A Circle With Centre C(-3, 2) Has Equation X2 + Y2 + 6x - 4y = 12 (a) Find The Y-coordinates Of The Points

A Circle With Centre C(-3, 2) Has Equation X2 + Y2 + 6x - 4y = 12 (a) Find The Y-coordinates Of The Points

Understanding the properties of circles and their equations is essential in coordinate geometry. In this article, we will analyze the given circle with the equation $x^2 + y^2 + 6x - 4y = 12$, identify its center and radius, and then determine the specific y-coordinates of points lying on this circle, especially those that satisfy particular conditions, such as intersecting with a given x-value or other geometric features.

Understanding the Equation of a Circle

Before delving into the specifics of our problem, it’s important to understand the general form of a circle's equation and how to interpret its components.

Standard Form of a Circle Equation

The standard form of a circle's equation in the coordinate plane is:

$$ (x - h)^2 + (y - k)^2 = r^2 $$

where:

- (h, k) represents the center of the circle.
- r is the radius of the circle.

Given an equation in the general quadratic form:

$$ x^2 + y^2 + Dx + Ey + F = 0 $$

we can rewrite it in the standard form by completing the square for both x and y.

Converting the Given Equation to Standard Form

Our goal is to find the circle's center and radius from the given equation:

$$ x^2 + y^2 + 6x - 4y = 12 $$

Step-by-Step Completion of the Square

1. Group x and y terms:

$$ (x^2 + 6x) + (y^2 - 4y) = 12 $$

2. Complete the square for the x-terms:

- Take half of 6, which is 3, and square it: $3^2 = 9$.
- Rewrite:

$$ x^2 + 6x + 9 - 9 $$

3. Complete the square for the y-terms:

- Take half of -4, which is -2, and square it: $(-2)^2 = 4$.
- Rewrite:

$$ y^2 - 4y + 4 - 4 $$

4. Rearranged equation:

$$ (x^2 + 6x + 9) + (y^2 - 4y + 4) = 12 + 9 + 4 $$

which simplifies to:

$$ (x + 3)^2 + (y - 2)^2 = 25 $$

Identifying the Center and Radius

From the standard form:

$$ (x + 3)^2 + (y - 2)^2 = 25 $$

- The center is $(-3, 2)$.
- The radius is $\sqrt{25} = 5$.

This confirms our initial information about the circle's center C(-3, 2).

Finding the Y-Coordinates of Points on the Circle

The core of the problem involves determining the y-coordinates of points on the circle. Depending on the specific question, this could involve:

- Finding y-coordinates for given x-values.
- Finding all y-values where the circle intersects a particular line.
- General solutions for y in terms of x.

General Equation for Y in Terms of X

From the standard form:

$$ (x + 3)^2 + (y - 2)^2 = 25 $$

We can solve for y:

$$ (y - 2)^2 = 25 - (x + 3)^2 $$

$$ y - 2 = \pm \sqrt{25 - (x + 3)^2} $$

$$ y = 2 \pm \sqrt{25 - (x + 3)^2} $$

This formula gives the y-coordinates for any x-value within the domain where the expression under the square root is non-negative:

$$ 25 - (x + 3)^2 \geq 0 $$

which simplifies to:

$$ (x + 3)^2 \leq 25 $$

or

$$ -8 \leq x \leq 2 $$

Calculating the Y-Coordinates for Specific Points

To find the y-coordinates for particular points, plug in specific x-values within the domain.

Example 1: Find y when x = -3

1. Substitute:

$$ y = 2 \pm \sqrt{25 - (-3 + 3)^2} $$

2. Simplify:

$$ y = 2 \pm \sqrt{25 - 0} $$

$$ y = 2 \pm 5 $$

3. Calculate:

$$ y = 2 + 5 = 7 $$
$$ y = 2 - 5 = -3 $$

Y-coordinates: $\boxed{7 \text{ and } -3}$

Example 2: Find y when x = 0

1. Substitute:

$$ y = 2 \pm \sqrt{25 - (0 + 3)^2} $$

$$ y = 2 \pm \sqrt{25 - 9} $$

$$ y = 2 \pm \sqrt{16} $$

$$ y = 2 \pm 4 $$

2. Calculate:

$$ y = 2 + 4 = 6 $$
$$ y = 2 - 4 = -2 $$

Y-coordinates: $\boxed{6 \text{ and } -2}$

Example 3: Find y when x = 2

1. Substitute:

$$ y = 2 \pm \sqrt{25 - (2 + 3)^2} $$

$$ y = 2 \pm \sqrt{25 - 25} $$

$$ y = 2 \pm 0 $$

$$ y = 2 $$

Y-coordinate: $\boxed{2}$

Graphical Interpretation of the Y-Coordinates

Plotting the points corresponding to these x-values and their y-coordinates can help visualize the circle:

- When $x = -3$, y-values are 7 and -3.
- When $x = 0$, y-values are 6 and -2.
- When $x = 2$, y-value is 2 (a single point on the circle).

This demonstrates the symmetry of the circle about its center $(-3, 2)$, with pairs of points equidistant from the center along horizontal lines.

Special Cases: Y-Coordinates at the Center

Notice that at the circle's center $(-3, 2)$, the y-coordinate is simply 2. At this point, the circle's x-value is -3, and the y-value is exactly the y-coordinate of the center.

Applications of Finding Y-Coordinates on a Circle

Knowing how to find y-coordinates for given x-values on a circle has many applications:

- Graphing Circles: Accurate plotting requires calculating specific points.
- Solving Geometry Problems: Finding intersections with lines or other shapes.
- Calculus Applications: Deriving equations of tangents or normals at specific points.
- Physics and Engineering: Modeling circular motion or waveforms.

Additional Tips for Solving Circle Equations

- Always complete the square to convert quadratic forms into standard circle equations.
- Verify the domain of x-values to ensure the square root yields real solutions.
- Use symmetry properties to double-check solutions.
- When solving for y, consider both positive and negative roots.

Summary and Conclusion

In this article, we've thoroughly analyzed the circle with the equation $x^2 + y^2 + 6x - 4y = 12$. We determined its center at $(-3, 2)$ and radius of 5 units. The key to finding the y-coordinates of points on this circle involves rewriting the equation in standard form, solving for y, and understanding the domain constraints.

By applying the formula:

$$ y = 2 \pm \sqrt{25 - (x + 3)^2} $$

we can compute y-coordinates for any x-value within the permissible range $[-8, 2]$. For particular x-values, such as -3, 0, and 2, we've calculated the corresponding y-values explicitly, demonstrating the symmetry and shape of the circle.

Mastering these techniques enhances your ability to work with circle equations in coordinate geometry, vital for solving complex mathematical problems, preparing for exams, and applying geometry concepts in real-world scenarios.

Keywords: circle equation, standard form, center of circle, radius, coordinate geometry, y-coordinates, completing the square, graphing circles, circle intersection, geometric solutions

Frequently Asked Questions

What is the standard form of the circle's equation given as $x^2 + y^2 + 6x - 4y = 12$?

The given equation is in general form. To find the center and radius, we need to complete the square for both x and y terms.

How do you find the center of the circle from the equation $x^2 + y^2 + 6x - 4y = 12$?

By completing the square, the center can be identified as (-3, 2), which matches the given point C(-3, 2).

What is the radius of the circle with the equation $x^2 + y^2 + 6x - 4y = 12$?

After completing the square, the radius r is found to be $\sqrt{13}$.

How do you find the y-coordinates of the points on the circle with the given equation?

Substitute x-values into the circle's equation and solve the resulting quadratic for y to find the y-coordinates.

What are the steps to find the y-coordinates of the intersection points of the circle?

Rewrite the equation in standard form, substitute specific x-values, and solve for y using algebraic methods.

Can you provide the y-coordinates when x = 0 for this circle?

Yes. Substituting $x=0$ into the equation yields $y^2 - 4y = 0$, which solves to $y = 0$ or $y = 4$.

What are the y-coordinates of the points on the circle when x = -3?

Substituting $x = -3$ simplifies the equation to $y^2 - 4y = 0$, giving $y = 0$ or $y = 4$.

Are there any points with the same y-coordinate but different x-coordinates on this circle?

Yes, because for $y = 0$ and $y = 4$, multiple x-values satisfy the circle's equation, indicating symmetric points.

How can the y-coordinates be summarized for points on this circle?

The y-coordinates are y = 0 and y = 4 for specific x-values, and solving the quadratic for various x-values yields all points' y-coordinates.

Additional Resources

A Circle With Centre C(-3, 2) Has Equation $X^2 + Y^2 + 6x - 4y = 12$ is a fascinating geometric problem that combines algebraic manipulation with analytical geometry. This particular question not only tests one's understanding of circle equations but also challenges students to interpret and extract specific features such as the y-coordinates of points lying on the circle. In this comprehensive review, we will delve into the process of analyzing the given circle equation, transforming it into standard form, and ultimately finding the y-coordinates of the points on the circle, supported by step-by-step explanations, mathematical insights, and practical tips.

Understanding the Equation of a Circle

General Form of a Circle Equation

The equation of a circle in the Cartesian plane can be written in the general quadratic form:

$$ x^2 + y^2 + Dx + Ey + F = 0 $$

where $D, E,$ and F are constants. The given problem presents the circle in this form:

$$ x^2 + y^2 + 6x - 4y = 12 $$

which already hints at the need for completing the square to find the circle's center and radius.

Standard Form of a Circle Equation

The standard form simplifies the understanding of a circle's properties:

$$ (x - h)^2 + (y - k)^2 = r^2 $$

where (h, k) is the center, and r is the radius. Converting the given equation into this form allows us to identify the circle's geometric characteristics straightforwardly.

Transforming the Equation into Standard Form

Step-by-Step Completion of the Square

Given equation:

$$ x^2 + y^2 + 6x - 4y = 12 $$

Let's reorganize the terms:

$$ (x^2 + 6x) + (y^2 - 4y) = 12 $$

Now, complete the square for the x and y parts separately.

For $x^2 + 6x$:

- Take half of 6, which is 3, and square it: $3^2 = 9$.

- Rewrite as:

$$ x^2 + 6x + 9 - 9 $$

Similarly, for $y^2 - 4y$:

- Half of -4 is -2, square it: $(-2)^2 = 4$.

- Rewrite as:

$$ y^2 - 4y + 4 - 4 $$

Adding these to the original equation:

$$ (x^2 + 6x + 9) + (y^2 - 4y + 4) = 12 + 9 + 4 $$

which simplifies to:

$$ (x + 3)^2 + (y - 2)^2 = 25 $$

Result:

The circle's equation in standard form is:

$$ (x + 3)^2 + (y - 2)^2 = 25 $$

Features:

- Center $C(-3, 2)$: derived directly from the form.

- Radius $r = \sqrt{25} = 5$:

The circle has a radius of 5 units.

Pros:

- The conversion makes geometric features transparent.

- Simplifies the process of finding specific points, such as y-coordinates.

Cons:

- Requires careful algebraic manipulation.

- Small mistakes in completing the square can lead to incorrect conclusions.

Finding the Y-Coordinates of Points on the Circle

Problem Restatement

Given the circle's equation, we are interested in the y-coordinates of points on the circle, i.e., the possible y-values for points (x, y) lying on the circle.

Approach:

- For specific x-values, solve for y.

- Alternatively, express y in terms of x or vice versa.

- Find all y-values corresponding to the circle's points, considering the geometric constraints.

Method 1: Fixing x and Solving for y

Suppose we choose a specific x-value, say $x = x_0$, then:

$$(x_0 + 3)^2 + (y - 2)^2 = 25$$

Rearranged as:

$$ (y - 2)^2 = 25 - (x_0 + 3)^2 $$

To find y:

$$ y - 2 = \pm \sqrt{25 - (x_0 + 3)^2} $$

$$ y = 2 \pm \sqrt{25 - (x_0 + 3)^2} $$

Note: For y to be real, the radicand must be non-negative:

$$ 25 - (x_0 + 3)^2 \geq 0 $$

which implies:

$$ (x_0 + 3)^2 \leq 25 $$

or

$$ -5 \leq x_0 + 3 \leq 5 $$

and consequently,

$$ -8 \leq x_0 \leq 2 $$

Example:

- For $x = -3$, then $x + 3 = 0$:

$$ y = 2 \pm \sqrt{25 - 0} = 2 \pm 5 $$

- So,

$$ y = 2 + 5 = 7 $$
$$ y = 2 - 5 = -3 $$

Thus, at $x = -3$, y-coordinates are 7 and -3.

Pros:

- Straightforward method for specific x-values.

- Provides exact y-coordinates for particular x.

Cons:

- Not a direct method for finding all y-coordinates independently; it depends on choosing x.

Method 2: Finding All Y-Coordinates for the Entire Circle

Alternatively, since the circle's center and radius are known, the maximum and minimum y-values can be directly identified, corresponding to the topmost and bottommost points:

- Topmost point: $y = k + r = 2 + 5 = 7$

- Bottommost point: $y = k - r = 2 - 5 = -3$

Explanation:

On a circle centered at (h, k) with radius r, the y-coordinates of points on the circle range from $k - r$ to $k + r$. Therefore, the y-coordinates of all points on this circle are within:

$$-3 \leq y \leq 7$$

Features:

- The y-coordinates of the points on the circle are continuous within this interval.

- The extremal y-values provide the maximum and minimum y-coordinates.

Pros:

- Quick and intuitive.

- No need for algebraic calculations for the extremal points.

Cons:

- Does not explicitly find the corresponding x-values.

Applications and Visualizations

Understanding the y-coordinates of points on the circle is crucial in various applications:

- Graphing: Accurately sketching the circle relies on knowing the extremal y-values.

- Intersection Points: Finding points where the circle intersects other curves or lines often involves solving for y.

- Analytical Geometry Problems: Many problems involve calculating the highest or lowest points, or the y-coordinates at specific x-values.

Visual Representation:

Plotting the circle with center $(-3, 2)$ and radius 5 will visually confirm the y-coordinates:

- Top point at $(-3, 7)$

- Bottom point at $(-3, -3)$

Any other point (x, y) on the circle will satisfy the equation, with y-values between -3 and 7.

Conclusion and Final Remarks

The exploration of the circle with the equation $x^2 + y^2 + 6x - 4y = 12$ exemplifies the powerful interplay between algebra and geometry. Converting the general quadratic form into standard form reveals the circle's center at $(-3, 2)$ and radius 5, which simplifies many subsequent calculations. When focusing on the y-coordinates of points on this circle, the key insights include recognizing that these y-values vary between -3 and 7, corresponding to the extremal points along the vertical axis.

Summary of key features:

- The standard form conversion is crucial for geometric interpretation.

- The maximum and minimum y-coordinates are directly derived from the center and radius.

- Specific y-coordinates at particular x-values can be calculated using the circle's equation, provided the radicand is non-negative.

Final thoughts:

This problem reinforces essential skills in algebraic manipulation, geometric interpretation, and analytical problem-solving. Whether for academic purposes or practical applications like engineering and physics, understanding how to manipulate circle equations and extract specific points is fundamental. Mastery of these concepts enables students and professionals to analyze complex geometric situations with confidence, making this a valuable topic in the broader context of coordinate geometry.

Note: For students and educators, practicing with varying circle equations and exploring different points enhances intuition and problem-solving proficiency.

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