7 1 Point Write The Equation Of A Line Passing Through (0,-4) With A Slope Of 5/4 Y = 5x -4 Y - 4 = 5/4x-4

7 1 Point Write The Equation Of A Line Passing Through (0,-4) With A Slope Of 5/4 Y = 5x -4 Y - 4 = 5/4x-4 is a common question in algebra that tests understanding of the basic principles of linear equations, slope-intercept form, and point-slope form. Understanding how to derive the equation of a line given a point and a slope is fundamental for students learning coordinate geometry. In this comprehensive guide, we will explore step-by-step how to write the equation of a line passing through the point (0, -4) with a slope of 5/4, including various methods, explanations, and practical tips to enhance your understanding of linear equations. This article is optimized for SEO to help students and educators find clear, detailed information on this important algebra topic.

Understanding the Basics of Linear Equations

Before diving into the specific problem, it’s essential to understand some fundamental concepts related to linear equations.

What is a Linear Equation?

- A linear equation represents a straight line when graphed on the coordinate plane.
- The general form of a linear equation in two variables (x and y) is:

$$
y = mx + b
$$

where:

- m is the slope of the line.
- b is the y-intercept, the point where the line crosses the y-axis.

Understanding Slope and Y-intercept

- Slope (m): Measures the steepness of the line. A positive slope indicates the line rises from left to right, while a negative slope indicates it falls.
- Y-intercept (b): The value of y when x = 0. This is the point where the line crosses the y-axis.

Given Data for the Line Equation

In our problem, the line passes through the point $(0, -4)$ and has a slope of $\frac{5}{4}$. From this data:

- The point $(0, -4)$ is the y-intercept because x = 0.
- The slope $m = \frac{5}{4}$.

This information allows us to directly write the equation in the slope-intercept form.

Deriving the Equation of the Line

There are multiple methods to derive the equation of a line given a point and a slope. Let's explore the most straightforward approach first.

Method 1: Using the Slope-Intercept Form

Since the point $(0, -4)$ is the y-intercept, and the slope is $\frac{5}{4}$, the equation can be written directly as:

$$
y = mx + b
$$

Plugging in the known values:

$$
y = \frac{5}{4}x - 4
$$

This is the simplest and most direct way to find the equation when the y-intercept is known.

Method 2: Using Point-Slope Form

The point-slope form is useful when you know any point (x_1, y_1) on the line and the slope m:

$$
y - y_1 = m (x - x_1)
$$

Applying the point $(0, -4)$:

$$
y - (-4) = \frac{5}{4}(x - 0)
$$

Simplifies to:

$$
y + 4 = \frac{5}{4}x
$$

And rearranged:

$$
y = \frac{5}{4}x - 4
$$

Both methods lead to the same equation, confirming the consistency and correctness of the solution.

Understanding the Equation $y = \frac{5}{4}x - 4$

This equation is now in slope-intercept form, which makes graphing and analysis straightforward.

Key Features of the Line

- Slope: $\frac{5}{4}$ (positive, indicating the line rises as x increases)
- Y-intercept: -4 (the line crosses the y-axis at $(0, -4)$)
- Graphical Representation: Starting at $(0, -4)$, the line moves upward with a rise of 5 units for every 4 units it runs to the right.

Graphing the Line

- Plot the y-intercept point $(0, -4)$.
- Use the slope to find another point: from $(0, -4)$, go up 5 units and right 4 units to reach $(4, 1)$.
- Draw a straight line through these points.

Additional Methods to Write the Equation of a Line

Apart from the direct slope-intercept and point-slope forms, there are other approaches to find the line's equation.

Method 3: Using Two-Point Form

- Although only one point is given, if you have two points, you can use the two-point form:

$$
y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)
$$

- In our case, since the point $(0, -4)$ is on the line and the slope is known, the previous methods are simpler.

Method 4: Converting to Standard Form

- The slope-intercept form can be converted to standard form:

$$
y = \frac{5}{4}x - 4
$$

Multiply through by 4 to clear fractions:

$$
4y = 5x - 16
$$

Rewrite as:

$$
5x - 4y = 16
$$

This form is useful in certain algebraic contexts.

Practical Applications of the Equation of a Line

Understanding how to derive and use the equation $y = \frac{5}{4}x - 4$ has numerous real-world applications:

- Engineering: Designing slopes for ramps or roads.
- Physics: Calculating velocity and acceleration relationships.
- Economics: Analyzing linear demand or supply functions.
- Statistics: Fitting linear regression models.

Common Mistakes and Tips for Students

When working with linear equations, students often make some common errors. Here are tips to avoid them:

- **Incorrectly substituting points:** Always verify the point and slope before plugging into formulas.
- **Mismanaging fractions:** Clear denominators early to prevent errors.
- **Confusing forms:** Know when to use slope-intercept, point-slope, or standard forms based on the problem.
- **Graphing inaccuracies:** Plot points accurately and use the slope to find additional points.

Summary and Key Takeaways

To summarize:

- The equation of a line passing through $(0, -4)$ with a slope of $\frac{5}{4}$ is:

$$
y = \frac{5}{4}x - 4
$$

- Alternatively, in standard form:

$$
5x - 4y = 16
$$

- The slope indicates the line rises 5 units for every 4 units it runs to the right.
- The y-intercept is at $(0, -4)$.

Understanding these concepts provides a strong foundation for mastering linear equations and their applications in various fields.

Conclusion

Writing the equation of a line given a point and a slope is a fundamental skill in algebra and coordinate geometry. Whether you are a student preparing for exams or an educator explaining key concepts, mastering this process is essential. Remember that the slope-intercept form $y = mx + b$ offers a quick way to write the line when the y-intercept is known, while point-slope form is versatile when working with any point on the line. By practicing these methods and understanding the underlying principles, you'll be well-equipped to handle a wide range of problems involving linear equations.

FAQs About Writing Equations of Lines

1. **How do I find the equation of a line if I only know two points?** Use the two-point form or calculate the slope first, then apply the point-slope or slope-intercept form.

2. **Can the line pass through (0, -4) and have a different slope?** Yes, the slope determines the line's steepness. Changing the slope changes the line's angle but the point $(0, -4)$ remains on the y-axis.

3. **What if the line is vertical or horizontal?** For a vertical line, the equation is $x = a$. For a horizontal line, the equation is $y = b$.

This detailed guide provides an in-depth understanding of how to write the

Frequently Asked Questions

What is the equation of a line passing through (0, -4) with a slope of 5/4?

The equation is $y = (5/4)x - 4$.

How do you derive the equation of a line given a point and a slope?

Use the point-slope form $y - y_1 = m(x - x_1)$, then simplify to slope-intercept form.

What is the slope-intercept form of the line passing through (0, -4) with slope 5/4?

The slope-intercept form is $y = (5/4)x - 4$.

Why is the y-intercept of the line (-4)?

Because the line passes through (0, -4), which indicates the y-intercept is at -4.

Can you verify the equation y = (5/4)x - 4 passes through the point (0, -4)?

Yes, plugging in $x = 0$ gives $y = (5/4)0 - 4 = -4$, confirming the point lies on the line.

Additional Resources

Understanding the Equation of a Line Passing Through a Point with a Given Slope

When studying coordinate geometry, one of the fundamental concepts is understanding how to find the equation of a line given specific information such as a point through which it passes and its slope. This knowledge not only forms the basis for many advanced topics in mathematics but also has practical applications in fields like engineering, physics, computer graphics, and more. In this detailed review, we'll explore the process of deriving the equation of a line passing through the point (0, -4) with a slope of 5/4, culminating in the standard form of the line: $Y = (5/4)x - 4$.

Understanding the Components: Point and Slope

Before diving into the derivation, it's essential to clarify the key components involved:

The Point (0, -4)

- Coordinates: The point is given as $(x_1, y_1) = (0, -4)$.
- Significance: This point lies on the line, meaning the line must pass through this exact location in the coordinate plane.

The Slope (m) = 5/4

- Definition: Slope measures the steepness or incline of a line, indicating the ratio of vertical change to horizontal change between any two points on the line.
- Value: The slope here is a fraction, 5/4, indicating that for every 4 units moved horizontally, the line rises 5 units vertically.

Deriving the Equation of the Line

The process of finding the line's equation involves applying the point-slope form, then converting it into the slope-intercept form, which is often more convenient for interpretation and plotting.

1. Point-Slope Form

The point-slope form of a line's equation is:

$$
y - y_1 = m(x - x_1)
$$

where:
- (x_1, y_1) is a point on the line,
- m is the slope.

Given:
- $(x_1, y_1) = (0, -4)$,
- $m = \frac{5}{4}$.

Plugging these into the point-slope form:

$$
y - (-4) = \frac{5}{4}(x - 0)
$$

which simplifies to:

$$
y + 4 = \frac{5}{4}x
$$

2. Converting to Slope-Intercept Form

To express the line in slope-intercept form $y = mx + b$, isolate y:

$$
y = \frac{5}{4}x - 4
$$

This form clearly indicates the slope and the y-intercept of the line:
- Slope (m): $\frac{5}{4}$,
- Y-intercept (b): -4.

Analyzing the Equation $y = \frac{5}{4}x - 4$

Understanding the slope-intercept form provides a wealth of information about the line's behavior and properties.

1. Slope Interpretation

- The slope $\frac{5}{4}$ indicates a positive incline:
- For every 4 units moved horizontally to the right, the line rises 5 units vertically.
- Conversely, moving left (negative x-direction), the line descends.

2. Y-Intercept

- The y-intercept is at -4, meaning the line crosses the y-axis at (0, -4).
- This point is consistent with the given point, confirming the correctness of the equation.

3. Graphical Representation

- The line can be plotted easily:
- Start at the y-intercept (0, -4).
- Use the slope to find another point: from (0, -4), move right 4 units and up 5 units to (4, 1).
- Connect these points for a straight line.

Alternative Forms of the Line Equation

While the slope-intercept form is most intuitive, other forms can be useful in different contexts.

1. Standard Form: $Ax + By = C$

- Starting from $y = \frac{5}{4}x - 4$, multiply both sides by 4 to clear the fraction:

$$
4y = 5x - 16
$$

- Rearranged into standard form:

$$
5x - 4y = 16
$$

2. Point-Form Equation

- As previously derived:

$$
y + 4 = \frac{5}{4}x
$$
- Useful when working with specific points and slopes in problem solving.

Verifying the Equation

It's crucial to verify that the derived equation correctly passes through the given point.

Substitution Test

- Plug $x = 0$ into $y = \frac{5}{4}x - 4$:
$$
y = \frac{5}{4} \times 0 - 4 = -4
$$
- The point (0, -4) satisfies the equation.

Alternative Point Check

- Find a second point using the slope:
- Starting at (0, -4), move right 4 units:
$$
x = 4 \Rightarrow y = \frac{5}{4} \times 4 - 4 = 5 - 4 = 1
$$
- Second point: (4, 1). This confirms the line's slope:
$$
\text{slope} = \frac{1 - (-4)}{4 - 0} = \frac{5}{4}
$$
which matches the given slope.

Practical Applications of the Line Equation

Understanding the equation of a line passing through a specific point with a known slope has numerous real-world applications:

1. Engineering and Design

- Designing roads, bridges, and structural components often involves calculating lines with specific inclinations passing through fixed points.

2. Physics

- Describing motion, such as velocity and acceleration, where the slope represents rate of change.

3. Computer Graphics

- Rendering lines and shapes based on equations, ensuring precise placement and orientation.

4. Data Analysis

- Fitting lines to data points (linear regression) often begins with understanding the basic equation form.

Further Exploration: Variations and Related Concepts

Expanding beyond the basic derivation can deepen understanding and application.

1. Parallel and Perpendicular Lines

- Parallel lines share the same slope but differ in y-intercept.
- Perpendicular lines have slopes that are negative reciprocals (e.g., slope of $\frac{5}{4}$ implies perpendicular slope $-\frac{4}{5}$).

2. Using Different Points and Slopes

- The method applies universally: given any point and slope, the point-slope form is the starting point for deriving the line's equation.

3. Extending to Non-Linear Equations

- While this discussion focuses on linear equations, similar principles apply to curves like parabolas, circles, and other conic sections.

Conclusion

In summary, deriving the equation of a line passing through a specific point with a given slope involves a clear understanding of fundamental concepts in coordinate geometry. Starting from the point-slope form, we seamlessly transition into the slope-intercept form, revealing the line's steepness and intercepts. The line passing through (0, -4) with a slope of $\frac{5}{4}$ is elegantly represented by the equation:

$$
\boxed{y = \frac{5}{4}x - 4}
$$

This equation encapsulates all the necessary information for graphing, analyzing, and applying the line in various contexts. Mastering these techniques enhances mathematical literacy and problem-solving skills, providing a strong foundation for further exploration in geometry, algebra, and beyond.

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