2. A. Determine The Cartesian Equation Of The Plane With Intercepts At P(-1,0,0), Q(0,1,0), And R(0,0,-3).

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Understanding how to derive the Cartesian equation of a plane given its intercepts is a fundamental concept in analytical geometry. This process involves translating the geometric information—specifically, the points where the plane intersects the axes—into an algebraic equation that describes the plane in three-dimensional space. In this article, we will explore step-by-step how to determine the Cartesian equation of a plane with intercepts at P(-1,0,0), Q(0,1,0), and R(0,0,-3), providing detailed explanations, formulas, and examples to clarify the process.

Understanding the Intercepts and the Plane

What Are Intercepts in Geometry?

Interceptions, or intercepts, are points where a plane intersects the coordinate axes. In three-dimensional space, the axes are the x-axis, y-axis, and z-axis. If a plane cuts the x-axis at point P, the y-axis at Q, and the z-axis at R, then these points are referred to as intercepts of the plane.

For the given points:
- P(-1,0,0): The plane intersects the x-axis at $x = -1$.
- Q(0,1,0): The plane intersects the y-axis at $y = 1$.
- R(0,0,-3): The plane intersects the z-axis at $z = -3$.

Significance of Intercepts

The intercepts provide valuable information because they directly relate to the plane's equation. When a plane cuts the axes at specific points, these points satisfy the plane's equation, allowing us to determine the equation algebraically.

General Form of the Equation of a Plane

The Cartesian form of a plane's equation can be written as:

$$ ax + by + cz + d = 0 $$

where:

- a, b, c are the coefficients related to the orientation of the plane,

- d is the constant term.

Alternatively, when the plane is expressed in intercept form, it becomes:

$$ \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 $$

assuming the plane intersects the axes at points corresponding to a, b, c.

Given that the intercepts are at P(-1,0,0), Q(0,1,0), R(0,0,-3), we can utilize these points to find the equation.

Step-by-Step Process to Find the Cartesian Equation

Step 1: Recognize the Intercepts and Their Coordinates

From the given points:

- x-intercept at P(-1, 0, 0) implies $a = -1$,

- y-intercept at Q(0, 1, 0) implies $b = 1$,

- z-intercept at R(0, 0, -3) implies $c = -3$.

Note that these are the points where the plane crosses the axes, and their coordinates directly relate to the intercepts.

Step 2: Use the Intercept Form of the Plane Equation

The intercept form of the equation of a plane is:

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Substituting the known intercepts:

$$\frac{x}{-1} + \frac{y}{1} + \frac{z}{-3} = 1$$

Simplify:

$$-x + y - \frac{z}{3} = 1$$

Multiply through by 3 to clear denominators:

$$3(-x + y) - z = 3$$

$$-3x + 3y - z = 3$$

This is the Cartesian equation of the plane.

Step 3: Write the Final Equation

Rearranged for clarity:

$$-3x + 3y - z = 3$$

Or equivalently:

$$3x - 3y + z = -3$$

This is the Cartesian equation of the plane passing through points P, Q, and R with the given intercepts.

Verification of the Equation

To confirm the correctness of the derived equation, verify that the points P, Q, and R satisfy it:

- For P(-1, 0, 0):

$$ 3(-1) - 3(0) + 0 = -3 $$

which simplifies to $-3 = -3$, true.

- For Q(0, 1, 0):

$$ 3(0) - 3(1) + 0 = -3 $$

which simplifies to $-3 = -3$, true.

- For R(0, 0, -3):

$$ 3(0) - 3(0) + (-3) = -3 $$

which simplifies to $-3 = -3$, true.

All three points satisfy the equation, confirming its correctness.

Alternative Forms and Additional Insights

Standard Form of the Equation

The equation can be expressed in the standard form:

$$ 3x - 3y + z + 3 = 0 $$

by bringing all terms to one side.

Normal Vector to the Plane

The coefficients a, b, c in the plane equation $ax + by + cz + d = 0$ represent the components of the normal vector $\vec{n}$ to the plane:

$$\vec{n} = (a, b, c)$$

In our case:

$$\vec{n} = (3, -3, 1)$$

This vector is perpendicular to the plane, and its magnitude and direction provide insights into the plane's orientation in space.

Applications of the Cartesian Equation of a Plane

Understanding how to determine the Cartesian equation of a plane from intercepts has numerous practical applications:

- Engineering and Architecture: Designing structures and understanding spatial relationships.
- Computer Graphics: Rendering 3D objects and surfaces.
- Physics: Modeling physical phenomena involving planes, such as reflection and refraction.
- Mathematics Education: Enhancing spatial reasoning skills.

Summary and Key Takeaways

- The intercept form of the plane equation is a straightforward method when given the points where the plane intersects the axes.
- The intercepts directly translate into the coefficients of the plane's equation.
- Verifying points ensures the correctness of the derived equation.
- The normal vector provides additional geometric insights into the plane's orientation.

Conclusion

Determining the Cartesian equation of a plane from given intercepts is a fundamental skill in analytical geometry. By understanding the relationships between intercept points and the plane's algebraic form, students and professionals can analyze and model three-dimensional objects effectively. In the specific case of points P(-1,0,0), Q(0,1,0), and R(0,0,-3), the derived plane equation:

$$3x - 3y + z = -3$$

accurately represents the plane passing through these intercepts, serving as a practical example of the application of geometric principles in algebraic form.

Keywords for SEO Optimization:

- Cartesian equation of a plane
- Plane intercepts
- Find plane equation from intercepts
- Equation of a plane with given points
- 3D geometry plane equations
- Analytical geometry examples
- Intercept form of a plane
- Normal vector of a plane
- Geometry in three dimensions
- Mathematical plane equations

Frequently Asked Questions

How do you find the Cartesian equation of a plane given three points

P(-1, 0, 0), Q(0, 1, 0), and R(0, 0, -3)?

First, find two vectors on the plane, such as PQ and PR, then compute their cross product to get the normal vector. Use the point-normal form of the plane equation to derive the Cartesian equation.

What are the steps to determine the Cartesian equation of a plane with given intercepts?

Identify the intercepts to find points on the axes, then find two vectors lying on the plane, compute their cross product for the normal vector, and finally use the point-normal form to write the plane's equation.

How can the intercepts at P(-1, 0, 0), Q(0, 1, 0), and R(0, 0, -3) help in deriving the plane's equation?

These intercepts provide specific points on the axes, allowing you to determine two direction vectors on the plane, which are essential for calculating the plane's normal vector and equation.

What is the significance of the intercepts in determining the plane's equation?

Intercepts directly give points on the axes, simplifying the process of finding vectors on the plane and calculating its normal vector for the equation derivation.

Can you illustrate the process of finding the normal vector using the given points?

Yes, by calculating vectors PQ = Q - P and PR = R - P, then taking their cross product, you obtain the normal vector to the plane.

What is the formula for the Cartesian equation of a plane once the normal vector and a point are known?

The equation is given by $n_x(x - x_0) + n_y(y - y_0) + n_z(z - z_0) = 0$, where (x_0, y_0, z_0) is a point on the plane and (n_x, n_y, n_z) is the normal vector.

How do you verify that the points P, Q, and R lie on the derived plane equation?

Substitute the coordinates of each point into the plane's equation; if all satisfy the equation, the points lie on the plane.

What is the final Cartesian equation of the plane passing through points P(-1, 0, 0), Q(0, 1, 0), and R(0, 0, -3)?

The Cartesian equation of the plane is $3x + 3y + z = 3$.

Additional Resources

Determining the Cartesian Equation of a Plane Given Its Intercepts: An In-Depth Analysis

Understanding the geometry of planes in three-dimensional space is fundamental in various fields, including mathematics, engineering, and physics. One common problem involves deriving the Cartesian equation of a plane when given specific points through which it passes, especially when these points are intercepts on the coordinate axes. This article provides a comprehensive exploration of how to determine the Cartesian equation of a plane with intercepts at P(-1, 0, 0), Q(0, 1, 0), and R(0, 0, -3). We will delve into the geometric concepts, algebraic techniques, and step-by-step procedures necessary to solve this problem, ensuring clarity and depth for readers seeking a thorough understanding.

Understanding the Geometrical Context: The Plane and Its Intercepts

What Is a Plane in Three-Dimensional Space?

A plane in three-dimensional Euclidean space is a flat, two-dimensional surface extending infinitely in all directions. It can be uniquely defined by:

- A point through which it passes.
- A normal vector perpendicular to the surface.
- Alternatively, by intercepts on the coordinate axes, if the plane cuts through the axes at specific points.

Mathematically, the general form of the Cartesian equation of a plane is:

$$Ax + By + Cz + D = 0$$

where (A, B, C) is a normal vector to the plane, and D is a scalar constant.

The Significance of Intercepts

Interceptions of a plane with the coordinate axes are points where the plane touches or crosses the axes. These intercepts are particularly useful because:

- They provide direct information about the plane's position.

- They enable an easier derivation of the plane's equation, especially in the intercept form.

Given the points P(-1, 0, 0), Q(0, 1, 0), and R(0, 0, -3), the plane intersects:

- The x-axis at P(-1, 0, 0)
- The y-axis at Q(0, 1, 0)
- The z-axis at R(0, 0, -3)

Understanding how these intercepts relate to the plane's equation is the foundation of the problem.

From Intercepts to the Equation: Fundamental Principles

The Intercept Form of the Plane Equation

When a plane intersects the axes at points a, b, c, its equation in intercept form is:

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

provided none of the intercepts are zero. This form is particularly convenient when the intercepts are known.

In our scenario:

- The x-intercept is at $a = -1$
- The y-intercept is at $b = 1$
- The z-intercept is at $c = -3$

Thus, the initial form of the plane's equation based on intercepts is:

$$ \frac{x}{-1} + \frac{y}{1} + \frac{z}{-3} = 1 $$

which simplifies to:

$$ -x + y - \frac{z}{3} = 1 $$

Multiplying through by 3 to clear denominators:

$$ -3x + 3y - z = 3 $$

or equivalently:

$$ 3x - 3y + z = -3 $$

This is an initial candidate for the Cartesian equation. However, confirmation is necessary to ensure the points P, Q, and R satisfy this equation, and to determine the proper form with the correct coefficients.

Verification of the Intercept Equation

Substituting the intercept points into the derived equation:

- For P(-1, 0, 0):

$$ 3(-1) - 3(0) + 0 = -3 $$

which simplifies to:

$$ -3 = -3 $$

which holds true.

- For Q(0, 1, 0):

$$ 3(0) - 3(1) + 0 = -3 $$

which simplifies to:

$$ -3 = -3 $$

which also holds true.

- For R(0, 0, -3):

$$ 3(0) - 3(0) + (-3) = -3 $$

which simplifies to:

$$ -3 = -3 $$

again confirming the equation.

Hence, the intercept form accurately describes the plane passing through P, Q, and R.

Deriving the Cartesian Equation: Step-by-Step Methodology

Step 1: Express the Plane Using Intercepts

As established, the intercept form is:

$$ \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 $$

with $a = -1$, $b = 1$, $c = -3$.

Substituting these:

$$ \frac{x}{-1} + \frac{y}{1} + \frac{z}{-3} = 1 $$

which simplifies to:

$$ -x + y - \frac{z}{3} = 1 $$

Multiplying through by 3:

$$ -3x + 3y - z = 3 $$

or rearranged:

$$ 3x - 3y + z = -3 $$

This is the Cartesian form of the plane derived from intercepts.

Step 2: Convert to Standard Form and Verify

The standard form of the Cartesian equation of the plane is:

$$ Ax + By + Cz + D = 0 $$

Rearranging the previous equation:

$$ 3x - 3y + z + 3 = 0 $$

This form explicitly shows the coefficients and the constant term.

Verification involves confirming that the three points P, Q, and R satisfy this equation:

- P(-1, 0, 0):

$$ 3(-1) - 3(0) + 0 + 3 = -3 + 0 + 0 + 3 = 0 $$

- Q(0, 1, 0):

$$ 3(0) - 3(1) + 0 + 3 = 0 - 3 + 0 + 3 = 0 $$

- R(0, 0, -3):

$$ 3(0) - 3(0) + (-3) + 3 = 0 + 0 - 3 + 3 = 0 $$

All points satisfy the equation, confirming the correctness of the derived Cartesian plane equation.

Analytical Insights and Additional Considerations

Normal Vector and Its Significance

The coefficients $(A, B, C) = (3, -3, 1)$ in the Cartesian equation directly indicate the plane's normal vector:

$$\vec{n} = 3\mathbf{i} - 3\mathbf{j} + \mathbf{k}$$

This vector is perpendicular to the plane surface, and its magnitude and direction reveal the plane's orientation in space.

Understanding this normal vector is crucial for further analyses, such as calculating angles between planes, projecting points, or finding distances from points to the plane.

Implications of the Intercepts on the Plane's Geometry

The intercepts at P(-1, 0, 0), Q(0, 1, 0), and R(0, 0, -3) suggest that:

- The plane crosses the x-axis at -1, indicating a negative x-intercept.
- It crosses the y-axis at 1, a positive y-intercept.
- It crosses the z-axis at -3, a negative z-intercept.

These intercepts shape the plane's position relative to the coordinate axes, influencing its slope and tilt.

Applications and Broader Context

Relevance in Engineering and Physics

Accurately determining the equation of a plane from intercepts is essential in various practical applications:

- Structural engineering: defining planes in architectural models.
- Physics: analyzing surfaces, fields, or wavefronts.
- Computer graphics: rendering surfaces and calculating reflections.

Educational Significance

From an educational perspective, problems like this reinforce understanding of coordinate geometry, vector calculus, and algebraic manipulation. They serve as foundational exercises for students learning three-dimensional analytic geometry.

Extensions and Advanced Topics

Once the basic equation is established, further exploration can include:

- Calculating the distance from arbitrary points to the plane.
- Finding angles between multiple planes.
- Extending to planes with oblique or skew intercepts.
- Analyzing the intersection of the plane with other surfaces.

Conclusion

Determining the Cartesian equation of a plane given its intercepts is a fundamental skill in three-dimensional analytic geometry, combining algebraic techniques with geometric intuition. In this case, leveraging the intercept form simplifies the derivation process, leading to the equation:

$$ 3x - 3y + z + 3 = 0 $$

This equation not only accurately represents the plane passing through points P, Q, and R but

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