A(n) 85-9 Ice Cube At 0C Is Placed In 790 G Of Water At 30C. What Is The Final Temperature Of The Mixture?

Introduction

A(n) 85-9 Ice Cube At 0°C Is Placed In 790 G Of Water At 30°C. What Is The Final Temperature Of The Mixture?

This question involves analyzing a classic heat transfer problem in thermodynamics, where an ice cube is introduced into warmer water, and the system reaches thermal equilibrium. To solve such a problem accurately, it is essential to understand the principles of heat transfer, specific heat capacities, the latent heat of fusion, and the conservation of energy. In this article, we will explore the step-by-step process to determine the final temperature of the mixture, considering all relevant thermodynamic concepts.

Understanding the Components of the Problem

The Ice Cube

- Mass of Ice Cube: 85 grams (assuming the notation 85-9 refers to 85 g)
- Initial Temperature of Ice: 0°C
- Phase State: Solid (ice)
- Properties Needed:
- Specific heat capacity of ice, c_{ice}
- Latent heat of fusion for water, L_f

The Water

- Mass of Water: 790 grams
- Initial Temperature of Water: 30°C
- Properties Needed:
- Specific heat capacity of water, c_{water}

The Goal

- To find the final equilibrium temperature, T_f, of the mixture after heat exchange.

Fundamental Concepts in Heat Transfer

Specific Heat Capacity

- The amount of heat required to raise the temperature of 1 gram of a substance by 1°C.
- For water, $c_{water} \approx 4.18 \, \text{J/g°C}$
- For ice, $c_{ice} \approx 2.09 \, \text{J/g°C}$

Latent Heat of Fusion

- The heat required to convert ice at its melting point to water at the same temperature, without changing temperature.
- $L_f \approx 334 \, \text{J/g}$

Principle of Conservation of Energy

- The heat lost by the warmer water will be gained by the ice (for melting and warming) until thermal equilibrium is reached.
- No heat is lost to the surroundings in an idealized system.

Step-by-Step Solution Approach

Step 1: Determine if the ice will completely melt

- Calculate the maximum heat available from the warm water to melt the ice.
- Compare the heat the water can supply to the energy required to melt the entire ice cube and warm the resulting water and the remaining ice (if any).

Step 2: Calculate heat exchange for melting and warming the ice

- If the ice melts completely, the resulting water will warm from 0°C to the final temperature.
- If not, only part of the ice melts, and the system reaches a different equilibrium.

Step 3: Set up energy balance equations

- Write expressions for heat lost and gained:
- Heat lost by water: $Q_{water} = m_{water} \times c_{water} \times (T_{initial, water} - T_f)$
- Heat gained by ice to melt: $Q_{melt} = m_{ice} \times L_f$
- Heat gained by melted ice to warm from 0°C to T_f: $Q_{ice} = m_{ice} \times c_{water} \times (T_f - 0)$

Step 4: Solve for the final temperature T_f

- Based on whether all ice melts or not, derive the equation and solve for T_f.

Detailed Calculations

Initial Data Summary

- $m_{ice} = 85\, \text{g}$
- $m_{water} = 790\, \text{g}$
- $T_{initial, water} = 30^\circ C$
- $T_{initial, ice} = 0^\circ C$

Calculating Heat Available from Water

$$Q_{water} = m_{water} \times c_{water} \times (T_{initial, water} - T_f)$$
$$Q_{water} = 790 \times 4.18 \times (30 - T_f)$$

Energy Needed to Melt Ice Completely

$$Q_{melt} = m_{ice} \times L_f = 85 \times 334 = 28,390 \, \text{J}$$

Energy Needed to Warm Melted Ice from 0°C to T_f

$$Q_{ice} = 85 \times 4.18 \times T_f$$

Case 1: Complete Melting of Ice

- The water must provide enough heat to:
1. Melt the entire ice cube
2. Warm the resulting water from 0°C to T_f

- Total heat required:
$$Q_{total} = Q_{melt} + Q_{ice} = 28,390 + 85 \times 4.18 \times T_f$$

- Equate to heat lost by water:
$$790 \times 4.18 \times (30 - T_f) = 28,390 + 85 \times 4.18 \times T_f$$

- Simplify:
$$3303.2 \times (30 - T_f) = 28,390 + 355.3 \times T_f$$

- Expand:
$$3303.2 \times 30 - 3303.2 \times T_f = 28,390 + 355.3 \times T_f$$
$$99,096 - 3303.2 T_f = 28,390 + 355.3 T_f$$

- Bring all T_f terms to one side:
$$99,096 - 28,390 = 3303.2 T_f + 355.3 T_f$$
$$70,706 = 3658.5 T_f$$

- Solve for T_f:
$$T_f = \frac{70,706}{3658.5} \approx 19.33^\circ C$$

- Since $T_f \approx 19.33^\circ C$, which is above 0°C, the assumption that all ice melts is valid.

Case 2: Verify if the ice could remain partially unmelted

- Because the calculated T_f is above 0°C, complete melting occurs, and the ice fully transitions into water.

Final Result and Interpretation

- The equilibrium temperature after the process reaches thermal balance is approximately 19.33°C.
- The calculation indicates that the warm water has enough energy to melt all the ice and warm the resulting water to about 19.33°C.
- The system stabilizes at this temperature, with all the ice melted, and no remaining unmelted ice.

Additional Considerations

Assumptions Made

- No heat loss to surroundings.
- The system is perfectly insulated.
- The specific heat capacities are constant over the temperature range.
- The ice is initially at 0°C and behaves as a pure substance without impurities.

Possible Deviations in Real Systems

- Heat loss to environment.
- Variations in specific heat capacities at different temperatures.
- Impurities affecting melting point.

Conclusion

In conclusion, when an 85-gram ice cube at 0°C is placed into 790 grams of water at 30°C, the final temperature of the mixture, after reaching thermal equilibrium, is approximately 19.33°C. This temperature reflects the energy exchanges involved: the warm water cooling down, melting the ice, and warming the resulting water to the equilibrium temperature. Such problems exemplify fundamental thermodynamic principles and illustrate how conservation of energy governs heat transfer processes in real-world systems.

Frequently Asked Questions

What is the initial temperature of the ice cube in the problem?

The initial temperature of the ice cube is 0°C.

How much water is present in the mixture and what is its initial temperature?

There are 790 grams of water initially at 30°C.

What is the specific heat capacity of water used in calculations?

The specific heat capacity of water is typically 4.18 J/g°C.

What is the heat of fusion for ice, and why is it relevant here?

The heat of fusion for ice is approximately 334 J/g, and it is relevant because melting ice absorbs heat without changing temperature.

How do you determine whether the ice melts completely or only partially?

By calculating the heat transfer from water to ice and comparing it to the energy needed to melt all the ice, you can determine if the ice melts completely or not.

What is the method to find the final temperature of the mixture?

Set up an energy balance where heat lost by water equals heat gained by ice (for melting and warming), then solve for the final temperature.

What assumptions are typically made in this type of heat transfer problem?

Assumptions include no heat loss to surroundings, constant specific heat capacities, and that ice starts at 0°C.

What is the approximate final temperature of the mixture?

The approximate final temperature is around 0°C, indicating the water cools down to near freezing after ice melts.

How can this problem be used to understand real-world thermal processes?

It illustrates principles of heat transfer, phase changes, and energy conservation, applicable in cooling systems and climate studies.

Additional Resources

A(n) 85-9 Ice Cube At 0°C Is Placed In 790 G Of Water At 30°C. What Is The Final Temperature Of The Mixture? This classic thermodynamics problem challenges us to understand heat transfer principles, phase changes, and specific heat capacities. It's a common scenario encountered in physics and engineering where two substances at different temperatures interact, and the goal is to find the equilibrium temperature after thermal exchange. In this detailed guide, we will walk through the problem step-by-step, explaining the physical concepts involved, setting up the necessary equations, and performing the calculations needed to determine the final temperature of the mixture.

Understanding the Problem

Imagine you have an ice cube of mass 85 grams at 0°C. You place it into 790 grams of water initially at 30°C. The question is: What is the final temperature of the mixture after thermal equilibrium is reached?

The key factors include:
- The ice cube undergoes a phase change from solid to liquid,
- The water cools down as heat transfers to the ice,
- Heat exchange continues until both reach a common temperature,
- No heat is lost to the surroundings.

Fundamental Concepts to Consider

1. Heat Transfer and Conservation of Energy
The principle of conservation of energy states that the heat lost by the warmer water will be gained by the ice and the melting process. Since no external heat is added or lost, the total heat exchanged sums to zero:

Heat lost by water = Heat gained by ice (melting + warming the meltwater)

2. Specific Heat Capacities
- Water: approximately 4.18 J/g°C
- Ice: approximately 2.09 J/g°C

- Latent heat of fusion of ice: approximately 334 J/g

3. Phase Changes
- Melting ice: requires energy (latent heat) without changing temperature,
- Warming the melted ice (liquid water): increases temperature from 0°C to the final temperature.

Step-by-Step Solution Approach

Step 1: Identify Possible Outcomes
Depending on the initial conditions and quantities, there are two possible outcomes:
- The ice melts completely, and the final temperature is above 0°C.
- The ice does not melt completely, and the final temperature remains at 0°C.

Given the initial parameters, we need to determine which scenario applies.

Step 2: Calculate the Heat Needed to Melt the Ice
$$ Q_{melt} = m_{ice} \times L_f $$
Where:
- $m_{ice} = 85\, \text{g}$
- $L_f = 334\, \text{J/g}$

$$
Q_{melt} = 85\, \text{g} \times 334\, \text{J/g} = 28,390\, \text{J}
$$

This is the minimum heat required to convert all ice at 0°C into liquid water at 0°C.

Step 3: Calculate the Heat Available from Water Cooling
If the water cools from 30°C to some final temperature T_f, the heat lost by water is:

$$
Q_{water} = m_{water} \times c_{water} \times (T_{initial} - T_f)
$$

Where:
- $m_{water} = 790\, \text{g}$
- $c_{water} = 4.18\, \text{J/g°C}$
- $T_{initial} = 30\,°C$

At the final temperature T_f, the water has cooled to T_f.

Analyzing the Scenarios

Scenario 1: The water’s heat is sufficient to melt all the ice, and the final temperature exceeds 0°C.

In this case, the heat lost by water is enough to:
- melt all the ice,

- warm the resulting water from 0°C to T_f.

The total heat required to melt all ice and warm the melted water from 0°C to T_f:

$$
Q_{total} = Q_{melt} + m_{ice} \times c_{water} \times T_f
$$

The heat lost by water:

$$
Q_{water} = m_{water} \times c_{water} \times (30 - T_f)
$$

Set heat lost equal to heat gained:

$$
m_{water} \times c_{water} \times (30 - T_f) = Q_{melt} + m_{ice} \times c_{water} \times T_f
$$

Plugging in known values:

$$
790 \times 4.18 \times (30 - T_f) = 28,390 + 85 \times 4.18 \times T_f
$$

Simplify:

$$
3302.2 \times (30 - T_f) = 28,390 + 355.3 \times T_f
$$

Distribute:

$$
3302.2 \times 30 - 3302.2 \times T_f = 28,390 + 355.3 \times T_f
$$

Calculate:

$$
99,066 - 3302.2 T_f = 28,390 + 355.3 T_f
$$

Bring like terms together:

$$
99,066 - 28,390 = 3302.2 T_f + 355.3 T_f
$$

\[

$$
70,676 = 3657.5 T_f
$$

Solve for T_f:

$$
T_f = \frac{70,676}{3657.5} \approx 19.33°C
$$

Since this is above 0°C, the scenario where all ice melts is physically consistent.

Final Answer: The mixture’s final temperature is approximately 19.33°C.

Additional Considerations

What if the water lacked enough heat to melt all the ice?

If the heat available from cooling water from 30°C to some temperature T_f is less than Q_{melt}, then not all ice melts. The ice would then reach a temperature below 0°C, and some ice would remain unmelted.

To check this:

Calculate the maximum heat water can lose when cooling from 30°C to 0°C:

$$
Q_{max} = 790 \times 4.18 \times 30 \approx 99,066\, \text{J}
$$

Since $Q_{melt} = 28,390\, \text{J}$, the water has enough energy to melt all ice and warm the resulting water to about 19.33°C, confirming our earlier conclusion.

Summary of Key Steps

1. Calculate the heat required to melt all ice: $Q_{melt} = 28,390\, \text{J}$.
2. Calculate the maximum heat water can lose when cooling from 30°C to 0°C: approximately 99,066 J.
3. Determine if the water can melt all ice: since 28,390 J < 99,066 J, the water has enough energy.
4. Set up energy balance equations considering melting and warming of the ice.
5. Solve for the final temperature: approximately 19.33°C.

Final Thoughts

This problem elegantly demonstrates the principles of thermodynamics, phase change energy, and heat transfer. It highlights how the quantities of substances and their thermal properties influence the outcome of thermal interactions. When approaching such problems, always consider the possible scenarios, perform energy calculations step-by-step, and verify whether assumptions are consistent with physical constraints.

By mastering these principles, you can confidently analyze similar problems involving heat exchange, phase changes, and thermal equilibrium, which are fundamental in both academic studies and practical engineering applications.

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