A Circular Loop Of Wire Of Radius 10 Cm Carries A Current Of 6.0 A . What Is The Magnitude Of The Magnetic

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Introduction

Understanding the magnetic field generated by a current-carrying conductor is fundamental in electromagnetism. In this article, we explore the magnetic field produced by a circular loop of wire with a radius of 10 centimeters carrying a current of 6.0 amperes. We will derive the magnitude of the magnetic field at the center of the loop using the Biot–Savart Law, discuss relevant concepts, and provide practical insights into the calculation process.

Basics of Magnetic Fields in Current-Carrying Loops

Magnetic Field Due to a Circular Loop

A circular loop of wire carrying an electric current generates a magnetic field. The shape and strength of this magnetic field depend on various factors such as:

- The current flowing through the wire
- The radius of the loop
- The position where the magnetic field is measured

The magnetic field at the center of the loop is of particular interest because it is symmetric and has a well-defined magnitude, which can be calculated precisely using classical electromagnetism principles.

Relevance of the Radius and Current

- Radius (r): Affects the spatial distribution and magnitude of the magnetic field
- Current (I): Determines the strength of the magnetic field; higher currents produce stronger fields

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Calculating the Magnetic Field at the Center of the Loop

Theoretical Foundation: Biot–Savart Law

The Biot–Savart Law states that the magnetic field d at a point due to a small segment of current-carrying wire is proportional to the current, the length of the segment, and the sine of the angle between the segment and the line connecting the segment to the point, and inversely proportional to the square of the distance.

Mathematically, it is expressed as:

$$
\mathbf{dB} = \frac{\mu_0}{4\pi} \frac{I\, d\mathbf{l} \times \mathbf{\hat{r}}}{r^2}
$$

Where:

- μ_0 = permeability of free space ($4\pi \times 10^{-7}\, \text{T}\cdot\text{m/A}$)
- I = current in the wire (6.0 A)
- $d\mathbf{l}$ = differential element of the wire
- $\mathbf{\hat{r}}$ = unit vector from the wire element to the point of observation
- r = distance from the wire element to the point

Applying the law to a circular loop simplifies the calculation significantly at the center due to symmetry.

Magnetic Field at the Center of a Circular Loop

For a circular loop, the magnetic field at its center is given by the formula:

$$
B = \frac{\mu_0 I}{2 r}
$$

Where:

- B = magnitude of the magnetic field at the center
- μ_0 = permeability of free space ($4\pi \times 10^{-7}\, \text{T}\cdot\text{m/A}$)
- I = current (6.0 A)
- r = radius of the loop (convert to meters)

This formula is derived from integrating the Biot–Savart Law over the entire loop.

Step-by-Step Calculation

Given Data

- Radius of the loop, r: 10 cm = 0.10 meters
- Current, I: 6.0 A
- Magnetic permeability of free space, μ_0: $4\pi \times 10^{-7}\, \text{T}\cdot\text{m/A}$

Calculating the Magnetic Field

Applying the formula:

$$
B = \frac{\mu_0 I}{2 r}
$$

Substituting the known values:

$$
B = \frac{(4\pi \times 10^{-7}) \times 6.0}{2 \times 0.10}
$$

Simplify numerator:

$$
(4\pi \times 10^{-7}) \times 6.0 = 24\pi \times 10^{-7}
$$

Calculate denominator:

$$
2 \times 0.10 = 0.20
$$

Therefore,

$$
B = \frac{24\pi \times 10^{-7}}{0.20}
$$

Divide numerator and denominator:

$$
B = (24\pi \times 10^{-7}) / 0.20
$$

$$
B = (24\pi / 0.20) \times 10^{-7}
$$

Calculate $24\pi / 0.20$:

$$
24\pi / 0.20 = (24 / 0.20) \times \pi = 120 \times \pi
$$

Using $\pi \approx 3.1416$:

$$
120 \times 3.1416 \approx 376.99
$$

Finally, the magnetic field magnitude:

$$
B \approx 376.99 \times 10^{-7} \text{Tesla}
$$

Expressed in standard form:

$$
B \approx 3.77 \times 10^{-5}\, \text{T}
$$

or approximately 37.7 microteslas (μT).

Understanding the Result and Its Implications

Magnetic Field Strength in Context

The calculated magnetic field of approximately 37.7 μT at the center of the loop is comparable to Earth's magnetic field, which typically ranges from 25 to 65 μT. This demonstrates that a simple current-carrying loop can generate a magnetic field of similar magnitude, relevant in various practical applications.

Applications of Magnetic Fields in Circuits and Devices

- Electromagnetic Induction: Magnetic fields are essential in transformers and inductors.
- Magnetic Resonance Imaging (MRI): Strong magnetic fields are used to produce detailed images.
- Wireless Power Transfer: Magnetic coupling enables transfer of energy without wires.
- Motor and Generator Design: Magnetic fields are fundamental in the operation of electric motors and generators.

Factors Affecting the Magnetic Field in Practical Scenarios

While the idealized calculation assumes a perfect circle and uniform current distribution, real-world factors can influence the magnetic field:

- Material Properties: Conductivity and permeability of the wire material.
- Loop Geometry: Deviations from perfect circularity.
- External Magnetic Fields: Interference from other magnetic sources.
- Temperature Variations: Affecting resistance and current flow.

Understanding these factors is essential for designing devices that rely on precise magnetic field control.

Conclusion

Calculating the magnetic field generated by a current-carrying loop is a fundamental aspect of electromagnetism with broad practical relevance. For a circular loop of radius 10 cm carrying a current of 6.0 A, the magnetic field at the center is approximately 37.7 microteslas. This value highlights the strength of magnetic fields generated in simple circuits and underscores their importance in technological applications ranging from medical imaging to electrical engineering.

By mastering the principles of the Biot–Savart Law and related formulas, students and professionals can accurately predict magnetic fields in various configurations, enabling the design and analysis of magnetic devices with precision.

Frequently Asked Questions

What is the magnetic field at the center of a circular loop of radius 10 cm carrying a 6.0 A current?

The magnetic field at the center is given by $B = (\mu_0 I) / (2 R)$. Using $\mu_0 = 4\pi \times 10^{-7}$ T·m/A, R = 0.10 m, and I = 6.0 A, $B \approx 7.2 \times 10^{-6}$ T.

How do you calculate the magnetic field at the center of a circular current loop?

Use the formula $B = (\mu_0 I) / (2 R)$, where μ_0 is the permeability of free space, I is the current, and R is the radius of the loop.

What is the significance of the radius in determining the magnetic field of a current loop?

The radius determines the distance from the loop's center to the wire, directly influencing the magnitude of the magnetic field produced at the center; larger radius results in a weaker field at the center.

If the current in the wire increases, how does that affect the magnetic field at the center?

The magnetic field at the center increases proportionally with the current; doubling the current doubles the magnetic field strength.

Can the magnetic field be calculated at points other than the center of the loop?

Yes, the magnetic field at any point along the axis of the loop can be calculated using the appropriate formula involving the distance from the loop's center.

What units are used for the magnetic field in this calculation?

The magnetic field is expressed in teslas (T), which is the SI unit for magnetic flux density.

What is the role of the permeability of free space (μ_0) in calculating magnetic fields?

μ_0, the permeability of free space, is a constant that relates magnetic field strength to current and geometry; its value is $4\pi \times 10^{-7}$ T·m/A.

How does the magnetic field change if the radius of the loop is doubled?

If the radius is doubled, the magnetic field at the center is halved, because $B \propto 1 / R$.

What practical applications rely on the magnetic field generated by circular current loops?

Applications include electromagnets, MRI machines, inductors, transformers, and wireless charging devices, all utilizing magnetic fields generated by current loops.

Additional Resources

A Circular Loop of Wire of Radius 10 Cm Carries a Current of 6.0 A. What Is the Magnitude of the Magnetic Field at Its Center?
Understanding the magnetic field generated by current-carrying conductors is fundamental in electromagnetism, with applications spanning from electric motors to magnetic resonance imaging. When a circular loop of wire with a specified radius carries a steady current, it produces a magnetic field that can be precisely calculated at various points, especially at the center of the loop. In this detailed guide, we will explore the principles behind this phenomenon, derive the relevant formulas, and work through the calculation for a specific case: a circular loop of radius 10 cm carrying a 6.0 A current.

Introduction to Magnetic Fields of Current Loops

A current flowing through a conductor creates a magnetic field around it, as described by Ampère's Law and the Biot-Savart Law. For a simple, circular loop of wire, the magnetic field at the center is particularly significant because it offers insight into the nature of magnetic field distribution and serves as a foundational concept for more complex magnetic systems.

Why Focus on the Center of the Loop?

The center of a circular current loop provides a symmetrical point where the magnetic field lines converge, and the calculation simplifies due to symmetry. The magnetic field at this point is maximized and directly proportional to the current and the number of turns in the loop, if multiple loops are involved.

Fundamental Concepts and Formulas

To analyze the magnetic field at the center of a circular loop, we rely on the Biot-Savart Law, which relates the magnetic field contribution from a small segment of current to its position relative to the point of observation.

Biot-Savart Law

The Biot-Savart Law states:

\[\mathbf{B} = \frac{\mu_0}{4\pi} \int \frac{I\, d\mathbf{l} \times \mathbf{\hat{r}}}{r^2} \]

where:

- $\mathbf{B}$ is the magnetic field,
- μ_0 is the permeability of free space ($4\pi \times 10^{-7} \, \text{T}\cdot\text{m/A}$),
- I is the current,
- $d\mathbf{l}$ is an element of the wire,
- $\mathbf{\hat{r}}$ is the unit vector from the wire element to the point of observation,
- r is the distance from the wire element to the point.

Magnetic Field at the Center of a Circular Loop

Due to symmetry, the calculation simplifies significantly at the center of the loop, resulting in the well-known formula:

$$
B = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}
$$

where:
- R is the radius of the loop,
- x is the distance along the axis from the center; at the center, $x = 0$.

Thus, the magnetic field at the center simplifies to:

$$
B = \frac{\mu_0 I}{2 R}
$$

This is a standard and widely used formula for the magnetic field at the center of a single circular loop.

Step-by-Step Calculation

Let's now apply this formula to our specific problem:

Given Data
- Radius of the loop, $R = 10\, \text{cm} = 0.10\, \text{m}$
- Current, $I = 6.0\, \text{A}$

Constants
- Permeability of free space, $\mu_0 = 4\pi \times 10^{-7}\, \text{T}\cdot\text{m/A}$

Calculation

Plugging the values into the formula:

$$
B = \frac{\mu_0 I}{2 R}
$$

$$

$$
B = \frac{(4\pi \times 10^{-7}) \times 6.0}{2 \times 0.10}
$$

Calculating numerator:

$$
(4\pi \times 10^{-7}) \times 6.0 = 4 \times 3.1416 \times 10^{-7} \times 6
$$

$$
= 12.5664 \times 10^{-7} \times 6 = 75.3984 \times 10^{-7}
$$

Now, dividing by $2 \times 0.10 = 0.20$:

$$
B = \frac{75.3984 \times 10^{-7}}{0.20} = 376.992 \times 10^{-7}
$$

Expressing in Tesla:

$$
B \approx 3.77 \times 10^{-5}\, \text{T}
$$

Final Result:
The magnitude of the magnetic field at the center of the loop is approximately 3.77×10^{-5} Tesla.

Additional Insights and Considerations

Magnetic Field Dependence on Radius and Current

- The magnetic field strength is directly proportional to the current I. Doubling the current doubles the magnetic field.
- It is inversely proportional to the radius R. Increasing the radius decreases the magnetic field at the center.

Effects of Multiple Loops

In practical applications, multiple loops or coils are often used to generate stronger magnetic fields. The total field is then multiplied by the number of turns N:

$$
B_{total} = N \times \frac{\mu_0 I}{2 R}
$$

Field Along the Axis

While this calculation is for the field at the center, the magnetic field at points along the axis (away from the center) can also be calculated using the more general Biot-Savart Law:

$$
B(x) = \frac{\mu_0 I R^2}{2 (R^2 + x^2)^{3/2}}
$$

where x is the axial distance from the center.

Practical Applications and Relevance

Understanding the magnetic field produced by current loops is crucial in designing electromagnets, inductors, and magnetic sensors. For instance:

- Electromagnetic Induction: Fields from loops induce voltages in nearby conductors.
- Magnetic Resonance Imaging (MRI): Strong, uniform magnetic fields are generated by carefully shaped coils.
- Electric Motors and Generators: Magnetic fields interact with current-carrying conductors to produce motion.

In our specific example, a small loop with a 10 cm radius and 6 A current produces a magnetic field on the order of 10^{-5} Tesla, which is comparable to the Earth's magnetic field (~10^{-5} Tesla). This highlights how even modest currents in small loops can generate measurable magnetic fields.

Summary

- The magnetic field at the center of a circular loop of radius R carrying current I is given by:

$$
B = \frac{\mu_0 I}{2 R}
$$

- For a loop of radius 10 cm with a 6.0 A current, the magnetic field magnitude is approximately 3.77×10^{-5} Tesla.
- The calculation illustrates the direct proportionality between current and magnetic field strength and the inverse relationship with radius.

Final Thoughts

Mastering the calculation of magnetic fields generated by current loops is essential for understanding electromagnetic phenomena and designing magnetic devices. By grasping the underlying principles and formulas, you can analyze complex systems, optimize magnetic field strengths, and innovate in fields ranging from electronics to medical imaging. Remember, always consider symmetry and boundary conditions to simplify your calculations, and recognize how fundamental constants like \(

\mu_0 \) influence the results.

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