A Box With A Square Base And Open Top Must Have A Volume Of 2500 Cm3. What Is The Minimum Possible Surface

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When designing containers, optimizing material usage is often a crucial consideration. Whether constructing packaging, storage boxes, or custom-designed containers, minimizing surface area while meeting volume requirements can lead to cost savings and material efficiency. In this article, we explore the problem: a box with a square base and an open top must have a volume of 2500 cm^3. Our goal is to determine the minimum possible surface area of such a box. We will systematically analyze the problem using principles of calculus and geometric optimization to find the most efficient design.

Understanding the Problem and Setting Up the Variables

Before diving into calculations, it’s essential to clarify the parameters and variables involved.

Defining the Dimensions

- Let the side length of the square base be x centimeters.
- Let the height of the box be h centimeters.

Since the box has an open top, the surface area will include only the base and the four vertical sides.

Volume Constraint

The volume V of the box is given by:

$$ V = \text{area of base} \times \text{height} = x^2 \times h $$

Given that:

$$ V = 2500\, \text{cm}^3 $$

we have:

$$ x^2 \times h = 2500 $$

which allows us to express h in terms of x:

$$ h = \frac{2500}{x^2} $$

This relation will be fundamental in expressing the surface area in terms of a single variable.

Deriving the Surface Area Formula

The surface area S of the box consists of:

- The base: a square with area x^2
- The four sides: each a rectangle with dimensions x (width) and h (height), totaling $4 \times x \times h$

Since the top is open, it does not contribute to the surface area.

Expressed mathematically:

$$ S = x^2 + 4xh $$

Using the relation for h from above:

$$ S = x^2 + 4x \times \frac{2500}{x^2} $$

Simplify:

$$ S = x^2 + \frac{10000}{x} $$

Our goal is to find the value of x that minimizes S, and consequently find the minimum surface area.

Optimizing the Surface Area

To find the minimum surface area, we employ calculus techniques, specifically taking the derivative of S with respect to x and finding where it equals zero.

Calculating the Derivative

Differentiate S with respect to x:

$$ \frac{dS}{dx} = 2x - \frac{10000}{x^2} $$

Set the derivative equal to zero to find critical points:

$$ 2x - \frac{10000}{x^2} = 0 $$

Multiply through by x^2 to clear the denominator:

$$ 2x^3 - 10000 = 0 $$

Solve for x:

$$ 2x^3 = 10000 $$
$$ x^3 = 5000 $$
$$ x = \sqrt[3]{5000} $$

Calculate the cube root:

$$ x \approx \sqrt[3]{5000} \approx 17.1\, \text{cm} $$

This is the critical point where the surface area could be minimized.

Determining the Corresponding Height

Recall:

$$ h = \frac{2500}{x^2} $$

Plugging in the value of x:

$$ h = \frac{2500}{(17.1)^2} \approx \frac{2500}{292.41} \approx 8.55\, \text{cm} $$

Thus, the dimensions that minimize the surface area are approximately:

- Base side length: 17.1 cm
- Height: 8.55 cm

Calculating the Minimum Surface Area

Substitute x back into the surface area formula:

$$ S = x^2 + \frac{10000}{x} $$

Compute:

$$ x^2 \approx (17.1)^2 \approx 292.41 $$
$$ \frac{10000}{x} \approx \frac{10000}{17.1} \approx 584.8 $$

Add these:

$$ S_{\text{min}} \approx 292.41 + 584.8 \approx 877.21\, \text{cm}^2 $$

Therefore, the minimum possible surface area of the box with volume 2500 cm^3 is approximately

877.21 cm^2.

Conclusion: Optimized Box Design

By applying calculus-based optimization, we have determined that the most material-efficient design for a box with a square base and open top, given a volume of 2500 cm^3, involves:

- A base side length of approximately 17.1 centimeters
- A height of approximately 8.55 centimeters
- A minimum surface area of approximately 877.21 square centimeters

This optimized design balances the dimensions to use the least amount of material possible, which can be crucial in manufacturing, packaging, and storage applications.

Additional Considerations and Practical Applications

While the calculations provide an idealized mathematical solution, real-world applications might require considering factors such as:

- Material thickness
- Manufacturing tolerances
- Structural stability
- Aesthetic preferences

However, understanding the mathematical principles behind surface area minimization remains foundational in engineering and design disciplines.

Summary of Key Steps

- Define variables for base side length x and height h.
- Express volume constraint to relate h and x.
- Formulate the surface area function in terms of x.
- Differentiate and find critical points to minimize surface area.
- Calculate the optimal dimensions and the minimum surface area.

By mastering these steps, designers and engineers can optimize various structural elements for efficiency and cost-effectiveness.

In conclusion, the problem of minimizing the surface area of a box with a square base and open top, given a fixed volume, demonstrates the powerful application of calculus in real-world design challenges. The key takeaway is that optimal dimensions can be determined systematically, leading to material savings and efficient designs.

Frequently Asked Questions

What are the dimensions of the box with a square base and open top that minimizes surface area for a volume of 2500 cm^3?

The dimensions are a square base of side length approximately 12.6 cm and a height of approximately 16 cm.

How do you derive the dimensions of the box to minimize its surface area given the volume constraint?

By expressing volume as $V = x^2h$ and surface area as $S = x^2 + 4xh$, then using calculus to minimize S with respect to x, subject to $V = 2500$ cm^3, leading to $x \approx 12.6$ cm and $h \approx 16$ cm.

What is the minimum possible surface area of the box with the specified volume?

The minimum surface area is approximately 1024.8 cm^2.

Why does the surface area of the box reach a minimum at certain dimensions?

Because optimizing the dimensions involves balancing the base and side areas, and calculus shows the critical point where the surface area is minimized occurs at specific x and h values.

How is the constraint volume of 2500 cm^3 incorporated into the minimization process?

By expressing height h in terms of side length x, using $h = 2500 / x^2$, then substituting into the surface area formula to minimize S as a function of x.

What mathematical techniques are used to solve for the minimum surface area?

Calculus, specifically differentiation and setting the derivative of the surface area function to zero to find critical points.

Can you provide the step-by-step calculation to find the optimal dimensions?

Yes. First, express $h = 2500 / x^2$. Then, surface area $S = x^2 + 4x(2500 / x^2) = x^2 + 10000 / x$. Differentiating S with respect to x, setting derivative to zero, solving for x yields approximately 12.6 cm, and $h \approx 16$ cm.

Is the minimum surface area a global minimum or could there be other solutions?

In this context, it is a global minimum because the surface area function has a single critical point that corresponds to the minimum, given the physical constraints.

How does changing the volume affect the minimum surface area?

Increasing the volume increases the optimal dimensions and the minimum surface area, while decreasing volume has the opposite effect, following a proportional relationship derived from the formulas.

What real-world applications require minimizing surface area for a given volume?

Applications include packaging design, container manufacturing, aerodynamics, and material cost reduction, where minimizing surface area reduces material usage and costs.

Additional Resources

A Box With A Square Base And Open Top Must Have A Volume Of 2500 Cm^3. What Is The Minimum Possible Surface?

When approaching optimization problems in geometry, especially those involving volume and surface area, the key idea is to find the shape that minimizes or maximizes a particular property under given constraints. The problem of designing a box with a square base and an open top, which has a fixed volume of 2500 cm^3, and determining the minimum possible surface area, is a classic example of such an optimization challenge. This type of problem not only has practical applications in manufacturing and packaging but also provides insight into the principles of calculus and geometric reasoning. In this article, we will thoroughly analyze this problem, breaking down the components, deriving the necessary formulas, and ultimately finding the minimum surface area that satisfies the volume constraint.

Understanding the Problem

The problem states that we need to construct a box with the following conditions:

- The box has a square base.
- The box has no top (open at the top).
- The volume of the box must be 2500 cm^3.
- The goal is to minimize the surface area of the box under these conditions.

This problem involves two main variables:

- The length of the side of the square base, which we will denote as x (in centimeters).
- The height of the box, which we will denote as h (in centimeters).

Given these variables, the problem can be mathematically formulated and solved using calculus.

Mathematical Formulation

Variables and Constraints

- Let x be the length of one side of the square base.
- Let h be the height of the box.

Since the box has a square base and no top:

- Volume constraint:
The volume V is given by:
$V = x^2 h = 2500$ cm^3

- Surface area:
The surface area S includes the base and the four vertical sides, but no top:
S = area of base + area of four sides
$S = x^2 + 4(x h)$

Our goal is to minimize S subject to the volume constraint.

Expressing Surface Area in Terms of a Single Variable

From the volume constraint:

$x^2 h = 2500$
$\Rightarrow h = 2500 / x^2$

Substituting this into the surface area formula:

$S = x^2 + 4(x h)$
$\Rightarrow S = x^2 + 4x (2500 / x^2)$
$\Rightarrow S = x^2 + 4 2500 / x$
$\Rightarrow S = x^2 + 10000 / x$

Now, the surface area S is expressed as a function of x only:

$S(x) = x^2 + 10000 / x$

Finding the Minimum Surface Area

To find the minimum value of S(x), we differentiate with respect to x and set the derivative to zero.

Calculating the Derivative

$dS/dx = 2x - 10000 / x^2$

Solving for Critical Points

Set $dS/dx = 0$:

$2x - 10000 / x^2 = 0$

Multiply through by x^2 to clear the denominator:

$2x x^2 - 10000 = 0$
$\Rightarrow 2x^3 = 10000$
$\Rightarrow x^3 = 5000$

Solve for x:

$x = (5000)^{(1/3)}$

Calculating the cube root:

$x \approx \sqrt[3]{(5000)} \approx 17.1$ cm

Now, find the corresponding height:

$h = 2500 / x^2$
$\Rightarrow h \approx 2500 / (17.1)^2$
$\Rightarrow h \approx 2500 / 292.4$
$\Rightarrow h \approx 8.55$ cm

Verifying the Nature of the Critical Point

To confirm that this critical point corresponds to a minimum, examine the second derivative:

$d^2S/dx^2 = 2 + 2\ 10000 / x^3$

Since $x \approx 17.1$ cm,

$d^2S/dx^2 \approx 2 + 2\ 10000 / (17.1)^3$

Calculate the denominator:

$(17.1)^3 \approx 17.1\ 17.1\ 17.1 \approx 5000$ (which matches our earlier cube root)

Thus,

$d^2S/dx^2 \approx 2 + 2\ 10000 / 5000$
$\Rightarrow 2 + 2\ 2$
$\Rightarrow 2 + 4$
$\Rightarrow 6\ (> 0)$

Since the second derivative is positive, the critical point corresponds to a minimum.

Calculating the Minimum Surface Area

Now, substitute $x \approx 17.1$ cm back into the surface area formula:

$S \approx x^2 + 10000 / x$
$\Rightarrow S \approx (17.1)^2 + 10000 / 17.1$
$\Rightarrow 292.4 + 10000 / 17.1$

Calculate 10000 / 17.1:

$\approx 10000 / 17.1 \approx 584.8$

Finally:

$S \approx 292.4 + 584.8 \approx 877.2 \text{ cm}^2$

Therefore, the minimum possible surface area of the box is approximately 877.2 cm^2.

Summary of Results

Parameter	Value	Explanation
Side of square base, x	$\approx$ 17.1 cm	Derived from the cube root of 5000
Height, h	$\approx$ 8.55 cm	Calculated using volume constraint
Minimum surface area, S	$\approx$ 877.2 cm^2	Calculated by substituting x into the surface area formula

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Features and Practical Implications

Pros:

- Optimal use of materials: Minimizing surface area reduces material costs or weight, which is beneficial in manufacturing.
- Simplicity of shape: The shape is straightforward to construct and analyze.
- Clear mathematical approach: The problem demonstrates the power of calculus in solving real-world design problems.

Cons:

- Idealized assumptions: Real-world factors like material thickness, structural stability, or manufacturing constraints are not considered.
- Limited to specific shape constraints: The solution applies strictly to a box with a square base and open top; different shapes or additional constraints would require reanalysis.

Conclusion

The problem of designing a box with a square base and open top, constrained to a volume of 2500 cm^3, exemplifies the application of calculus in optimization. By expressing the surface area as a function of one variable and applying calculus techniques, we find that the minimum surface area is approximately 877.2 cm^2 when the side of the square base is about 17.1 cm, and the height is approximately 8.55 cm. This solution not only satisfies the volume constraint but also ensures the material is used most efficiently, embodying principles that are widely applicable in engineering, packaging, and design.

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