A System Of Equations Is Given. Equation 1: 4x + 3y = 20 Equation 2: 3x + 2y = 12 Explain How To Eliminate

A System Of Equations Is Given. Equation 1: 4x + 3y = 20 Equation 2: 3x + 2y = 12 Explain How To Eliminate

Understanding how to solve systems of equations is a fundamental skill in algebra. When faced with multiple equations involving the same variables, various methods can be employed to find the solution. One of the most efficient and widely used techniques is the elimination method. This approach involves manipulating the equations to eliminate one variable, allowing you to solve for the remaining variable easily. In this article, we will explore how to eliminate a variable step-by-step using the given system of equations:

Equation 1: $4x + 3y = 20$
Equation 2: $3x + 2y = 12$

What Is the Elimination Method?

The elimination method aims to remove one variable by making the coefficients of that variable equal (or opposites) in both equations. Once the variable is eliminated, you can solve for the other variable directly. After finding one variable, substitute its value back into one of the original equations to find the second variable.

Why Use Elimination?

- Efficiency: It often simplifies calculations, especially when coefficients are easily manipulated.
- Versatility: It can be used with any system of linear equations, whether two or more variables.
- Clarity: It provides a clear path to the solution by directly eliminating variables.

Step-by-Step Guide to Eliminating Variables

Let's now examine how to apply the elimination method to our specific system:

Equation 1: $4x + 3y = 20$
Equation 2: $3x + 2y = 12$

Step 1: Identify the Coefficients of the Variable to Eliminate

Choose which variable to eliminate first. Usually, it's easiest to eliminate the variable with the smallest coefficients or the one that makes calculations easier.

In our case:

- Coefficients of x: 4 (Equation 1) and 3 (Equation 2)
- Coefficients of y: 3 (Equation 1) and 2 (Equation 2)

Let's choose to eliminate x first.

Step 2: Make the Coefficients of the Chosen Variable Equal (or Opposite)

To eliminate x, we need to make the coefficients of x in both equations equal (or negatives of each other). We can do this by finding the least common multiple (LCM) of the coefficients:

- LCM of 4 and 3 is 12.

Now, multiply each equation by a number that makes the coefficient of x equal to 12:

- Multiply Equation 1 by 3:
$3 \times (4x + 3y) = 3 \times 20$
$12x + 9y = 60$

- Multiply Equation 2 by 4:
$4 \times (3x + 2y) = 4 \times 12$
$12x + 8y = 48$

Now, the coefficients of x are both 12, ready for elimination.

Step 3: Subtract One Equation from the Other to Eliminate the Variable

Subtract the second scaled equation from the first:

$$
(12x + 9y) - (12x + 8y) = 60 - 48
$$

Simplify:

$$
12x - 12x + 9y - 8y = 12
$$
$$
0x + y = 12

\]

Thus, the variable y is isolated:

\[
\boxed{ y = 12 }
\]

Solving for the Remaining Variable

Having found \(y = 12 \), substitute this value back into one of the original equations to find x.

Let's use Equation 2:

\[
3x + 2y = 12
\]

Substitute \(y = 12 \):

\[
3x + 2 \times 12 = 12
\]
\[
3x + 24 = 12
\]
\[
3x = 12 - 24
\]
\[
3x = -12
\]
\[
x = -4
\]

Final Solution

The solution to the system of equations is:

\[
\boxed{ x = -4, \quad y = 12 }
\]

This point \((-4, 12)\) is where the two lines intersect, satisfying both equations simultaneously.

Additional Techniques for Eliminating Variables

While the above example demonstrates elimination by multiplication to equalize coefficients, there are other strategies and considerations:

Eliminating y Instead of x

If you prefer to eliminate y, follow similar steps:

- Find the LCM of the coefficients of y (3 and 2), which is 6.
- Multiply the equations to make the coefficients of y equal (or opposites):

Equation 1: multiply by 2:

\[
2 \times (4x + 3y) = 2 \times 20 \Rightarrow 8x + 6y = 40
\]

Equation 2: multiply by -3:

\[
-3 \times (3x + 2y) = -3 \times 12 \Rightarrow -9x - 6y = -36
\]

- Add these equations:

\[
(8x + 6y) + (-9x - 6y) = 40 + (-36)
\]

Simplify:

\[
(8x - 9x) + (6y - 6y) = 4
\]
\[
- x + 0 = 4
\]
\[

x = -4
\]

- Substitute \(x = -4 \) into one of the original equations to find y.

Benefits of Elimination

- Simplifies solving systems with larger coefficients.
- Avoids more complex algebraic manipulations like substitution when coefficients are conducive.
- Suitable for solving systems with multiple variables, extending beyond two equations.

Tips for Effective Elimination

- Always choose the variable that makes calculations easier, often the one with coefficients that are factors of each other.
- Ensure to multiply the equations appropriately to make the coefficients of the variable to be eliminated equal or opposite.
- Be cautious with signs — multiplying by negative numbers can help in making coefficients opposites.
- After elimination, double-check your calculations before substituting back to find the other variables.

Common Mistakes to Avoid

- Forgetting to multiply both sides of the equations when scaling.
- Not aligning the equations properly before subtraction or addition.
- Mixing up signs during multiplication or subtraction.
- Substituting into the wrong original equation, leading to incorrect solutions.

Summary

The elimination method is a straightforward and powerful technique to solve systems of linear equations like:

Equation 1: $4x + 3y = 20$
Equation 2: $3x + 2y = 12$

By following these key steps:

1. Choose a variable to eliminate.
2. Find the least common multiple of the coefficients.
3. Multiply the equations to make the coefficients equal or opposites.
4. Subtract or add the equations to eliminate one variable.
5. Solve for the remaining variable.
6. Substitute back into an original equation to find the other variable.

Mastering the elimination method equips you with a vital algebraic skill, enabling you to solve various systems efficiently and accurately. Whether tackling simple two-variable problems or more complex multi-variable systems, elimination remains an essential tool in your mathematical toolkit.

Final Thoughts

Understanding how to eliminate variables is fundamental in algebra, and practicing with different systems will improve your proficiency. With patience and attention to detail, you'll be able to solve systems of equations confidently, laying a strong foundation for more advanced topics in mathematics and related fields.

Frequently Asked Questions

What is the first step to eliminate a variable in the system of equations 4x + 3y = 20 and 3x + 2y = 12?

The first step is to align the coefficients of one variable by multiplying the equations by suitable numbers so that the coefficients of that variable are the same or opposites.

How do I decide which variable to eliminate in the equations 4x + 3y = 20 and 3x + 2y = 12?

You can choose the variable with coefficients that are easier to manipulate—often the one with smaller coefficients—and then multiply the equations to match the coefficients of that variable for elimination.

Can you demonstrate how to eliminate y in the given system of equations?

Yes. Multiply Equation 1 by 2 and Equation 2 by 3:
$(4x + 3y)\times 2 = 8x + 6y = 40$
$(3x + 2y)\times 3 = 9x + 6y = 36$

Then subtract the second from the first: $(8x + 6y) - (9x + 6y) = 40 - 36$, which simplifies to $-x = 4$, so $x = -4$.

After eliminating one variable, how do you find the other variable in the system?

Once one variable is eliminated, you substitute the known value back into one of the original equations to solve for the remaining variable.

What are common mistakes to avoid when eliminating variables in a system of equations?

Common mistakes include not multiplying both equations correctly, losing track of signs, or forgetting to perform the same operation on both sides of the equations. Double-check each step to ensure accuracy.

Is elimination always the best method for solving systems of equations?

Not always. Elimination is ideal when coefficients are easy to align. For some systems, substitution or graphing may be more straightforward, especially when one variable is already isolated.

How can I verify the solution obtained after elimination?

Plug the solution values for x and y back into the original equations to check if both equations are satisfied. If they are, the solution is correct.

What is the significance of multiplying equations during elimination?

Multiplying equations helps create equal or opposite coefficients for the variable to be eliminated, facilitating the addition or subtraction of equations to eliminate that variable.

Can elimination be used for systems with more than two equations?

Yes, elimination can be extended to larger systems by systematically eliminating variables through combination of equations, but it can become more complex and may require matrix methods like Gaussian elimination.

Additional Resources

A System of Equations Is Given: How to Eliminate

In the realm of algebra, solving systems of equations is a fundamental skill that enables mathematicians and students alike to find the values of variables that satisfy multiple conditions simultaneously. Among the various methods available for solving these systems, the elimination method — also known as the addition method — stands out for its clarity and efficiency, especially when variables can be canceled out directly through strategic manipulation. This article explores in depth how to eliminate variables from a system of equations, using a specific example for clarity and demonstrating step-by-step procedures, common pitfalls, and best practices.

Understanding the System of Equations

Before diving into elimination, it's essential to comprehend the structure and nature of the system under consideration.

Given the system:

Equation 1: $4x + 3y = 20$
Equation 2: $3x + 2y = 12$

This pair of linear equations involves two variables, x and y. The goal is to find the pair (x, y) that satisfies both simultaneously.

The Principle Behind Elimination Method

At its core, elimination involves manipulating the equations such that one variable's coefficients are equal (or additive inverses), allowing us to add or subtract the equations to eliminate that variable. The process hinges on the following principles:

- Equalizing coefficients of one variable (by multiplying equations by suitable constants).
- Adding or subtracting the equations to cancel out the chosen variable.
- Solving the resulting single-variable equation.
- Back-substituting to find the other variable.

This method is particularly effective when the coefficients of a variable are already or can be made to be opposites or equal through multiplication.

Step-by-Step Process to Eliminate Variables

Applying the elimination method involves a systematic approach:

Step 1: Identify the Variable to Eliminate

Choose which variable to eliminate — typically, the one with coefficients that are easier to manipulate, or based on simplicity. In our example, the coefficients are 4 and 3 for x, and 3 and 2 for y. Let's choose to eliminate y because their coefficients are 3 and 2, which can be made equal with minimal multiplication.

Step 2: Equalize Coefficients of the Chosen Variable

Determine least common multiples (LCM) of the coefficients of the variable to eliminate.

- For y: coefficients are 3 and 2.
- LCM of 3 and 2 is 6.

Multiply each equation to make the y-coefficients equal to 6:

- Equation 1: multiply both sides by 2:
$2(4x + 3y) = 2(20)$
$\Rightarrow 8x + 6y = 40$

- Equation 2: multiply both sides by 3:
$3(3x + 2y) = 3(12)$
$\Rightarrow 9x + 6y = 36$

Now, both equations have the same y-coefficient (6), setting the stage for elimination.

Step 3: Add or Subtract Equations to Eliminate the Variable

Since both equations contain +6y, subtracting one from the other will eliminate y:

$(8x + 6y) - (9x + 6y) = 40 - 36$

$\Rightarrow 8x + 6y - 9x - 6y = 4$

=> $(8x - 9x) + (6y - 6y) = 4$

=> $-x = 4$

Step 4: Solve for the Remaining Variable

- From $-x = 4$, multiply both sides by -1:

$x = -4$

Step 5: Back-Substitute to Find the Other Variable

Now, substitute $x = -4$ into one of the original equations to find y.

Using Equation 1: $4x + 3y = 20$

Substitute x:

$4(-4) + 3y = 20$

$-16 + 3y = 20$

Add 16 to both sides:

$3y = 36$

Divide both sides by 3:

$y = 12$

Final Solution

The solution to the system is:

$x = -4$
$y = 12$

This pair satisfies both equations, as can be verified by substitution.

Additional Considerations and Common Pitfalls

While the elimination method is straightforward, several nuances and potential errors can arise:

1. Choosing the Right Variable to Eliminate

- Sometimes, coefficients are more easily manipulated by choosing a different variable.
- Always check which variable's coefficients can be made equal with minimal effort.

2. Proper Multiplication of Equations

- Multiplying equations must be done carefully to maintain equality.
- Remember to multiply both sides of the entire equation, not just one term.

3. Sign Management

- Be cautious with signs during addition or subtraction.
- Incorrect handling of negatives can lead to erroneous solutions.

4. Verification of Solutions

- Always substitute the solutions back into the original equations.
- This ensures the accuracy of the solution and confirms the system's consistency.

Alternative Methods and Their Relationship to Elimination

While elimination is effective, it is one of several methods for solving systems:

- Substitution Method: Solving one equation for one variable and substituting into the other.
- Graphical Method: Plotting both equations and identifying intersection points.
- Matrix Method (Gaussian Elimination): Using matrices and row operations for larger systems.

Each method has its advantages depending on the system's complexity.

Conclusion

The elimination method provides a systematic and reliable approach for solving systems of linear equations like:

- $4x + 3y = 20$
- $3x + 2y = 12$

By strategic multiplication to align coefficients, followed by addition or subtraction to cancel variables, solutions emerge efficiently. Mastery of this method involves understanding the underlying principles, careful arithmetic manipulation, and verification of solutions. As with all algebraic techniques, practice enhances proficiency, reducing errors and increasing confidence in solving more complex systems. Whether applied in academic settings, engineering problems, or real-world scenarios, elimination remains a cornerstone of algebraic problem-solving.

In summary, eliminating variables in a system of equations involves:

- Selecting the variable to eliminate.
- Multiplying equations to align coefficients.
- Adding or subtracting to cancel the variable.
- Solving the resulting equation.
- Back-substituting to find all variables.
- Verifying solutions.

This method exemplifies the elegance and efficiency of algebraic strategies for system solving, emphasizing the importance of strategic thinking and meticulous calculation.

A System Of Equations Is Given Equation 1 4x 3y 20 Equation

2 3x 2y 12 Explain How To Eliminate

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