ABChas The VerticesA(3,2),B(5,1)andC(6,4). It Undergoes A Dilation Of Factor 2 From Thex-axis And A Factor

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Introduction

Understanding the transformations of geometric figures is fundamental in the study of coordinate geometry. One such transformation, dilation, alters the size of a figure relative to a specific point or axis. In this article, we explore the dilation of triangle ABC with vertices at A(3, 2), B(5, 1), and C(6, 4). The triangle undergoes a dilation of factor 2 from the x-axis, followed by an additional dilation with a specified factor. These transformations exemplify core concepts in coordinate geometry, including how points shift and scale in the Cartesian plane.

This detailed guide aims to clarify the process of performing these dilations step-by-step, elucidate the mathematical principles involved, and demonstrate how to compute the new vertices after each transformation. Whether you're a student preparing for exams or a geometry enthusiast seeking deeper understanding, this article will provide comprehensive insights into how dilations affect geometric figures in a coordinate plane.

Understanding Dilation in Coordinate Geometry

What Is Dilation?

Dilation is a transformation that produces a similar figure to the original, scaled by a certain factor, and centered at a fixed point called the center of dilation. The key characteristics of dilation include:

- Scaling Factor (k): Determines how much the figure enlarges or reduces. A factor greater than 1 enlarges the figure, while a factor between 0 and 1 reduces it.
- Center of Dilation: The point about which the figure is expanded or contracted. Common centers include the origin (0,0), axes, or other specific points.

Dilation from the x-axis

When dilation occurs from the x-axis, the x-axis acts as the center of dilation or the reference line, depending on the context. Typically, dilation from the x-axis involves scaling points relative to that axis, affecting their y-coordinates primarily, especially when the transformation is described as "from the x-axis."

Mathematical Representation

For a dilation of a point (x, y) with center at (x_c, y_c) and dilation factor k, the new point (x', y') is given by:

$$
x' = x_c + k \times (x - x_c)
$$

$$
y' = y_c + k \times (y - y_c)
$$

When dilation is from the x-axis (which can be considered as the line $y=0$), the center is at $(0,0)$. Thus, the transformation simplifies to:

$$
x' = k \times x
$$

$$
y' = k \times y
$$

This means each point is scaled away from the origin by the factor k.

Step-by-Step Dilation of Triangle ABC

Initial Coordinates

Let's restate the vertices for clarity:

- A: (3, 2)
- B: (5, 1)
- C: (6, 4)

First Dilation: Factor of 2 from the x-axis

Since the dilation is "from the x-axis," and the x-axis is the line $y=0$, the transformation affects only the y-coordinates, scaling them by a factor of 2, while the x-coordinates remain unchanged.

Transformation:

$$
x' = x
$$

$$
y' = 2 \times y
$$

Calculations:

- For A(3, 2):

$$
x' = 3
$$

$$
y' = 2 \times 2 = 4
$$

New A: (3, 4)

- For B(5, 1):

$$
x' = 5
$$

$$
y' = 2 \times 1 = 2
$$

New B: (5, 2)

- For C(6, 4):

$$
x' = 6
$$

$$
y' = 2 \times 4 = 8
$$

New C: (6, 8)

Result after first dilation:

Vertices of triangle ABC are now at:

- A': (3, 4)
- B': (5, 2)
- C': (6, 8)

This transformation effectively stretches the triangle vertically, doubling its height from the x-axis.

Second Dilation: Additional factor (specify the factor)

The original prompt mentions "and a factor," implying a second dilation with another factor. Since the exact second factor isn't fully specified in the prompt, for demonstration purposes, let's assume the second dilation is also from the x-axis with a factor of 3. If needed, this can be adjusted.

Applying the second dilation

Assuming the second dilation has a factor of 3 from the x-axis:

- The transformation again scales the y-coordinates by 3, with x-coordinates remaining unchanged.

Calculations:

- For A'(3, 4):

$$
x'' = 3
$$
$$
y'' = 3 \times 4 = 12
$$
New A'': (3, 12)

- For B'(5, 2):

$$
x'' = 5
$$
$$
y'' = 3 \times 2 = 6
$$
New B'': (5, 6)

- For C'(6, 8):

$$
x'' = 6
$$
$$
y'' = 3 \times 8 = 24
$$
New C'': (6, 24)

Final vertices after both dilations:

- A'': (3, 12)
- B'': (5, 6)
- C'': (6, 24)

This sequence of transformations illustrates how multiple dilations can significantly alter the size and position of a geometric figure in the coordinate plane.

Visualizing the Transformations

Visual aids significantly enhance understanding of geometric transformations. Here's how the triangle evolves through each step:

Original Triangle ABC

- Vertices at (3, 2), (5, 1), and (6, 4).
- The triangle has a certain size and shape in the coordinate plane.

After First Dilation (factor 2 from x-axis)

- Vertices at (3, 4), (5, 2), and (6, 8).
- The triangle is stretched vertically, doubling its height relative to the x-axis.

After Second Dilation (factor 3 from x-axis)

- Vertices at (3, 12), (5, 6), and (6, 24).
- The triangle enlarges further, with all y-coordinates scaled by an additional factor of 3.

Visualizing these steps helps in comprehending how dilations affect size, shape, and position.

Applications of Dilation in Coordinate Geometry

Dilation transformations are not just theoretical exercises; they have practical applications across various fields:

- Engineering and Design: Scaling models and prototypes.
- Computer Graphics: Resizing objects while maintaining proportions.
- Mapping and Geographical Information Systems (GIS): Adjusting map features proportionally.
- Mathematics Education: Demonstrating properties of similar figures and scale invariance.

Understanding how to perform and interpret dilations enhances problem-solving skills in these areas.

Common Mistakes and Tips

While working with dilations, students often encounter pitfalls. Here are some tips to avoid them:

- Identify the correct center of dilation: Whether from the origin, x-axis, y-axis, or another point, the center determines how points are scaled.
- Apply the correct scale factor: Remember that the same factor applies uniformly to all relevant coordinates during each dilation.
- Check whether the dilation is from a line or point: This affects the formula used. For dilation from the x-axis, the center is at $y=0$; from the y-axis, at $x=0$.
- Perform calculations step-by-step: Break down transformations into manageable steps to minimize errors.
- Visualize the transformation: Sketching the figure before and after helps in understanding the effects.

Summary

In this article, we've explored the dilation of triangle ABC with vertices at A(3, 2), B(5, 1), and C(6, 4). The process involved:

1. First dilation with a factor of 2 from the x-axis, which scaled the y-coordinates by 2 while leaving the x-coordinates unchanged.
2. Second dilation, assumed here with a factor of 3 from the x-axis, further scaled the y-coordinates by 3.

The resulting vertices after both transformations are:

- A: (3, 12)
- B: (5, 6)
- C: (6, 24)

These steps demonstrate how dilations modify the size and position of figures in the coordinate plane, emphasizing the importance of understanding centers of dilation, scale factors, and the direction of transformation.

Final Thoughts

Frequently Asked Questions

What are the original coordinates of the vertices of the triangle ABC?

The original vertices are A(3, 2), B(5, 1), and C(6, 4).

How does a dilation with a factor of 2 from the x-axis affect the vertices of triangle ABC?

Dilation with a factor of 2 from the x-axis stretches each vertex's y-coordinate by a factor of 2, while the x-coordinates remain unchanged.

What are the new coordinates of triangle ABC after the dilation of factor 2 from the x-axis?

The new vertices are A'(3, 4), B'(5, 2), and C'(6, 8).

Does the dilation with respect to the x-axis alter the shape or size of the triangle?

Yes, it increases the size of the triangle vertically by a factor of 2, effectively doubling the height while maintaining the shape.

Can the dilation of triangle ABC be represented mathematically? If so, what is the transformation rule?

Yes. The transformation rule is (x, y) □ (x, 2y), since the dilation is from the x-axis with a factor of 2.

What is the significance of choosing the x-axis as the center of dilation in this problem?

Dilation from the x-axis means the y-coordinates are scaled while the x-coordinates remain constant, affecting the shape's height but not its horizontal position.

Additional Resources

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Introduction

Geometry, the branch of mathematics concerned with the properties and relations of points, lines, surfaces, and solids, offers a fascinating exploration of transformations that alter figures while preserving certain attributes. Among these transformations, dilation (or scaling) plays a pivotal role in understanding how figures change size and position relative to specific points or axes. In this article, we analyze the transformation of a triangle with vertices at A(3,2), B(5,1), and C(6,4), which undergoes a dilation of factor 2 from the x-axis. We will delve into the mathematical principles behind this transformation, interpret its geometric implications, and examine the resulting figure in detail.

Understanding the Initial Triangle

Coordinates and Geometric Significance

The triangle ABC is defined by three points in the coordinate plane:

- A(3, 2): Located 3 units along the x-axis and 2 units up the y-axis.
- B(5, 1): Located 5 units along the x-axis and 1 unit up the y-axis.
- C(6, 4): Located 6 units along the x-axis and 4 units up the y-axis.

This triangle's shape and size are determined by these vertices, with side lengths and angles that could be computed for further analysis. For now, understanding their positions relative to the axes provides a baseline for assessing how the figure transforms.

Visual Representation

While visual sketches aid understanding, even in textual form, it's important to note that:

- Point A is relatively close to the origin, slightly above the x-axis.
- Point B is to the right of A, closer to the x-axis.
- Point C is further to the right and higher up, forming a diverse triangle that spans both axes.

The initial configuration sets the stage for transformations that will modify the triangle's size and position.

The Concept of Dilation in Geometry

What is Dilation?

Dilation is a transformation that enlarges or reduces a figure by a scale factor relative to a fixed point known as the center of dilation. Unlike translations or rotations, dilation changes the size of the figure but preserves the shape's similarity.

Key Components of Dilation

- Center of Dilation: The fixed point about which the figure is scaled.
- Scale Factor: A positive real number indicating how much the figure is enlarged (>1) or reduced (<1).

In our scenario, the dilation occurs from the x-axis, which functions as the center of dilation, with a factor of 2.

Dilation from the x-axis

This is a less common but interesting case where the transformation is centered on an entire line rather than a single point. The line in question here is the x-axis (the line y=0). The dilation will:

- Scale points vertically relative to the x-axis.
- Leave points on the x-axis unchanged (as they are on the center line).
- Scale points above or below the x-axis proportionally, based on their vertical distance from the line.

Mathematical Framework of the Dilation

Dilation with a Line as Center

Typically, dilation is centered at a point, but when the center is a line, the transformation can be viewed as a series of individual dilations about points on the line. For points not on the x-axis, the process involves:

- Finding the perpendicular distance from the point to the x-axis.
- Scaling this distance by the factor (here, 2).
- Moving the point along the line perpendicular to the x-axis accordingly.

Coordinates Transformation

Given a point $P(x, y)$, and the x-axis as the line of dilation:

- The x-coordinate remains unchanged because the transformation is vertical.
- The y-coordinate transforms as:

$$
y' = k \times y
$$

where $k=2$ is the dilation factor.

Application to Vertices

Applying this to each vertex:

- Vertex A(3, 2):

$$
A' (x' = 3, y' = 2 \times 2 = 4)
$$

- Vertex B(5, 1):

$$
B' (x' = 5, y' = 1 \times 2 = 2)
$$

- Vertex C(6, 4):

$$
C' (x' = 6, y' = 4 \times 2 = 8)
$$

This transformation effectively doubles the vertical distances of each point from the x-axis, resulting in a scaled-up version of the original triangle.

Geometric Interpretation of the Transformation

Shape Preservation and Size Change

The dilation from the x-axis with a factor of 2 preserves the similarity of the triangle. The angles remain unchanged, and the shape is proportionally enlarged. The key change is in the size:

- The vertical height of each point from the x-axis doubles.
- The horizontal positions stay fixed.
- The overall structure of the triangle is maintained, but its size is scaled vertically.

Visualization of the Transformation

Imagine stretching the triangle vertically along lines perpendicular to the x-axis:

- Points above the x-axis move further away from it, doubling their original vertical distances.
- Points below the x-axis would move further below, but since all points are above or on the axis in this case, only upward movement occurs.

This kind of dilation is often visualized as a "stretch" along the y-direction, with the x-axis acting as a "mirror" and pivot point.

Calculating the New Coordinates and the Resulting Triangle

New Coordinates

Applying the dilation:

Vertex	Original Coordinates	Transformed Coordinates
A	(3, 2)	(3, 4)

B	(5, 1)	(5, 2)
C	(6, 4)	(6, 8)

New Triangle ABC'

- Vertices:

- $A' (3, 4)$
- $B' (5, 2)$
- $C' (6, 8)$

Analyzing the New Triangle

- The base segment AB has an original length:

$$
AB = \sqrt{(5-3)^2 + (1-2)^2} = \sqrt{2^2 + (-1)^2} = \sqrt{4 + 1} = \sqrt{5} \approx 2.236
$$

- After dilation, the new segment $A'B'$:

$$
A'B' = \sqrt{(5-3)^2 + (2-4)^2} = \sqrt{2^2 + (-2)^2} = \sqrt{4 + 4} = \sqrt{8} \approx 2.828
$$

- The length increased roughly by a factor of 1.414 (square root of 2), consistent with the vertical scaling.

- Similarly, the side lengths involving point C will be scaled proportionally.

This analysis confirms the dilation's effect on the triangle's dimensions.

Geometric Implications and Properties

Preservation of Shape and Ratios

Since the dilation is uniform along the vertical direction, the shape of the triangle remains similar to its original form. The ratios of corresponding sides are preserved, with all vertical distances scaled by a factor of 2.

Changes in Area

The area of the original triangle:

$$
\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}
$$

Choosing AB as the base:

- Original height: vertical distance from $C(6,4)$ to the line AB (which may be calculated or approximated).
- Transformed area: since vertical distances double, the area increases by a factor of 2 (from the vertical scaling), implying the new area is approximately twice the original.

Implications for Coordinate Geometry and Real-World Applications

Understanding such transformations is crucial in fields like engineering, computer graphics, and physics, where scaling figures or models is commonplace. Recognizing how the shape and size change under dilation helps with designing scaled models, simulations, and visualizations.

Broader Context and Mathematical Significance

Dilation in Coordinate Geometry

Dilation represents a fundamental transformation that demonstrates the concept of similarity and proportionality. It reinforces the idea that figures can be scaled without altering their shape, a principle fundamental to similarity geometry.

Transformations and Composition

In practice, dilation can be combined with other transformations such as translation, rotation, or reflection to achieve complex manipulations of figures. Understanding dilation from a line (like the x-axis) expands the toolkit for geometric transformations.

Educational Value

Analyzing this problem helps students grasp the concept of line-based dilations, coordinate transformations, and the preservation of angles and shape ratios. It also emphasizes the importance of understanding the geometric significance behind algebraic formulas.

Conclusion

The transformation of triangle ABC with vertices at A(3, 2), B(5, 1), and C(6, 4), undergoing a dilation of factor 2 from the x-axis, exemplifies the elegance and

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