

A Basket Contains 3 Oranges And 7 Apples. Two Pieces Of Fruit Are Selected At Random, With Replacement. Find

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Introduction

Imagine a simple basket filled with a mixture of oranges and apples, specifically containing 3 oranges and 7 apples. Suppose you randomly select two pieces of fruit from this basket, with the process involving replacement — meaning after each selection, the chosen piece is put back into the basket, keeping the total number of fruits constant. This scenario, although straightforward, introduces interesting probability concepts that are fundamental in understanding random selection, probability calculations, and the principles of sampling with replacement.

Whether you're a student preparing for exams, a teacher designing engaging math problems, or just a curious mind exploring probability theory, understanding how to analyze such problems is essential. This article aims to provide a comprehensive, detailed explanation of how to approach and solve this problem, including relevant formulas, step-by-step calculations, and real-world applications.

Understanding the Problem Statement

Before diving into calculations, it's important to clearly understand what the problem entails. The key elements are:

- The basket contains 3 oranges and 7 apples.
- The total number of fruits in the basket is 10.
- Two pieces of fruit are selected at random.
- The selection is with replacement, meaning:
 - After the first selection, the fruit is put back.
- The second selection is made from the same original composition.

The question typically asks for probabilities associated with different outcomes, such as:

- The probability that both selected fruits are oranges.
- The probability that at least one apple is selected.
- The probability that exactly one orange and one apple are selected.
- The probability of selecting two apples, etc.

Depending on the specific query, various probability concepts come into play.

Fundamental Concepts in Probability

Sample Space and Events

- Sample Space (S): All possible outcomes of the experiment.
- Event (E): A specific outcome or set of outcomes within the sample space.

Probability of an Event

$$P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

Sampling with Replacement

In this context, sampling with replacement ensures that the probability of selecting an orange or an apple remains the same for each draw because the composition of the basket remains unchanged after each pick.

Step-by-Step Solution Approach

Step 1: Determine Probabilities of Individual Selections

Since the process involves replacement, the probabilities for each draw remain constant:

- Probability of selecting an orange ($P(O)$):

$$P(O) = \frac{\text{Number of oranges}}{\text{Total fruits}} = \frac{3}{10}$$

- Probability of selecting an apple ($P(A)$):

$$P(A) = \frac{\text{Number of apples}}{\text{Total fruits}} = \frac{7}{10}$$

Step 2: Identify Possible Outcomes

When two pieces are selected with replacement, the possible outcomes are:

1. Orange - Orange (O, O)
2. Orange - Apple (O, A)
3. Apple - Orange (A, O)
4. Apple - Apple (A, A)

Step 3: Calculate Probabilities for Each Outcome

Because each draw is independent (due to replacement), the probability of each combined outcome is

the product of individual probabilities:

Outcome	Calculation	Probability
Both oranges (O, O)	$P(O) \times P(O)$	$\frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$
First orange, second apple (O, A)	$P(O) \times P(A)$	$\frac{3}{10} \times \frac{7}{10} = \frac{21}{100}$
First apple, second orange (A, O)	$P(A) \times P(O)$	$\frac{7}{10} \times \frac{3}{10} = \frac{21}{100}$
Both apples (A, A)	$P(A) \times P(A)$	$\frac{7}{10} \times \frac{7}{10} = \frac{49}{100}$

Calculating Specific Probabilities

This section explores various probability questions based on the above outcomes.

1. Probability that both selected fruits are oranges

$$P(\text{both oranges}) = P(O, O) = \frac{9}{100} = 0.09$$

Interpretation: There is a 9% chance that both selections will be oranges.

2. Probability that both selected fruits are apples

$$P(\text{both apples}) = P(A, A) = \frac{49}{100} = 0.49$$

Interpretation: There is a 49% chance that both selections will be apples.

3. Probability that at least one orange is selected

This can be approached in two ways:

- Direct calculation: Sum probabilities of outcomes with at least one orange.
- Complement approach: Subtract the probability of no oranges (both apples) from 1.

Using the complement:

$$P(\text{at least one orange}) = 1 - P(\text{both apples}) = 1 - \frac{49}{100} = \frac{51}{100} = 0.51$$

Interpretation: There's a 51% chance that at least one orange is selected in two picks.

4. Probability that exactly one orange and one apple in two selections

This includes two outcomes:

- First orange, second apple: $P(O, A) = \frac{21}{100}$
- First apple, second orange: $P(A, O) = \frac{21}{100}$

Adding these:

$$P(\text{exactly one orange and one apple}) = P(O, A) + P(A, O) = \frac{21}{100} + \frac{21}{100} = \frac{42}{100} = 0.42$$

Interpretation: There is a 42% chance that the two selected fruits will consist of one orange and one apple in any order.

5. Probability that both selected fruits are oranges

Already calculated as $\frac{9}{100}$.

6. Probability of not selecting any oranges (i.e., both apples)

Already calculated as $\frac{49}{100}$.

Advanced Concepts: Expected Value and Variance

While the problem primarily involves calculating probabilities, understanding expected values can deepen insights.

Expected Number of Oranges in Two Selections

Since each draw is independent with a probability $P(O) = \frac{3}{10}$, the expected number of oranges in two picks:

$$E(\text{number of oranges}) = 2 \times P(O) = 2 \times \frac{3}{10} = \frac{6}{10} = 0.6$$

Interpretation: On average, you can expect to select 0.6 oranges in two draws with replacement.

Variance of the Number of Oranges

The variance for the number of oranges in two independent Bernoulli trials:

$$\begin{aligned} \text{Var} &= n \times p \times (1 - p) = 2 \times \frac{3}{10} \times \frac{7}{10} = 2 \times \frac{21}{100} = \frac{42}{100} = 0.42 \end{aligned}$$

This provides insights into the variability in the number of oranges selected.

Real-World Applications of Such Probability Scenarios

Understanding probability in daily life and various fields is crucial. Here are some applications related to the problem:

- Quality Control: Random sampling of products (e.g., oranges and apples) to determine defect rates.
- Gaming and Gambling: Calculating the odds of certain outcomes in games involving random draws.
- Decision Making: Assessing risks when making choices involving uncertain outcomes.
- Education: Teaching probability concepts through simple, relatable examples.

Summary and Key Takeaways

- When selecting items with replacement, the probabilities for each draw remain constant.
- Probabilities of combined events can be calculated by multiplying individual probabilities due to independence.
- Complement rules are useful for calculating probabilities of "at least" events.
- Expected value and variance offer insights into the average outcomes and variability in repeated trials.
- Practical understanding of such probability problems aids in decision-making and risk assessment in various fields.

Final Thoughts

The problem of selecting two pieces of fruit from a basket containing oranges and apples, with replacement, encapsulates fundamental probability principles. Through systematic analysis—identifying individual probabilities, considering all possible outcomes, and applying probability rules—you can solve a wide range of similar problems. Mastery of these concepts not only enhances mathematical skills but also prepares you to analyze real-world scenarios involving randomness and uncertainty.

By understanding how to approach such problems, students, educators, and professionals can develop critical thinking skills applicable across diverse disciplines, from statistics and data science to economics and engineering.

Remember: The core idea in problems like this is recognizing independence, calculating individual probabilities, and applying the appropriate rules to find combined probabilities. Whether dealing with fruits,

Frequently Asked Questions

What is the probability of selecting two oranges in a row with replacement from a basket containing 3 oranges and 7 apples?

The probability of selecting an orange in one draw is $\frac{3}{10}$. Since replacement is involved, the probability of selecting two oranges consecutively is $(\frac{3}{10})(\frac{3}{10}) = \frac{9}{100}$.

What is the probability of selecting one orange and one apple in any order with replacement from the basket?

The probability of selecting an orange then an apple is $(\frac{3}{10})(\frac{7}{10}) = \frac{21}{100}$, and for an apple then an orange is also $\frac{21}{100}$. Therefore, the total probability is $\frac{21}{100} + \frac{21}{100} = \frac{42}{100}$ or $\frac{21}{50}$.

If two pieces of fruit are selected with replacement, what is the probability that both are apples?

The probability of selecting an apple in one draw is $\frac{7}{10}$. With replacement, the probability that both draws are apples is $(\frac{7}{10})(\frac{7}{10}) = \frac{49}{100}$.

What is the expected number of oranges selected in two draws with replacement?

The expected number of oranges in one draw is $\frac{3}{10}$. For two independent draws, the expected number is $2(\frac{3}{10}) = \frac{6}{10}$ or $\frac{3}{5}$.

What is the probability of selecting at least one orange in two draws with replacement?

The probability of selecting no oranges (both picks are apples) is $(\frac{7}{10})(\frac{7}{10}) = \frac{49}{100}$. Therefore, the probability of selecting at least one orange is $1 - \frac{49}{100} = \frac{51}{100}$.

Additional Resources

Fruit Selection Probability Analysis

When it comes to understanding probability in everyday scenarios, the simple act of selecting fruits from a basket can serve as an excellent illustrative example. Whether you're a student learning the fundamentals of probability or a curious individual exploring how chance works in real life, analyzing such problems provides valuable insights into statistical reasoning. In this article, we will delve into a specific problem: a basket containing 3 oranges and 7 apples, from which two pieces of fruit are selected with replacement. Our goal is to determine various probabilities associated with these selections, such as the likelihood of selecting certain fruits or combinations, while providing comprehensive explanations along the way.

Understanding the Basic Setup

Before we explore the probabilities, let's clarify the initial conditions and terminology:

The Composition of the Basket

- Number of Oranges: 3
- Number of Apples: 7
- Total Fruits: 10

The total number of fruits is the sum of oranges and apples: $3 + 7 = 10$.

The Nature of the Selection Process

- Selection Type: Random selection of two pieces of fruit.
- Replacement: After each pick, the fruit is put back into the basket, restoring the original composition.
- Implication of Replacement: The probability remains constant for each selection, since the basket's contents are unchanged after each pick.

Key Concepts in Probability for This Scenario

To analyze the problem thoroughly, we need to understand some fundamental probability concepts:

1. Sample Space

The total number of possible outcomes when selecting two fruits with replacement:

- For each pick, there are 10 options.
- Since picks are independent (due to replacement), total outcomes:

$$\text{Total outcomes} = 10 \times 10 = 100$$

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2. Event Probability

The probability of an event is calculated as:

$$P(\text{Event}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}}$$

Given the symmetry and independence of picks, probabilities for specific outcomes can be broken down accordingly.

Calculating Basic Probabilities

Let's examine the probabilities of selecting oranges and apples in a single pick, which will serve as building blocks for more complex calculations.

Probability of Selecting an Orange ($P(O)$)

$$P(O) = \frac{\text{Number of oranges}}{\text{Total fruits}} = \frac{3}{10} = 0.3$$

Probability of Selecting an Apple ($P(A)$)

$$P(A) = \frac{\text{Number of apples}}{\text{Total fruits}} = \frac{7}{10} = 0.7$$

Since each selection is independent with replacement, these probabilities stay the same for both picks.

Analyzing Outcomes for Two Picks

Now that we know individual probabilities, we can analyze various events involving two fruit selections.

1. Probability that both fruits are oranges

Since each pick is independent:

$$P(\text{both oranges}) = P(O) \times P(O) = \left(\frac{3}{10}\right)^2 = \frac{9}{100} = 0.09$$

2. Probability that both fruits are apples

Similarly:

$$P(\text{both apples}) = P(A) \times P(A) = \left(\frac{7}{10}\right)^2 = \frac{49}{100} = 0.49$$

3. Probability of selecting one orange and one apple in any order

Since order matters in the sequence of picks, there are two favorable sequences:

- Orange first, then apple (O-A)
- Apple first, then orange (A-O)

Calculating each:

$$P(O-A) = P(O) \times P(A) = \frac{3}{10} \times \frac{7}{10} = \frac{21}{100} = 0.21$$

$$P(A-O) = P(A) \times P(O) = \frac{7}{10} \times \frac{3}{10} = \frac{21}{100} = 0.21$$

Total probability for one orange and one apple in any order:

$$P(\text{one orange, one apple}) = P(O-A) + P(A-O) = 0.21 + 0.21 = 0.42$$

4. Probability that at least one orange is selected

This includes combinations where:

- Exactly one orange and one apple
- Both oranges

Complementary probability approach:

$$P(\text{at least one orange}) = 1 - P(\text{no oranges}) = 1 - P(\text{both apples})$$

$$P(\text{both apples}) = 0.49 \rightarrow P(\text{at least one orange}) = 1 - 0.49 = 0.51$$

Advanced Probabilistic Events

Beyond simple calculations, we can explore more nuanced probabilities.

1. Probability that the first fruit is an orange and the second is an apple

$$\begin{aligned} P(\text{first orange, second apple}) &= P(O) \times P(A) = \frac{3}{10} \times \frac{7}{10} = 0.21 \end{aligned}$$

2. Probability that the first fruit is an apple and the second is an orange

$$\begin{aligned} P(A \text{ then } O) &= P(A) \times P(O) = 0.21 \end{aligned}$$

3. Probability that both fruits are different

This includes the cases where one is orange and the other is apple, in either order:

$$\begin{aligned} P(\text{different}) &= P(O-A) + P(A-O) = 0.42 \end{aligned}$$

4. Probability that both fruits are the same (either both oranges or both apples)

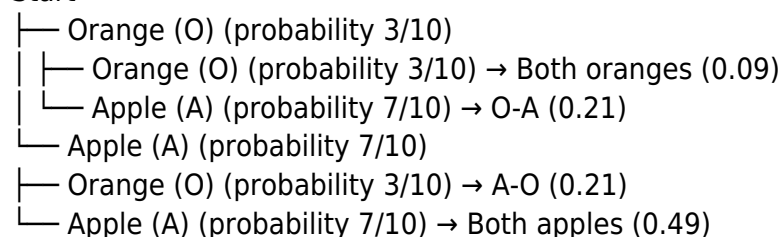
$$\begin{aligned} P(\text{both same}) &= P(\text{both oranges}) + P(\text{both apples}) = 0.09 + 0.49 = 0.58 \end{aligned}$$

Visualizing Outcomes Using a Probability Tree

Constructing a probability tree helps visualize all possible outcomes and their associated probabilities:

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Start



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This visual approach aids in comprehending the independent probabilities and the sum of all outcomes equaling 1.

Real-Life Implications and Applications

Understanding probabilities in such simple models has broad applications:

- Quality Control: Estimating the likelihood of selecting defective fruits in a batch.
- Game Design: Creating fair chances in probability-based games or simulations.
- Decision Making: Informing strategies based on chances of particular outcomes.
- Educational Tools: Teaching fundamental probability concepts through tangible examples.

In the context of the basket problem, grasping the probabilities of different combinations enhances decision-making, such as choosing fruits for a recipe or predicting outcomes in a game involving chance.

Conclusion and Summary

The problem of selecting two pieces of fruit from a basket containing 3 oranges and 7 apples, with replacement, offers a rich exploration of probability concepts. To summarize:

- The probability of selecting an orange in a single pick is 0.3.
- The probability of selecting an apple in a single pick is 0.7.
- The probability of selecting two oranges is 0.09.
- The probability of selecting two apples is 0.49.
- The probability of selecting one orange and one apple (in any order) is 0.42.
- The probability of selecting at least one orange in two picks is 0.51.
- The probability of both fruits being the same (either oranges or apples) is 0.58.

This comprehensive analysis exemplifies how basic probability rules can be applied to everyday situations, providing clarity and insight into the nature of chance and randomness. Whether used for academic purposes or practical decision-making, understanding these principles enhances our ability to interpret and predict outcomes in various contexts.

In essence, a simple basket of fruits becomes a microcosm of probability theory, illustrating how quantifying uncertainty can be both accessible and profoundly informative.

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