A Certain Number Of 1-cm Square Tiles Form A Rectangular Region With A Perimeter Of 30 Cm. The Same Number

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Understanding geometric problems involving tiles and perimeters is a fundamental aspect of mathematics, especially in the realm of area and perimeter calculations. This particular problem involves determining the number of 1-centimeter square tiles that form a rectangle with a given perimeter, and understanding the relationship between the dimensions of the rectangle and the total number of tiles. In this article, we will explore the problem in detail, develop methods to solve it, and discuss related concepts to deepen your understanding of geometric shapes and algebraic reasoning.

Problem Overview and Key Concepts

Before diving into the solution, it's essential to understand the problem's core components and relevant mathematical principles.

The Core Problem

- You have a collection of square tiles, each measuring 1 cm by 1 cm.
- These tiles are arranged to form a rectangular region.
- The perimeter of this rectangle is 30 cm.
- The total number of tiles is equal to the area of the rectangle, which is the same as the number of tiles.

Key Concepts Involved

- Perimeter of a rectangle: the total length around the rectangle, calculated as $P = 2 \times (\text{length} + \text{width})$.
- Area of a rectangle: the number of unit squares covering the region, calculated as $\text{Area} = \text{length} \times \text{width}$.
- Relationship between area and number of tiles: Since each tile is $1\ \text{cm}^2$, the total number of tiles equals the area of the rectangle.
- Algebraic representation: Using variables to represent rectangle dimensions and formulating equations.

Mathematical Formulation

Let's formalize the problem with variables and equations to facilitate solving.

Defining Variables

- Let L = length of the rectangle in centimeters.
- Let W = width of the rectangle in centimeters.

Since the tiles are 1 cm by 1 cm, the number of tiles equals the area:

$$\text{Number of tiles} = L \times W$$

The perimeter of the rectangle is:

$$P = 2 \times (L + W)$$

Given:

$$P = 30 \text{ cm}$$

and

$$\text{Number of tiles} = L \times W$$

The problem states that the number of tiles is equal to the number of tiles, which is inherently true, but more precisely, the number of tiles equals the area $L \times W$, and this same number is also related to the perimeter.

Establishing the Equations

From the given perimeter:

$$2 \times (L + W) = 30$$

which simplifies to:

$$L + W = 15$$

The total number of tiles:

$$N = L \times W$$

Since the problem states "the same number" (implying the total tiles and perhaps another quantity are equal), the most logical interpretation is that:

$$N = L \times W$$

and the total number of tiles (area) is equal to the number of tiles, which is trivial, but perhaps the problem emphasizes that the total number of tiles is also equal to the total perimeter or another measure.

However, based on typical problems of this nature, the key relation is:

- The number of tiles N equals the area $L \times W$.
- The perimeter is 30 cm, leading to $L + W = 15$.

The goal is to find all possible pairs (L, W) such that both are positive integers (since tiles can't be fractional), satisfying:

$$L + W = 15$$
$$L \times W = N$$ (the total number of tiles)

Finding Possible Dimensions of the Rectangle

To proceed, we analyze the possible integer pairs (L, W) that satisfy the sum condition.

Integer Solutions for the Perimeter Equation

Since both L and W are positive integers:

$$L + W = 15$$

possible pairs are:

L	W	$L \times W$ (area)
1	14	14
2	13	26
3	12	36
4	11	44
5	10	50
6	9	54
7	8	56
8	7	56
9	6	54
10	5	50
11	4	44
12	3	36
13	2	26
14	1	14

Note: Since length and width are interchangeable, pairs like (1,14) and (14,1) are equivalent in shape. To avoid duplication, consider only pairs where $L \leq W$.

Possible Rectangles and Their Areas

The unique pairs (with $L \leq W$) are:

- (1,14): area = 14
- (2,13): area = 26
- (3,12): area = 36
- (4,11): area = 44
- (5,10): area = 50
- (6,9): area = 54
- (7,8): area = 56

Since the tiles are 1 cm^2 each, the total number of tiles for each rectangle equals its area:

- For (1,14): 14 tiles
- For (2,13): 26 tiles
- For (3,12): 36 tiles
- For (4,11): 44 tiles
- For (5,10): 50 tiles
- For (6,9): 54 tiles

- For (7,8): 56 tiles

Interpreting the "Same Number" in the Problem

The phrase "the same number" could imply several interpretations:

1. The number of tiles equals the total perimeter: Is the total number of tiles the same as the perimeter? Let's check.

- For each rectangle, compare the total perimeter (which is given as 30 cm) with the number of tiles.

2. The total number of tiles equals the total perimeter: Since the total perimeter is 30 cm, and the number of tiles is area, which varies, this is unlikely.

3. The number of tiles equals the total area (which is already established): This seems to be the core of the problem.

Given the problem statement, it's most reasonable to interpret that:

- The total number of tiles $N = L \times W$,
- The perimeter $P = 30$ cm,
- And perhaps, the total number of tiles equals the total perimeter in some form, or that the total number of tiles is equal to some value derived from the dimensions.

But, the problem seems to be emphasizing that the total number of tiles (area) is equal to some quantity related to the shape, with the key given being the perimeter.

Summary of the Solution Approach

Based on the above analysis, the key steps to solve similar problems are:

- Step 1: Express the perimeter as $2(L + W) = 30$,
- Step 2: Simplify to $L + W = 15$,
- Step 3: Identify all positive integer pairs (L, W) satisfying the sum,
- Step 4: Calculate the area $L \times W$ for each pair,
- Step 5: Interpret what "the same number" refers to—likely, the area (or the number of tiles) is equal to the perimeter in some context.

Conclusion: The Possible Rectangular Regions

Based on the calculations, the rectangles that satisfy the perimeter condition are:

Length (cm)	Width (cm)	Area (Number of Tiles)
1	14	14
2	13	26

3	12	36
4	11	44
5	10	50
6	9	54
7	8	56

These configurations demonstrate the possible arrangements of tiles with a perimeter of 30 cm, each forming a rectangle with the corresponding number of tiles.

Related Concepts and Extensions

Understanding this problem opens doors to exploring more advanced topics:

- Factors and divisibility: Since the dimensions are integers, the problem relates to factors of the total area.
- Algebraic equations: Formulating and solving for

Frequently Asked Questions

How many 1-cm square tiles are used to form the rectangular region?

The number of tiles is the area of the rectangle, which is determined by its length and width in centimeters. Since the perimeter is 30 cm, and the tiles are 1 cm each, possible dimensions can be found that satisfy both conditions.

What are the possible length and width combinations for the rectangle?

Possible combinations are pairs of integers (length and width) such that 2(length + width) = 30, which simplifies to length + width = 15. The pairs are (1,14), (2,13), (3,12), (4,11), (5,10), (6,9), and (7,8).

How do you calculate the number of tiles in each rectangular arrangement?

The number of tiles equals the area, which is length multiplied by width for each pair. For example, for (3,12), the number of tiles is $3 \times 12 = 36$.

Are there multiple rectangular arrangements with the same number of tiles?

Yes, different pairs of length and width that sum to 15 will produce different rectangular regions, each with a specific number of tiles. For example, (5,10) and (6,9) both sum to 15, but have areas of 50 and 54 tiles respectively.

What is the maximum number of tiles that can be formed given the perimeter constraint?

The maximum number of tiles occurs when the rectangle is as close to a square as possible. For perimeter 30 cm, the dimensions closest to a square are (7,8), resulting in 56 tiles.

Can the rectangle be a square? Why or why not?

A square with perimeter 30 cm would have sides of 7.5 cm, which is not possible with 1-cm tiles. Therefore, the rectangle cannot be a perfect square in this case.

How does the perimeter relate to the dimensions of the rectangle?

The perimeter is calculated as 2 times the sum of length and width. Given the perimeter, the sum of length and width is fixed, but individual dimensions can vary.

If the total number of tiles is fixed, how does changing the dimensions affect the perimeter?

Changing the dimensions while keeping the total number of tiles constant alters the perimeter since perimeter depends on the sum of length and width. To maintain the same number of tiles, the dimensions must satisfy both area and perimeter constraints simultaneously.

Additional Resources

A Certain Number Of 1-cm Square Tiles Form A Rectangular Region With A Perimeter Of 30 Cm. The Same Number is a fascinating geometric problem that combines elements of algebra, number theory, and spatial reasoning. This problem invites students and enthusiasts alike to explore how simple units—1-centimeter square tiles—can be arranged to form a specific rectangular shape with a given perimeter, all while maintaining a consistent number of tiles. It offers an excellent opportunity to develop problem-solving skills, understand the relationship between length, width, and perimeter, and appreciate the elegance of mathematical constraints. In this article, we will analyze the problem in depth, explore various solutions, and discuss the underlying mathematical principles.

Understanding the Problem

Restating the Problem

The problem states: "A certain number of 1-cm square tiles form a rectangular region with a perimeter of 30 cm. The same number of tiles." The key points here are:

- The tiles are 1 cm by 1 cm squares.
- The total number of tiles is fixed.
- The shape formed is a rectangle.
- The perimeter of this rectangle is 30 cm.
- The number of tiles remains the same for some related configuration (this phrase may be interpreted as either the same total number of tiles or related to the initial configuration).

Clarifying the phrase "The same number" is essential. Typically, in such problems, it suggests that the total number of tiles and the shape's perimeter are constrained, and the question is to find all possible arrangements where these conditions are met.

Mathematical Foundations

Key Variables and Equations

Let:

- L be the length of the rectangle in centimeters.
- W be the width of the rectangle in centimeters.
- N be the total number of tiles.

Since each tile is 1 cm by 1 cm:

$$
N = L \times W
$$

The perimeter P of a rectangle is given by:

$$
P = 2(L + W)
$$

Given $P = 30$ cm, we have:

$$
2(L + W) = 30 \implies L + W = 15
$$

Because tiles are discrete units, L and W are positive integers, and:

$$
L, W \in \mathbb{Z}^+, \quad L, W \geq 1
$$

From the above, we need to find all pairs (L, W) such that:

$$
L + W = 15
$$

and

$$
N = L \times W
$$

Finding All Possible Rectangular Arrangements

Integer Solutions for L and W

Since L and W are positive integers summing to 15, the possible pairs are:

L	W	$N = L \times W$

1	14	14
2	13	26
3	12	36
4	11	44
5	10	50
6	9	54
7	8	56
8	7	56
9	6	54
10	5	50
11	4	44
12	3	36
13	2	26
14	1	14

Note that (L, W) and (W, L) represent the same rectangle, so solutions are symmetric.

Key observations:

- The total number of tiles N varies depending on the pair.
- The maximum N occurs at $L = 7, W = 8$, yielding $N=56$.

Exploring the Feasible Solutions

Are All These Arrangements Possible?

Since the tiles are 1 cm squares, and the rectangle is formed by these tiles, the dimensions L and W must be integers. All pairs listed above satisfy this.

The number of tiles in each configuration is simply the product of the dimensions.

Important Note: The phrase "The same number" could also imply that the total number of tiles remains constant for multiple arrangements, but in this context, it's most consistent to interpret that the total number of tiles is determined by the dimensions satisfying the perimeter condition.

Visualizing the Arrangements

Examples of Arrangements

Let's consider some specific cases:

- Rectangle with $L=3, W=12$:

- Tiles: $3 \times 12 = 36$
- Perimeter: $2(3 + 12) = 30$ cm

- Rectangle with $L=5, W=10$:

- Tiles: $5 \times 10 = 50$
- Perimeter: $2(5 + 10) = 30$ cm

- Rectangle with $L=7, W=8$:

- Tiles: $7 \times 8 = 56$
- Perimeter: $2(7 + 8) = 30$ cm

These arrangements are straightforward to visualize: a set of tiles forming rectangles with the specified dimensions.

Implications and Further Questions

Are There Other Shapes with the Same Perimeter and Area?

While rectangles are the focus here, one might ask whether other shapes or arrangements could satisfy similar conditions. For example:

- Could irregular shapes formed by the same number of tiles have the same perimeter?
- Is it possible to form a shape with the same number of tiles but a different perimeter?

Given the constraints, only the rectangle with dimensions (L, W) that satisfy $L + W = 15$ and $N = L \times W$ are considered.

Is There a Unique Arrangement for Each N?

No. For each sum $L + W = 15$, multiple pairs exist (e.g., (3,12) and (12,3)), but these are geometrically identical when considering the rectangle's orientation. The key is the pair where $L \leq W$ to avoid duplication.

Mathematical Features and Educational Significance

Features of the Problem

- Simple yet rich: Involves basic algebra but offers multiple solution paths.
- Encourages visualization: Students can draw rectangles and verify dimensions.
- Connects to number theory: Solutions relate to factor pairs of the total area.
- Highlights constraints: The perimeter constraint limits possible dimensions.

Educational Benefits

- Reinforces understanding of perimeter and area formulas.
- Demonstrates the relationship between linear dimensions and total units.
- Develops problem-solving strategies, such as systematic enumeration.
- Encourages exploration beyond the initial problem scope.

Pros and Cons of the Problem

Pros:

- Accessible to high school students.
- Reinforces fundamental geometric and algebraic concepts.
- Stimulates critical thinking and pattern recognition.
- Can be extended to more complex shapes or constraints.

Cons:

- May seem trivial without deeper exploration.
- Could be misinterpreted if the phrase "The same number" isn't clarified.
- Limited complexity for advanced learners unless extended.

Extensions and Open Questions

- Varying the perimeter: What if the perimeter were different? How do the solutions change?
- Different tile sizes: How would the problem change if tiles were not 1 cm squares?
- Three-dimensional analogs: Can similar problems be formulated in 3D, e.g., stacking cubes?
- Optimization problems: Find the rectangle with maximum or minimum area given a fixed perimeter.

Conclusion

The problem involving a certain number of 1-cm square tiles forming a rectangle with a perimeter of 30 cm exemplifies how simple geometric constraints can lead to interesting mathematical exploration. By analyzing the relationships between perimeter, length, width, and area, students gain insight into fundamental concepts while developing problem-solving skills. The solutions are straightforward yet open doors to deeper questions about shape, number theory, and geometric arrangements. Whether used as an educational exercise or as a stepping stone to more complex problems, this scenario underscores the beauty and interconnectedness of mathematics, demonstrating that even the most basic units—square tiles—can lead to rich mathematical investigations.

In summary:

- The key relation is \(L + W = 15\).

- Possible arrangements range from small rectangles like (1,14) to larger ones like (7,8).
- The total number of tiles varies accordingly, from 14 to 56.
- The problem highlights fundamental geometric principles and encourages exploration of dimensions, perimeters, and areas.

This problem serves as

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