A Helium Ion Of Mass 4m And Charge 2e Is Accelerated From Rest Through A Potential Difference V In Vacuum.

A Helium Ion Of Mass 4m And Charge 2e Is Accelerated From Rest Through A Potential Difference V In Vacuum.
This scenario presents a classic problem in electromagnetism and particle physics, involving the acceleration of charged particles through electric potentials. Understanding the behavior of ions under such conditions is fundamental in fields like mass spectrometry, plasma physics, and accelerator physics. When a helium ion, with a specified mass and charge, is accelerated from rest by a potential difference V, it gains kinetic energy directly related to the electric potential energy provided by the voltage. This article explores the physics underlying this process, deriving key formulas, discussing energy considerations, and examining the implications for experimental and theoretical applications.

Basic Principles of Ion Acceleration in Vacuum

The acceleration of charged particles in vacuum involves fundamental electromagnetic principles. When a particle with charge q is subjected to an electric potential difference V, the work done by the electric field imparts kinetic energy to the particle. This interaction is governed mainly by the conservation of energy and the Lorentz force law.

Electric Potential and Work Done

The electric potential difference V between two points in space signifies the work required to move a unit charge from one point to another. For a particle with charge q moving through this potential difference, the work done W is:

$$ W = qV $$

This work translates into the particle’s kinetic energy, assuming negligible other forces or energy losses:

$$ KE = qV $$

Energy Conversion and Motion

As the particle accelerates from rest, its initial kinetic energy is zero. After passing through the potential difference V, its kinetic energy becomes:

$$ KE = qV $$

Since kinetic energy relates to the particle’s momentum and velocity, this energy conversion allows us to derive its velocity after acceleration.

Calculating the Final Velocity of the Helium Ion

Given the initial rest state and the work done by the electric field, the final velocity v of the helium ion can be obtained through energy conservation:

$$ KE = \frac{1}{2} m v^2 $$

Substituting the work done:

$$ qV = \frac{1}{2} m v^2 $$

Rearranged for v:

$$ v = \sqrt{\frac{2 q V}{m}} $$

For our helium ion with charge $2e$ and mass $4m$, the specific values are:

$$ v = \sqrt{\frac{2 \times 2e \times V}{4m}} = \sqrt{\frac{4eV}{4m}} = \sqrt{\frac{eV}{m}} $$

Thus, the final velocity of the helium ion after acceleration is:

$$ v = \sqrt{\frac{eV}{m}} $$

This formula indicates that the velocity depends on the square root of the ratio of the charge to mass and the potential difference.

Relativistic Considerations

While the above derivation assumes non-relativistic velocities, for sufficiently high potentials V, relativistic effects become significant. The particle’s velocity approaching the speed of light c requires adjustments to the energy calculations.

Relativistic Energy and Momentum

In relativistic mechanics, the total energy E of a particle is:

$$ E = \gamma m c^2 $$

where γ is the Lorentz factor:

$$ \gamma = \frac{1}{\sqrt{1 - v^2 / c^2}} $$

The kinetic energy KE is then:

$$ KE = (\gamma - 1) m c^2 $$

Equating the work done to the relativistic kinetic energy:

$$ qV = (\gamma - 1) m c^2 $$

Solving for γ:

$$ \gamma = 1 + \frac{qV}{m c^2} $$

The velocity:

$$ v = c \sqrt{1 - \frac{1}{\gamma^2}} = c \sqrt{1 - \frac{1}{(1 + \frac{qV}{m c^2})^2}} $$

For high voltage V, this relativistic correction ensures more accurate velocity predictions.

Numerical Example and Practical Implications

To illustrate, consider a helium ion with:

- Mass $4m$, where m is the atomic mass unit ($1\, \text{amu} \approx 1.66 \times 10^{-27}\, \text{kg}$)
- Charge $2e$, where e is the elementary charge ($1.60 \times 10^{-19}\, \text{C}$)
- Accelerating potential V

Suppose $V = 1000\, \text{V}$. Then, the velocity:

$$ v = \sqrt{\frac{eV}{m}} $$

Calculating:

$$ m = 4 \times 1.66 \times 10^{-27}\, \text{kg} = 6.64 \times 10^{-27}\, \text{kg} $$

$$ v = \sqrt{\frac{1.60 \times 10^{-19} \times 1000}{6.64 \times 10^{-27}}} $$

$$ v = \sqrt{\frac{1.60 \times 10^{-16}}{6.64 \times 10^{-27}}} $$

$$ v \approx \sqrt{2.41 \times 10^{10}} $$

$$ v \approx 1.55 \times 10^5\, \text{m/s} $$

This velocity is about 0.52% of the speed of light, so non-relativistic calculations are sufficiently accurate in this case.

Applications and Significance

Understanding how ions accelerate through electric potentials is crucial in various scientific and technological fields.

Mass Spectrometry

In mass spectrometers, ions are accelerated through known potentials, and their velocities are used to determine their mass-to-charge ratio. The precise calculation of ion velocities enables the identification and quantification of different species within a sample.

Ion Beams and Accelerators

Particle accelerators use electric potentials to propel ions to high energies for fundamental physics experiments. The principles outlined here underpin the design of such accelerators, ensuring accurate control of ion velocities and energies.

Plasma Physics and Space Science

In plasma environments and space physics, ions are naturally accelerated by electric fields, influencing phenomena like solar winds and cosmic rays. Understanding the relationship between potential differences and ion velocities helps interpret observational data.

Summary of Key Formulas

- Non-relativistic velocity:

$$ v = \sqrt{\frac{qV}{m}} $$

- Relativistic energy considerations:

$$ \gamma = 1 + \frac{qV}{m c^2} $$

$$ v = c \sqrt{1 - \frac{1}{\gamma^2}} $$

- For the specific case of a helium ion:

$$ v = \sqrt{\frac{eV}{m}} $$

where e is the elementary charge, V is the potential difference, and m is the mass of the ion.

Conclusion

The acceleration of a helium ion with mass $4m$ and charge $2e$ through a potential difference V exemplifies fundamental principles of electromagnetism and energy conservation. The derived formulas provide essential tools for predicting ion velocities, which are critical in various scientific applications. Whether in designing precise mass spectrometers, developing particle accelerators, or understanding space phenomena, these principles serve as foundational elements in modern physics and engineering.

References:
- Griffiths, D. J. (2017). Introduction to Electrodynamics. Cambridge University Press.
- Serway, R. A., & Jewett, J. W. (2014). Physics for Scientists and Engineers. Cengage Learning.
- Krane, K. S. (1988). Introductory Nuclear Physics. Wiley.

Frequently Asked Questions

How is the kinetic energy of the helium ion related to the potential difference V?

The kinetic energy gained by the helium ion is equal to the work done by the electric field, given by KE = 2eV, where e is the elementary charge. Since the ion has charge 2e, it gains energy 2eV upon acceleration through potential difference V.

What is the expression for the velocity of the helium ion after acceleration through potential difference V?

The velocity v of the helium ion is given by v = sqrt(2 KE / m) = sqrt((4eV) / m), where m = 4m (mass of the ion).

How does the mass of the helium ion influence its acceleration in the electric field?

The acceleration is inversely proportional to the mass of the ion, given by a = (charge electric field) / mass. A larger mass results in a lower acceleration for the same potential difference.

What is the significance of the charge being 2e for the helium ion in this scenario?

Having a charge of 2e means the ion experiences twice the force compared to a singly charged ion when subjected to the same electric field, leading to higher kinetic energy and velocity after acceleration.

If the helium ion is accelerated through a potential difference V, what is its de Broglie wavelength?

The de Broglie wavelength λ is given by $\lambda = h / p$, where $p = mv = \text{sqrt}(2m\,KE) = \text{sqrt}(8meV)$. Substituting the values yields $\lambda = h / \text{sqrt}(8meV)$.

What are some practical applications of accelerating helium ions through a potential difference?

Accelerated helium ions are used in ion beam analysis, materials science for surface modifications, in helium ion microscopy for high-resolution imaging, and in nuclear physics experiments.

Additional Resources

Helium Ion of Mass 4m and Charge 2e Accelerated Through Potential Difference V

Introduction

The behavior of charged particles in electric fields is fundamental to understanding various phenomena in physics, ranging from atomic interactions to large-scale applications such as particle accelerators and ion propulsion. Among these particles, ions—atoms or molecules that have gained or lost electrons—are of particular interest because their dynamics under electric fields reveal insights into atomic structure, plasma physics, and materials science.

In this article, we explore a specific case: a helium ion with a mass of 4m and a charge of 2e, accelerated from rest through a potential difference V in vacuum. This scenario encapsulates essential principles of classical electromagnetism, kinetic energy, and particle dynamics, offering a comprehensive understanding of ion acceleration mechanisms. We aim to dissect each aspect of this problem systematically, providing detailed explanations, mathematical derivations, and contextual insights.

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1. Fundamental Properties of the Helium Ion

1.1 Atomic and Ion Characteristics

Helium is a noble gas with atomic number 2, consisting of two protons, two neutrons, and two electrons in its neutral state. When ionized to form a helium ion with a charge of 2e, the atom loses both electrons, resulting in a doubly charged helium ion, often denoted as He^{2+}.

Mass of the Helium Ion:

- The mass of a neutral helium atom is approximately 4 atomic mass units (amu), where 1 amu ≈ 1.66×10^{-27} kg.
- Since the ion is essentially the nucleus with no electrons, its mass remains approximately 4 amu, i.e.,

$$
\text{Mass} \quad M \approx 4m,
$$

where m is the atomic mass unit in kilograms:

$$
m = 1.66 \times 10^{-27} \, \mathrm{kg}.
$$

Thus,

$$
M \approx 4 \times 1.66 \times 10^{-27} \, \mathrm{kg} \approx 6.64 \times 10^{-27} \, \mathrm{kg}.
$$

Charge of the Helium Ion:

- The charge of an electron is approximately $e = 1.602 \times 10^{-19} \, \mathrm{C}$.
- The helium ion carries a charge of $2e$:

$$
Q = 2e = 2 \times 1.602 \times 10^{-19} \, \mathrm{C} = 3.204 \times 10^{-19} \, \mathrm{C}.
$$

This high charge-to-mass ratio influences the ion's acceleration and trajectory significantly, especially in electric fields.

1.2 Significance in Physics and Applications

Understanding the acceleration of such ions is pivotal in multiple contexts:

- Mass Spectrometry: Differentiating ions based on their mass-to-charge ratio.
- Ion Thrusters: Propulsion systems that utilize accelerated ions for

spacecraft maneuvering.
- Nuclear Physics: Studying nuclear reactions where helium ions serve as projectiles.
- Fundamental Research: Probing atomic and subatomic structures via ion scattering experiments.

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2. Theoretical Framework for Ion Acceleration in Vacuum

2.1 Electric Field and Potential Difference

When an ion is placed in a potential difference V, it experiences an electric field E across the gap. The electric field is related to the potential difference by:

$$
E = \frac{V}{d},
$$

where d is the separation distance. However, in many cases, especially in idealized problems, the precise value of d is not specified; instead, the focus is on the energy gained by the particle when accelerated through V.

Work-Energy Principle:

The work done W by the electric field on the ion equals the change in its kinetic energy:

$$
W = \text{Charge} \times \text{Potential Difference} = QV.
$$

Since the ion starts from rest, the kinetic energy KE acquired is:

$$
KE = QV.
$$

This relation forms the cornerstone for calculating the final velocity of the ion after acceleration.

2.2 Kinetic Energy and Velocity Relationship

Given the kinetic energy:

$$
KE = \frac{1}{2} M v^2,
$$

and the work done:

$$
Q V = \frac{1}{2} M v^2,
$$

we can derive the final velocity v:

$$
v = \sqrt{\frac{2 Q V}{M}}.
$$

This fundamental expression links the potential difference to the ion's velocity, depending on its mass and charge.

---

3. Derivation of Final Velocity

Applying the known values:

$$
Q = 2e = 3.204 \times 10^{-19} \, \mathrm{C},
$$
$$
M \approx 6.64 \times 10^{-27} \, \mathrm{kg},
$$

we find:

$$
v = \sqrt{\frac{2 \times 3.204 \times 10^{-19} \times V}{6.64 \times 10^{-27}}}.
$$

Simplifying:

$$
v \approx \sqrt{\frac{6.408 \times 10^{-19} V}{6.64 \times 10^{-27}}} = \sqrt{9.64 \times 10^{7} V}.
$$

Thus, the velocity is:

$$
v \approx \sqrt{9.64 \times 10^{7} V} \quad \text{(m/s)}.
$$

Note: The velocity increases with the square root of the potential difference, indicating diminishing returns at very high voltages but still enabling significant speeds.

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4. Relativistic Considerations

4.1 When Is Relativity Important?

At high potentials, the ion's velocity can approach a significant fraction of the speed of light $c \approx 3 \times 10^{8} \, \mathrm{m/s}$. When v becomes comparable to c, relativistic effects must be considered, as classical mechanics no longer accurately describes the particle's behavior.

A rough criterion:

$$
v \gtrsim 0.1 c \approx 3 \times 10^{7} \, \mathrm{m/s}
$$

- If v exceeds this threshold, relativistic corrections are necessary.

4.2 Relativistic Energy and Momentum

The total energy E of the ion includes rest energy and kinetic energy:

$$
E = \gamma M c^2,
$$

where

$$
\gamma = \frac{1}{\sqrt{1 - v^2 / c^2}}.
$$

The kinetic energy:

$$
KE = (\gamma - 1) M c^2,
$$

which, in the non-relativistic limit, reduces to the classical $\frac{1}{2} M v^2$.

Relativistic relation between energy and charge:

$$
Q V = (\gamma - 1) M c^2,
$$

which allows solving for v considering relativistic effects:

\[

$$
\gamma = 1 + \frac{Q V}{M c^2}.
$$

Then,

$$
v = c \sqrt{1 - \frac{1}{\gamma^2}}.
$$

5. Practical Implications and Experimental Considerations

5.1 Expected Final Velocities

Let's examine a few scenarios:

- Low voltage case: $V = 10^3 \, \mathrm{V}$

$$
v \approx \sqrt{9.64 \times 10^{7} \times 10^3} \approx \sqrt{9.64 \times 10^{10}} \approx 9.82 \times 10^5 \, \mathrm{m/s}.
$$

This is about 0.0033c, well within the classical regime.

- High voltage case: $V = 10^6 \, \mathrm{V}$

$$
v \approx \sqrt{9.64 \times 10^{7} \times 10^6} \approx \sqrt{9.64 \times 10^{13}} \approx 9.82 \times 10^{6} \, \mathrm{m/s}.
$$

Approximately 0.033c, approaching the regime where relativistic effects should be considered.

5.2 Limitations and Real-World Factors

In practice, acceleration of ions through high voltages involves several considerations:

- Vacuum quality: To prevent collisions with residual gas molecules.
- Field uniformity: Ensuring uniform electric fields for predictable acceleration.
- Ion source stability: Consistent ion production and charge states.
- Energy spread: Variations in initial conditions lead to a distribution of velocities.
- Relativistic corrections: Necessary at high voltages to accurately predict velocities and energies.

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6. Applications and Broader Context

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6.1 Mass Spectrometry

The relation between ion velocity and potential difference underpins time-of-
flight mass spectrometry (TOF-MS), where ions are accelerated through known
voltages, and their velocities are measured to deduce their mass-to
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