A Lot Of 30 Watches Is 20% Defective. What Is The Probability That A Sample Of 3 Will Contain 2 Defectives

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Introduction

In manufacturing and quality control, understanding the probability of defective items in a lot is crucial for maintaining standards and making informed decisions. Suppose a company produces 30 watches, and 20% of these watches are defective. This scenario presents a common problem in probability theory and statistical sampling: what is the likelihood that, when randomly selecting a small sample from this lot, a specific number of defective items will be present?

Specifically, if we randomly select 3 watches from this lot, what is the probability that exactly 2 of these watches are defective? This question is not only fundamental in quality control but also demonstrates the application of probability distributions, particularly the hypergeometric distribution, in real-world contexts.

In this article, we will explore the problem comprehensively, covering the necessary background, detailed calculations, and implications. We will also discuss related concepts such as defective rates, sampling methods, and how to interpret the results for quality assurance purposes.

Understanding the Scenario

Before diving into calculations, it's essential to understand the context and the details of the problem:

- Total watches in the lot: 30
- Defective watches: 20% of 30 = 6 watches
- Non-defective watches: 30 - 6 = 24 watches
- Sample size: 3 watches
- Desired outcome: Exactly 2 defective watches in the sample

The question: What is the probability that when 3 watches are randomly selected from the lot, exactly 2 are defective?

Key Concepts and Terminology

To accurately calculate this probability, we must understand the following core concepts:

1. Probability and Random Sampling

Probability measures the likelihood of an event occurring, ranging from 0 (impossible) to 1 (certain). Random sampling involves selecting items from a population such that each item has an equal chance of being chosen, without replacement.

2. Types of Probability Distributions

- Hypergeometric Distribution: Applies when sampling without replacement, which is the case here. It calculates the probability of a specific number of successes (defectives) in a fixed number of draws from a finite population containing a known number of successes and failures.

- Binomial Distribution: Suitable when sampling with replacement or when each trial is independent with a fixed probability of success. Not appropriate here because sampling is without replacement.

3. Hypergeometric Distribution Formula

The probability of drawing exactly k defective items in a sample of size n from a lot containing K defectives and N - K non-defectives is given by:

$$
P(X = k) = \frac{\binom{K}{k} \times \binom{N - K}{n - k}}{\binom{N}{n}}
$$

Where:
- N = total population size (30 watches)
- K = total number of defectives (6 watches)
- n = sample size (3 watches)
- k = number of defectives in the sample (2 watches)

Calculating the Probability

Using the hypergeometric distribution, we can now compute the probability that exactly 2 out of the 3 selected watches are defective.

Step 1: Identify the Parameters

- $N = 30$
- $K = 6$
- $n = 3$
- $k = 2$

Step 2: Plug into the Hypergeometric Formula

$$
P(X=2) = \frac{\binom{6}{2} \times \binom{24}{1}}{\binom{30}{3}}
$$

Step 3: Calculate Each Binomial Coefficient

- $\binom{6}{2} = \frac{6!}{2!(6-2)!} = \frac{6 \times 5}{2 \times 1} = 15$
- $\binom{24}{1} = 24$
- $\binom{30}{3} = \frac{30!}{3!(30-3)!} = \frac{30 \times 29 \times 28}{3 \times 2 \times 1} = 4060$

Step 4: Compute the Probability

$$
P(X=2) = \frac{15 \times 24}{4060} = \frac{360}{4060} \approx 0.0884
$$

Thus, the probability that exactly 2 of the 3 randomly selected watches are defective is approximately 8.84%.

Interpreting the Results

An 8.84% chance indicates that, under random sampling without replacement, there's a relatively low probability of selecting exactly 2 defective watches out of 3. This insight is valuable for quality control teams, as it helps assess the likelihood of detecting multiple defectives in small samples, which can influence inspection strategies.

Implications:

- If the sampling process is repeated multiple times, about 8.84% of the samples of size 3 will contain exactly 2 defectives.
- This probability can help in estimating the effectiveness of sampling plans, especially in detecting defective batches.
- Quality assurance managers can use this information to decide on sample sizes that balance inspection costs and defect detection probabilities.

Related Probabilities and Variations

While the focus here is on the probability of exactly 2 defectives in a sample of 3, other related probabilities include:

1. Probability of 0 defectives

$$
P(X=0) = \frac{\binom{6}{0} \times \binom{24}{3}}{\binom{30}{3}}
$$

- $\binom{6}{0} = 1$
- $\binom{24}{3} = \frac{24 \times 23 \times 22}{6} = 2024$

Calculate:

$$
P(X=0) = \frac{1 \times 2024}{4060} \approx 0.498
$$

Approximately 49.8% chance.

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2. Probability of 1 defective

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$$
P(X=1) = \frac{\binom{6}{1} \times \binom{24}{2}}{\binom{30}{3}}
$$

- $\binom{6}{1} = 6$
- $\binom{24}{2} = \frac{24 \times 23}{2} = 276$

Calculate:

$$
P(X=1) = \frac{6 \times 276}{4060} = \frac{1656}{4060} \approx 0.4077
$$

Approximately 40.77% chance.

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3. Sum of all probabilities

Ensure the probabilities sum to 1:

$$
P(0) + P(1) + P(2) + P(3) = 1
$$

Calculate $P(3)$:

$$
P(X=3) = \frac{\binom{6}{3} \times \binom{24}{0}}{\binom{30}{3}} = \frac{20 \times 1}{4060}

\approx 0.0049
\]

Sum:

$$
0.498 + 0.4077 + 0.0884 + 0.0049 \approx 1.000
$$

This confirms the calculations are consistent.

Practical Applications in Quality Control

Understanding these probabilities enables companies to design effective inspection protocols. For example:

- Sampling Plans: Determining an optimal sample size to detect a certain level of defectives with high confidence.
- Batch Acceptance: Deciding whether to accept or reject a batch based on the likelihood of defective items.
- Risk Management: Quantifying the risks of passing defective products or rejecting good batches.

Using probability models like the hypergeometric distribution, companies can balance inspection costs with the need to maintain quality standards.

Conclusion

In conclusion, when selecting a sample of 3 watches from a lot of 30 that contains 6 defective watches (20% defective rate), the probability of finding exactly 2 defectives is approximately 8.84%. This calculation, based on the hypergeometric distribution, provides valuable insights for quality control and manufacturing processes.

By understanding and applying these probability principles, businesses can make data-driven decisions, optimize sampling strategies, and uphold high-quality standards. Whether for defect detection, process improvement, or compliance, leveraging probability theory plays a vital role in effective quality assurance.

Further Reading and Resources

- "Introduction to Probability" by Joseph K. Blitzstein and Jessica Hwang
- Online calculators for hypergeometric probabilities
- Industry standards for sampling plans, such as MIL-STD-105E or ISO 2859

Remember: Accurate probability calculations are essential for effective quality management and ensuring customer

satisfaction through consistent

Frequently Asked Questions

What is the probability that a single watch is defective if 20% of 30 watches are defective?

The probability that a randomly selected watch is defective is 0.20 or 20%.

How do you calculate the probability that exactly 2 out of 3 sampled watches are defective?

Use the binomial probability formula: P = C(3,2) (0.20)^2 (0.80)^1, where C(3,2) is the combination of 3 items taken 2 at a time.

What is the probability that exactly 2 watches out of 3 are defective in this scenario?

The probability is 3 (0.20)^2 (0.80) = 3 0.04 0.80 = 0.096 or 9.6%.

Does the calculation assume sampling with or without replacement?

The calculation assumes sampling without replacement, but since the sample size is small compared to the total, the

binomial approximation is appropriate.

What is the significance of using the binomial distribution in this problem?

The binomial distribution models the probability of a fixed number of successes (defective watches) in a fixed number of independent trials, which applies here.

If the total number of watches changes, how does that affect the probability calculation?

The probability calculation depends on the defect rate (20%) and the sample size; changing the total number may affect the probability if sampling without replacement is considered, but with large populations, the binomial approximation remains valid.

Can this probability be used to inform quality control decisions?

Yes, knowing the probability helps assess the likelihood of defective watches in samples, aiding in quality control and decision-making processes.

Additional Resources

A Lot Of 30 Watches Is 20% Defective. What Is The Probability That A Sample Of 3 Will Contain 2 Defectives

In the world of quality control and manufacturing, understanding the likelihood of defects within a batch is crucial. Suppose you are a quality inspector examining a shipment of 30 watches, where 20% of these watches are defective. A common question arises: if you randomly select 3 watches from this batch, what is the probability that exactly 2 of them are defective? This seemingly straightforward problem involves key principles from probability theory, particularly the hypergeometric distribution, and offers insights into the quality assurance process.

In this article, we will delve deep into the problem, breaking down the underlying concepts, calculations, and implications. Whether you're a student of statistics, a manufacturing professional, or simply a curious reader, this comprehensive guide aims to clarify the steps involved in solving such probability questions, emphasizing both the theoretical foundation and practical application.

Understanding the Scenario: The Basics of the Problem

Before jumping into calculations, it’s essential to grasp the fundamental details of the problem:

- **Total Watches (Population Size): 30 watches**
- **Number of Defective Watches: 20% of 30 = 0.20 × 30 = 6**

defective watches
- Number of Non-Defective Watches: 30 - 6 = 24 non-defective watches
- Sample Size: 3 watches chosen randomly without replacement
- Desired Outcome: Exactly 2 defective watches in the sample

The key question: What is the probability that, upon randomly selecting 3 watches from the batch, exactly 2 are defective?

This problem is a classic example of sampling without replacement from a finite population, which is modeled accurately by the hypergeometric distribution.

Introduction to the Hypergeometric Distribution

The hypergeometric distribution describes the probability of achieving a specific number of successes in a sequence of draws without replacement from a finite population containing a known number of successes and failures.

Parameters of the hypergeometric distribution:

- Population size (N): Total number of items (here, 30 watches)
- Number of successes in the population (K): Total defective watches (6)
- Sample size (n): Number of items drawn (3 watches)
- Number of observed successes (k): Number of defective

watches in the sample (here, 2)

Probability mass function (pmf):

$$
P(X = k) = \frac{\binom{K}{k} \times \binom{N - K}{n - k}}{\binom{N}{n}}
$$

where:

- $\binom{a}{b}$ is the binomial coefficient ("a choose b"), representing the number of ways to choose b items from a.

This formula calculates the probability that exactly k successes (defectives) are found in the sample.

Applying the Hypergeometric Distribution to Our Problem

Let's now translate the problem's specifics into the hypergeometric formula:

- $N = 30$
- $K = 6$ (defective watches)
- $n = 3$ (sample size)
- $k = 2$ (desired number of defectives in sample)

Plugging these into the formula:

$$
P(\text{exactly 2 defectives}) = \frac{\binom{6}{2} \times \binom{24}{1}}{\binom{30}{3}}
$$

This calculation involves three binomial coefficients:

1. $\binom{6}{2}$: Ways to choose 2 defective watches from the 6 available
2. $\binom{24}{1}$: Ways to choose 1 non-defective watch from the 24 available
3. $\binom{30}{3}$: Total ways to choose any 3 watches from the entire batch

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Calculating the Binomial Coefficients

Let's compute each binomial coefficient step-by-step:

- $\binom{6}{2}$:

$$
\binom{6}{2} = \frac{6!}{2!(6-2)!} = \frac{6 \times 5}{2 \times 1} = 15
$$

- $\binom{24}{1}$:

$$
\binom{24}{1} = 24
$$

\]

- $\binom{30}{3}$:

$$
\binom{30}{3} = \frac{30!}{3!(30-3)!} = \frac{30 \times 29 \times 28}{3 \times 2 \times 1} = \frac{24360}{6} = 4060
$$

Final Calculation of the Probability

Putting it all together:

$$
P = \frac{15 \times 24}{4060} = \frac{360}{4060}
$$

Simplify the fraction:

$$
P = \frac{360 \div 20}{4060 \div 20} = \frac{18}{203}
$$

Expressed as a decimal or percentage:

$$
P \approx \frac{18}{203} \approx 0.0887 \text{ or } 8.87\%
$$

Therefore, there is approximately an 8.87% chance that a randomly selected sample of 3 watches will contain exactly 2 defective watches.

Implications and Real-World Significance

Understanding this probability has practical value in quality control processes:

- Risk Assessment: An 8.87% chance might seem low, but in large-scale manufacturing, multiple samples could be taken, increasing the likelihood of encountering such a sample.
- Decision Making: If a batch is sampled and 2 defectives are found in a small subset, this probability helps assess whether the batch's defect rate is within acceptable limits.
- Quality Control Strategies: Recognizing the likelihood of defective watches in small samples aids in designing inspection procedures—whether to increase sample size or implement additional quality checks.

Additional considerations:

- For larger sample sizes, probabilities change, often increasing the chance of encountering more defectives.
- When the defect rate is higher, the probability of sampling multiple defectives also rises.
- The hypergeometric model assumes random sampling without replacement, aligning with real-world inspection practices.

Extensions and Related Probability Problems

The methodology applied here can extend to various scenarios, such as:

- Probability of at least a certain number of defectives: Calculating the probability that a sample contains 2 or more defectives.
- Sampling with replacement: Using the binomial distribution instead when the same item can be selected more than once.
- Different batch sizes and defect rates: Adjusting the parameters to fit different production contexts.

By mastering the hypergeometric distribution and understanding its applications, quality control professionals and statisticians can make informed decisions based on probabilistic insights.

Conclusion

In summary, the probability that a randomly selected sample of three watches from a batch of 30—where 6 are defective—contains exactly two defectives is approximately 8.87%. This calculation relies on the hypergeometric distribution, which accurately models sampling without

replacement from a finite population.

Such analyses are vital tools in manufacturing, quality assurance, and statistical quality control, enabling stakeholders to assess risks, make data-driven decisions, and maintain high standards. Whether dealing with watches, electronics, or any other products, understanding the probability of defects in samples ensures better oversight and improved product quality.

By translating theoretical formulas into practical calculations, professionals can better anticipate outcomes, optimize inspection protocols, and uphold the integrity of their production processes.

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