, 0) \cup (0, \infty)
$$

- For x_4:

$$
(-\infty, 0) \cup (0, \infty)
$$

Therefore, the set of points where F is differentiable can be written as:

$$
\left((-\infty, 0) \cup (0, \infty) \right) \times \left((-\infty, 0) \cup (0, \infty) \right)
$$

In terms of the entire 4-dimensional space:

$$
\{ (x_1, x_2, x_3, x_4) \mid x_1 \in (-\infty, 0) \cup (0, \infty), \quad x_4 \in (-\infty, 0) \cup (0, \infty) \}
$$

Since x_2 and x_3 do not affect differentiability, they can take any real values.

Summary of Differentiability Regions

Based on the above analysis, the function $F(x) = |x_1| \cdot |x_4|$ is differentiable exactly where:

- $x_1 \neq 0$
- $x_4 \neq 0$

Expressed in interval notation:

$$
\boxed{
\left((-\infty, 0) \cup (0, \infty) \right) \times \mathbb{R} \times
\mathbb{R} \times \left((-\infty, 0) \cup (0, \infty) \right)
}
$$

or, focusing solely on x_1 and x_4:

$$
\boxed{
\left((-\infty, 0) \cup (0, \infty) \right) \times \left((-\infty, 0) \cup
(0, \infty) \right)
}
$$

which indicates that F is differentiable for all points where x_1 and x_4 are non-zero, regardless of the values of x_2 and x_3.

Conclusion

In conclusion, the differentiability of the function $F(x) = |x_1| \cdot |x_4|$ hinges on the behavior of the absolute value functions involved. The points of non-differentiability are precisely where either x_1 or x_4 equals zero. Using interval notation, the set of points where F is differentiable can be precisely characterized as:

$$
\boxed{
\left((-\infty, 0) \cup (0, \infty) \right) \times \mathbb{R} \times
\mathbb{R} \times \left((-\infty, 0) \cup (0, \infty) \right)
}
$$

This analysis underscores the importance of understanding the subtleties of absolute value functions within multivariable calculus and provides a clear framework for identifying differentiability regions in higher dimensions.

Frequently Asked Questions

What is the function F(x) in the given problem?

$F(x) = |x_1|\ |x_4|$, where x_1 and x_4 are components of the vector x.

How do absolute value functions affect differentiability?

Absolute value functions are not differentiable at points where their arguments are zero, due to a cusp or sharp corner at that point.

At which points is the function F(x) not differentiable?

F(x) is not differentiable where either $x_1 = 0$ or $x_4 = 0$, because the absolute value functions are non-differentiable at zero.

How do you express the differentiability interval for F(x)?

The function is differentiable on the set of all x where $x_1 \neq 0$ and $x_4 \neq 0$.

What is the interval notation for the set where F(x) is differentiable?

F(x) is differentiable on the set $\{x \mid x_1 \neq 0 \text{ and } x_4 \neq 0\}$.

Is F(x) differentiable at points where x1 ≠ 0 but x4 = 0?

No, F(x) is not differentiable at points where $x_4 = 0$, regardless of x_1, because the absolute value function is not differentiable at zero.

Is the function F(x) differentiable at points where both x1 ≠ 0 and x4 ≠ 0?

Yes, at points where both x_1 and x_4 are non-zero, F(x) is differentiable.

How does the product of absolute value functions influence differentiability?

The product is differentiable wherever both functions are differentiable; hence, where neither x_1 nor x_4 is zero.

Can F(x) be differentiable at points where x1 = 0 or x4 = 0?

No, because absolute value functions are not differentiable at zero, so the product also fails to be differentiable there.

Summarize the differentiability of F(x) using interval notation.

F(x) is differentiable on {x | x1 ≠ 0 and x4 ≠ 0}.

Additional Resources

Understanding the Differentiability of the Function $F(x) = |x_1| \cdot |x_4|$

When analyzing the differentiability of functions involving absolute value expressions, such as $F(x) = |x_1| \cdot |x_4|$, it’s crucial to understand how the absolute value function behaves and how it influences the smoothness of the overall function. This review aims to explore the differentiability of this specific function thoroughly, using interval notation to delineate the regions where F is differentiable.

Introduction to the Function $F(x) = |x_1| \cdot |x_4|$

The function $F: \mathbb{R}^n \to \mathbb{R}$ is given by:

$$
F(x) = |x_1| \cdot |x_4|
$$

where:

- $x = (x_1, x_2, x_3, x_4, \dots, x_n)$, an n-dimensional vector.
- Only the first and fourth components, x_1 and x_4, impact the value of F; the other components do not affect the function directly.

The focus of this analysis is to determine the set of points x in $\mathbb{R}^n$ where F is differentiable, explicitly using interval notation for the relevant variables.

Fundamental Concepts Involved

Absolute Value Function and Its Differentiability

The absolute value function:

$$
|x| = \begin{cases}
x, & \text{if } x \geq 0 \\
-x, & \text{if } x < 0
\end{cases}
$$

has well-known properties:

- It is continuous everywhere.
- It is differentiable everywhere except at $x = 0$, where the derivative does not exist because of a "corner" or "sharp point" in the graph.

Specifically:

$$
\frac{d}{dx}|x| = \begin{cases}
1, & x > 0 \\
-1, & x < 0
\end{cases}
$$

and not defined at $x=0$.

Product of Absolute Values

Since $F(x)$ involves the product of two absolute values, it inherits properties related to their individual behavior:

$$
F(x) = |x_1| \cdot |x_4|
$$

- F is continuous everywhere because absolute value functions are continuous.
- Differentiability depends on the differentiability of each absolute value component, especially at points where either $x_1=0$ or $x_4=0$.

Analyzing Differentiability of $F(x)$

Step 1: Identify Critical Points Where Differentiability Might Fail

The function F could fail to be differentiable at points where:

- $x_1=0$
- $x_4=0$

since at these points, the absolute value functions are non-differentiable.

Outside these points:

- When $x_1 \neq 0$ and $x_4 \neq 0$, both $|x_1|$ and $|x_4|$ are differentiable functions.
- The product of differentiable functions is differentiable, so F is differentiable except possibly at the points where either $x_1=0$ or $x_4=0$.

Step 2: Formalize the Domain of Differentiability

The set of points where F is differentiable is:

$$
\{ x \in \mathbb{R}^n \mid x_1 \neq 0 \text{ and } x_4 \neq 0 \}
$$

Expressed in interval notation for the relevant variables, the domain of differentiability with respect to x_1 and x_4 is:

$$
(-\infty, 0) \cup (0, \infty)
$$

for each variable separately.

Explicit Characterization Using Interval Notation

Given the above, the set where F is differentiable can be expressed as:

$$
\left\{ x \in \mathbb{R}^n \mid x_1 \in (-\infty, 0) \cup (0, \infty), \quad x_4 \in (-\infty, 0) \cup (0, \infty) \right\}
$$

Since the other components $x_2, x_3, \dots, x_n$ do not influence the differentiability of F, they are unrestricted and span the entire $\mathbb{R}$.

In interval notation:

$$
\boxed{
\left\{ x \in \mathbb{R}^n \mid x_1 \in (-\infty, 0) \cup (0, \infty), \quad x_4 \in (-\infty, 0) \cup (0, \infty), \quad x_2, x_3, \dots, x_n \in \mathbb{R} \right\}
}
$$

This explicit description indicates that F is differentiable everywhere except along the hyperplanes:

- $x_1=0$
- $x_4=0$

which are the "singular" sets where the absolute value functions have corners.

Calculating the Gradient of F When Differentiable

Understanding the gradient of F gives insight into its differentiability properties.

For x where $x_1 \neq 0$ and $x_4 \neq 0$, the derivatives are:

$$
\frac{\partial F}{\partial x_1} = \frac{d}{dx_1} |x_1| \cdot |x_4| = \operatorname{sgn}(x_1) \cdot |x_4|
$$
$$
\frac{\partial F}{\partial x_4} = |x_1| \cdot \frac{d}{dx_4} |x_4| = |x_1| \cdot \operatorname{sgn}(x_4)
$$
$$
\frac{\partial F}{\partial x_j} = 0, \quad \text{for } j=2,3,\dots,n

\]

where:

$$
\operatorname{sgn}(x) = \begin{cases}
1, & x > 0 \\
-1, & x < 0
\end{cases}
$$

This confirms that the function is smooth (differentiable) where both x_1 and x_4 are non-zero.

Deeper Insights: Behavior at the Boundaries

What Happens at $x_1=0$ or $x_4=0$?

At these hyperplanes:

- The absolute value functions are non-differentiable.
- F has a "corner" or "kink" similar to the absolute value function at zero.
- The directional derivatives exist from certain directions but the total derivative (gradient) does not.

For example, consider fixing all variables except x_1:

$$
F(x) = |x_1| \cdot |x_4|
$$

- At $x_1=0$, the partial derivative with respect to x_1 does not exist because $\operatorname{sgn}(x_1)$ is undefined at zero.
- Similarly for $x_4=0$.

This leads to the conclusion that F is not differentiable at points where either $x_1=0$ or $x_4=0$, regardless of the values of the other variables.

Summary and Final Characterization

```
The differentiability set of \(F(x) = |x_1| \cdot |x_4|\) can be summarized
as:

\[
\boxed{
\left\{ x \in \mathbb{R}^n \mid x_1 \neq 0, \quad x_4 \neq 0, \quad x_2, x_3,
\dots, x_n \in \mathbb{R} \right\}
}
\]

This set includes all vectors in \(\mathbb{R}^n\) where the first and fourth
components are strictly non-zero, and the remaining components are
unrestricted.

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```

Implications and Practical Applications

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Understanding where \(F\) is differentiable is essential in various contexts,
including:

- Optimization Problems: When minimizing or maximizing \(F\),
differentiability determines the applicability of gradient-based methods.
- Sensitivity Analysis: Knowing the regions where the derivative exists helps
in understanding how small changes in \(x\) affect \(F\).
- Numerical Methods: Algorithms
```

1 Point Let F X X1 X 4 Use Interval Notation To Indicate The Values Of X Where F Is Differentiable

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