

A Line Contains The Points P (-1, 4) And Q (-4, -2). Write The Equation Of The Line In The Slope-intercept

A Line Contains The Points P (-1, 4) And Q (-4, -2). Write The Equation Of The Line In The Slope-intercept Form

Understanding the equation of a line that passes through specific points is a fundamental concept in coordinate geometry. When given two points, such as P (-1, 4) and Q (-4, -2), the goal is to determine the unique line that intersects both points and to express its equation in the slope-intercept form ($y = mx + b$). This process involves calculating the slope of the line, determining the y-intercept, and then formulating the final equation. This article provides a comprehensive guide to achieve this, emphasizing the importance of each step and offering practical examples to reinforce understanding.

Understanding the Fundamentals: Points and Line Equations

What Are Coordinates and Points?

In the Cartesian coordinate system, a point is represented by an ordered pair (x, y), where:

- **x** is the horizontal distance from the origin (0,0).
- **y** is the vertical distance from the origin.

For example, the point P (-1, 4) has an x-coordinate of -1 and a y-coordinate of 4, indicating it is located one unit left of the origin and four units above the x-axis.

Line Equation Basics

The most common form for a line's equation in algebra is the slope-intercept

form:

$$y = mx + b$$

where:

- **m** is the slope of the line, representing its steepness.
- **b** is the y-intercept, the point where the line crosses the y-axis.

Step-by-Step Guide to Find the Equation of the Line through P (-1, 4) and Q (-4, -2)

Step 1: Calculate the Slope (m)

The slope (m) of the line passing through two points P(x₁, y₁) and Q(x₂, y₂) is given by:

$$m = (y_2 - y_1) / (x_2 - x_1)$$

Applying the points P (-1, 4) and Q (-4, -2):

- x₁ = -1, y₁ = 4
- x₂ = -4, y₂ = -2

Calculating:

$$m = (-2 - 4) / (-4 - (-1)) = (-6) / (-3) = 2$$

So, the slope of the line is 2.

Step 2: Find the Y-Intercept (b)

Once the slope is known, substitute one of the points into the slope-intercept formula to find the y-intercept b. Let's use point P (-1, 4):

$$y = mx + b$$

$$4 = 2(-1) + b$$

$$4 = -2 + b$$

$$b = 4 + 2 = 6$$

Thus, the y-intercept is 6.

Step 3: Write the Equation in Slope-Intercept Form

Now, combine the slope and y-intercept to write the equation:

$$y = 2x + 6$$

Verifying the Equation

To ensure accuracy, check whether the line passes through both points.

Check with Point Q (-4, -2):

$$y = 2x + 6$$

$$y = 2(-4) + 6 = -8 + 6 = -2$$

Since $y = -2$ matches the y-coordinate of Q, the equation is correct.

Understanding the Significance of the Slope-Intercept Form

The slope-intercept form offers an immediate understanding of the line's behavior:

- $m = 2$ indicates the line rises two units vertically for each unit it moves horizontally.
- $b = 6$ shows the line crosses the y-axis at $(0, 6)$.

Additional Examples and Practice Problems

Example 1: Find the equation of a line passing

through points R (0, 3) and S (4, 11).

1. Calculate slope: $m = (11 - 3) / (4 - 0) = 8 / 4 = 2$
2. Use point R (0, 3) to find b: $y = mx + b \rightarrow 3 = 2 \cdot 0 + b \rightarrow b = 3$
3. Equation: $y = 2x + 3$

Practice Problem:

Determine the equation of the line passing through points A (2, -1) and B (5, 4) in slope-intercept form.

Common Mistakes to Avoid

- Incorrectly calculating the slope, especially mixing up the order of points.
- Forgetting to verify the equation with both points.
- Misidentifying the y-intercept when the line does not cross the y-axis at an integer point.
- Using the wrong point for substitution—ensure to check which point is used for calculation.

How to Use This Knowledge in Real-Life Contexts

Understanding how to find the equation of a line through two points is useful in various real-world scenarios:

- **Physics:** Calculating the trajectory or rate of change between two measurements.
- **Economics:** Modeling the relationship between two economic variables.
- **Engineering:** Analyzing the linear relationship between forces, distances, or other parameters.
- **Data Science:** Creating predictive models based on linear relationships.

Advanced Topics Related to Line Equations

Once mastered, students can explore more complex forms and concepts, including:

- **Point-Slope Form:** Useful when you know a point and the slope.
- **Standard Form:** $Ax + By = C$, which can be useful in certain applications.
- **Parallel and Perpendicular Lines:** Understanding slopes and their relationships.
- **Linear Regression:** Finding the best-fit line through data points.

Summary and Key Takeaways

To summarize, finding the equation of a line in slope-intercept form through two points involves a systematic approach:

1. Calculate the slope (m) using the two points.
2. Use one point and the slope to find the y-intercept (b).
3. Express the line's equation as $y = mx + b$.
4. Verify the equation with both points for accuracy.

This method provides a clear and efficient way to understand linear relationships, essential in mathematics and many applied sciences.

Final Thoughts

Mastering the process of deriving the line's equation from two points enhances your problem-solving skills and deepens your understanding of algebra and geometry. Whether you're preparing for exams, working on a project, or simply seeking to strengthen your mathematical foundation, practicing these steps will make you proficient in analyzing linear relationships and applying them in various contexts.

Frequently Asked Questions

What is the slope of the line passing through points P(-1, 4) and Q(-4, -2)?

The slope is $m = (-2 - 4) / (-4 - (-1)) = (-6) / (-3) = 2$.

How do you find the equation of a line in slope-intercept form given two points?

First, find the slope using the two points, then use one point and the slope to solve for y-intercept (b) in $y = mx + b$.

What is the slope of the line passing through points P(-1, 4) and Q(-4, -2)?

The slope is 2.

What is the y-intercept of the line passing through P(-1, 4) and Q(-4, -2)?

Using the slope $m = 2$ and point P, substitute into $y = mx + b$: $4 = 2(-1) + b$
 $\Rightarrow 4 = -2 + b \Rightarrow b = 6$.

What is the equation of the line in slope-intercept form passing through the given points?

The equation is $y = 2x + 6$.

How do you verify the line equation passes through both given points?

Substitute each point into $y = 2x + 6$: for P(-1, 4), $4 = 2(-1) + 6 = 4$; for Q(-4, -2), $-2 = 2(-4) + 6 = -8 + 6 = -2$. Both satisfy the equation.

What is the final slope-intercept form of the line containing points P(-1, 4) and Q(-4, -2)?

The line's equation is $y = 2x + 6$.

Additional Resources

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Introduction

In the realm of analytic geometry, understanding how to determine the equation of a line given specific points is a fundamental skill. Whether for academic purposes, engineering, or computer graphics, being able to derive a linear equation from coordinate points is essential for analyzing relationships and making predictions. Today, we focus on a practical example: given two points—P (-1, 4) and Q (-4, -2)—how can we find the equation of the line that passes through them, expressed in the slope-intercept form? This form, written as $y = mx + b$, succinctly captures the line's slope (m) and y-intercept (b), providing a clear picture of its behavior and position on the Cartesian plane.

Understanding the Fundamentals

Before diving into calculations, it's important to revisit some core concepts related to lines on a coordinate plane:

- Points: A point is identified by an ordered pair (x, y). In our case, P (-1, 4) and Q (-4, -2).
- Line: The set of all points that satisfy a linear equation.
- Slope (m): Measures the steepness or incline of the line, calculated as the change in y over the change in x between two points.
- Y-intercept (b): The point where the line crosses the y-axis; the value of y when x = 0.

Understanding these components allows us to systematically work through the process of deriving the line's equation.

Step 1: Calculating the Slope (m)

The slope is fundamental because it indicates how much y changes for a unit change in x. The formula for the slope between two points P(x₁, y₁) and Q(x₂, y₂) is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Applying the coordinates:

- (x₁ = -1), (y₁ = 4)
- (x₂ = -4), (y₂ = -2)

Plugging into the formula:

$$m = \frac{-2 - 4}{-4 - (-1)} = \frac{-6}{-3} = 2$$

Result: The slope of the line is 2.

This tells us that for every one-unit increase in x , y increases by 2 units.

Step 2: Finding the Y-Intercept (b)

With the slope known, the next step is to find the y -intercept, b , which is the y -coordinate where the line crosses the y -axis ($x = 0$). To do this, we utilize the slope-intercept form:

$$y = mx + b$$

Rearranged to solve for b :

$$b = y - mx$$

We can use either point P or Q for this calculation; both should yield the same b if the points lie on the same line. Let's use point $P (-1, 4)$:

$$b = y - mx = 4 - (2)(-1) = 4 - (-2) = 4 + 2 = 6$$

Result: The y -intercept is 6.

Step 3: Formulating the Equation

With the slope ($m = 2$) and y -intercept ($b = 6$), the line's equation in slope-intercept form is:

$$y = 2x + 6$$

This equation fully describes the line passing through points P and Q .

Verification of the Equation

To ensure accuracy, it's prudent to verify that both points satisfy the derived equation:

- For point $P (-1, 4)$:

$$y = 2(-1) + 6 = -2 + 6 = 4$$

- For point $Q (-4, -2)$:

$$y = 2(-4) + 6 = -8 + 6 = -2$$

Both points satisfy the equation, confirming its correctness.

Practical Applications and Significance

Understanding how to determine the equation of a line through two points is more than an academic exercise; it has numerous real-world applications:

- Engineering and Design: Engineers use line equations to model structural components, trajectories, and load distributions.
- Computer Graphics: Rendering images often involves calculating lines between points on a screen.
- Data Analysis: In statistics, lines of best fit are derived using similar principles to predict data trends.
- Navigation and Mapping: Coordinates and line equations help plot routes and geographic features.

Mastering the derivation of such equations equips professionals in these fields to analyze spatial relationships effectively.

Additional Insights: Alternative Forms of Line Equations

While the slope-intercept form is widely used for its simplicity, other forms like point-slope or standard form are also valuable:

- Point-Slope Form: Useful if you know a point on the line and the slope.

$$\backslash [y - y_1 = m(x - x_1) \backslash]$$

Using point P:

$$\backslash [y - 4 = 2(x + 1) \backslash]$$

which simplifies to:

$$\backslash [y - 4 = 2x + 2 \rightarrow y = 2x + 6 \backslash]$$

- Standard Form: Expressed as $Ax + By = C$, often used in algebraic contexts.

In this case, rearranged:

$$\backslash [y = 2x + 6 \rightarrow -2x + y = 6 \backslash]$$

Summary

In conclusion, determining the equation of a line given two points involves

calculating the slope and y-intercept systematically. For points P (-1, 4) and Q (-4, -2), the slope is 2, and the y-intercept is 6, leading to the line's equation:

$$\boxed{y = 2x + 6}$$

This equation not only confirms the points lie on the same line but also provides a straightforward way to analyze and graph the line's behavior. Mastery of these steps enhances spatial reasoning and mathematical fluency, essential skills across scientific, engineering, and technological disciplines.

Final Thoughts

Understanding the process of deriving a line's equation from two points is a cornerstone of coordinate geometry. It exemplifies how algebraic techniques translate geometric relationships into precise mathematical expressions. Whether for solving complex problems or visualizing data, the ability to navigate between points and equations empowers professionals and students alike to interpret the world through a mathematical lens.

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