

Adult Men Have Heights With A Mean Of 69.0 Inches And A Standard Deviation Of 2.8 Inches. Find The Z-score

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Understanding how individual heights compare to the average height of adult men is a fundamental aspect of statistics, especially in fields like health sciences, anthropology, and even fashion. Calculating the Z-score allows us to determine how many standard deviations a particular height is from the mean. This article aims to explain the concept of the Z-score, how to compute it using given data, and the practical applications of this statistical measure.

What Is a Z-score?

Definition and Significance

A Z-score, also known as a standard score, quantifies how many standard deviations an individual data point is from the population mean. It is a dimensionless number that enables comparisons across different datasets and distributions.

For example, if a man is 73 inches tall, and the average height is 69 inches with a standard deviation of 2.8 inches, the Z-score tells us how much taller he is compared to the average, scaled in terms of standard deviations.

Mathematical Formula

The Z-score for a data point X is calculated using the formula:

$$Z = \frac{X - \mu}{\sigma}$$

Where:

- Z = Z-score
- X = individual data point (height)
- μ = mean of the data (average height)
- σ = standard deviation

This formula subtracts the mean from the individual value and divides the result by the standard deviation, standardizing the data point.

Given Data and Problem Context

Let's restate the given information:

- Mean height of adult men, $(\mu = 69.0)$ inches
- Standard deviation, $(\sigma = 2.8)$ inches

Suppose we want to find the Z-score for a specific height, say 73 inches, or to analyze what percentage of men fall within certain height ranges.

Calculating the Z-score: Step-by-Step

Example 1: Finding the Z-score for a height of 73 inches

Using the formula:

$$Z = \frac{X - \mu}{\sigma}$$

Plugging in the values:

$$Z = \frac{73 - 69}{2.8} = \frac{4}{2.8} \approx 1.43$$

This means that a man who is 73 inches tall is approximately 1.43 standard deviations above the average height.

Interpretation of the Z-score

- A positive Z-score indicates a value above the mean.
- A Z-score of 0 indicates the value is exactly at the mean.
- A negative Z-score indicates a value below the mean.

In our example, 1.43 suggests that the individual is taller than about 92.4% of adult men, assuming a normal distribution (more on this later).

Understanding the Normal Distribution

Bell-Shaped Curve

The distribution of adult men's heights in a large, randomly selected population tends to follow a normal distribution, characterized by its bell-shaped curve. This means most men are close to the mean height, with fewer men being extremely tall or short.

Empirical Rule

According to the empirical rule:

- About 68% of data falls within 1 standard deviation of the mean.
- About 95% falls within 2 standard deviations.
- About 99.7% falls within 3 standard deviations.

Using this rule, a height with a Z-score of 1.43 falls within the 68% range, closer to the upper end, indicating a relatively taller stature.

Using Z-scores to Find Probabilities

One of the most valuable applications of Z-scores is to determine the probability of a data point falling within a certain range.

Standard Normal Distribution Table

This table provides the cumulative probability associated with a Z-score, representing the area under the curve to the left of that Z-score.

For a Z-score of 1.43, the cumulative probability is approximately 0.9236, meaning:

- About 92.36% of adult men are shorter than 73 inches.
- Conversely, about 7.64% are taller.

Calculating Percentiles

If you want to find the percentile rank of a particular height:

- Find the Z-score.
- Use the Z-table or a calculator to get the cumulative probability.
- Convert the probability into a percentile.

For example, a Z-score of 1.43 corresponds roughly to the 92nd percentile.

Finding the Z-score for Other Heights

Suppose you are interested in other heights:

| Height (in inches) | Z-score Calculation | Z-score | Interpretation |

|-----|-----|-----|-----|
| 65 inches | $\left(\frac{65 - 69}{2.8} \right)$ | -1.43 | Below average; about 7.64% taller than 65 inches |
| 71 inches | $\left(\frac{71 - 69}{2.8} \right)$ | 0.71 | Slightly above average; about 76% shorter than 71 inches |

Example Calculation for 65 inches:

$$\begin{aligned} Z &= \frac{65 - 69}{2.8} = \frac{-4}{2.8} \approx -1.43 \end{aligned}$$

This Z-score indicates that a height of 65 inches is approximately 1.43 standard deviations below the mean.

Example Calculation for 71 inches:

$$\begin{aligned} Z &= \frac{71 - 69}{2.8} = \frac{2}{2.8} \approx 0.71 \end{aligned}$$

This suggests the individual is slightly taller than average.

Practical Applications of Z-scores in Real Life

Health and Medical Research

Z-scores are used to assess growth in children, compare bone density, or evaluate the height of individuals relative to population norms.

Anthropological Studies

Researchers analyze the variations in height across different populations, regions, or ethnic groups to understand evolutionary and environmental influences.

Fashion and Apparel Industry

Manufacturers use height percentiles to design clothing sizes that fit the majority of the population.

Sports Science

Coaches and trainers might analyze athletes' heights relative to the population to tailor training and identify potential in certain sports.

Limitations and Assumptions

While Z-scores are powerful, they rely on certain assumptions:

- The data follows a normal distribution. For heights, this is generally a reasonable assumption due to their natural variation.
- The population parameters (μ) and (σ) are accurately known. In practice, estimates are used, which can introduce slight inaccuracies.
- Outliers or skewed data can affect the reliability of Z-scores.

Summary and Key Takeaways

- The Z-score is a standardized measure indicating how many standard deviations a data point is from the mean.
- Calculating the Z-score involves subtracting the mean from the data point and dividing by the standard deviation.
- In the context of adult men's heights, with a mean of 69.0 inches and a standard deviation of 2.8 inches, the Z-score helps determine how tall or short a particular individual is relative to the average.
- Z-scores facilitate probability calculations and percentile rankings, providing valuable insights into population distributions.
- Understanding Z-scores has wide-ranging applications in health, research, industry, and everyday life.

By mastering the concept of Z-scores, you can interpret data more effectively, compare individual values within a population, and make informed decisions based on statistical analysis.

Remember: Always consider the distribution and context when interpreting Z-scores to ensure accurate and meaningful insights.

Frequently Asked Questions

What is the formula to calculate the Z-score for adult men's heights?

The Z-score is calculated using the formula: $Z = (X - \mu) / \sigma$, where X is the individual height, μ is the mean height, and σ is the standard deviation.

Given an adult man who is 72 inches tall, what is his Z-score?

$$Z = (72 - 69.0) / 2.8 = 3 / 2.8 \approx 1.07.$$

What does a Z-score of 2.0 indicate about an adult man's

height?

It indicates that the man's height is 2 standard deviations above the mean, meaning he is taller than approximately 97.5% of adult men.

How can the Z-score help determine if an individual's height is typical?

A Z-score between -2 and 2 generally indicates a typical or normal height within the population distribution.

If an adult man has a height of 64 inches, what is his Z-score?

$$Z = (64 - 69.0) / 2.8 = -5 / 2.8 \approx -1.79.$$

Why is understanding Z-scores important in analyzing adult men's heights?

Z-scores help identify how unusual or typical a specific height is relative to the population's mean and variability.

What is the significance of a negative Z-score in this context?

A negative Z-score indicates the height is below the mean height of adult men.

Can the Z-score be used to compare heights across different populations?

Yes, Z-scores standardize individual data points, allowing comparisons across different populations with different means and standard deviations.

What is the Z-score for the mean height of 69 inches?

$$Z = (69 - 69.0) / 2.8 = 0 / 2.8 = 0, \text{ meaning the mean height has a Z-score of zero.}$$

How does knowing the Z-score assist in identifying outliers in adult men's heights?

Heights with Z-scores greater than 3 or less than -3 are considered outliers, indicating unusually tall or short individuals compared to the population.

Additional Resources

Understanding Z-Scores in the Context of Adult Male Heights: A Comprehensive Review

Introduction

Statistics serve as an essential tool in understanding and interpreting data across numerous fields, including health sciences, demographics, psychology, and social sciences. One of the fundamental concepts within statistics is the Z-score, which provides a standardized way to interpret individual or observed data points in relation to a population. When analyzing heights of adult men, calculating the Z-score allows us to determine how typical or atypical a specific height is within that population.

This review delves into the concept of Z-scores, especially focusing on adult male heights characterized by a mean (μ) of 69.0 inches and a standard deviation (σ) of 2.8 inches. We'll explore what Z-scores represent, how to calculate them, their significance in statistical analysis, and practical applications, all while emphasizing a deep understanding suitable for both students and professionals.

Fundamental Concepts of Z-Score

What is a Z-Score?

A Z-score is a statistical measurement that describes a data point's position relative to the mean of a group. It indicates how many standard deviations an individual data point is away from the population mean.

Mathematically, the Z-score for a data point (X) is calculated as:

$$Z = \frac{X - \mu}{\sigma}$$

Where:

- (X) = the individual data point (height in this context)
- (μ) = the population mean
- (σ) = the population standard deviation

Interpretation of Z-scores:

- $Z = 0$: The data point is exactly at the mean.
- $Z > 0$: The data point is above the mean.
- $Z < 0$: The data point is below the mean.
- Magnitude of Z : Indicates how far the point is from the mean, measured in standard deviations.

Why Are Z-Scores Important?

Z-scores are invaluable because they:

- Standardize data from different distributions for comparison.
- Enable calculation of probabilities associated with a data point (via standard normal distribution).
- Assist in identifying outliers or unusual observations.
- Facilitate decision-making based on statistical thresholds.

The Context: Adult Male Heights

Given the parameters:

- Mean height, $(\mu = 69.0)$ inches
- Standard deviation, $(\sigma = 2.8)$ inches

This data suggests that the heights of adult men are approximately normally distributed around 69 inches, with most individuals falling within a few inches of this mean. Understanding the distribution allows us to interpret individual heights in terms of their Z-scores.

Calculating Z-Score: Step-by-Step

Suppose we wish to find the Z-score for a specific adult man's height. Let's explore the general process, followed by a concrete example.

Step 1: Identify the Height of Interest

Assume an adult man is 74 inches tall. We want to determine how this height compares to the average.

Step 2: Apply the Z-Score Formula

$$Z = \frac{X - \mu}{\sigma} = \frac{74 - 69.0}{2.8}$$

Calculating numerator:

$$74 - 69.0 = 5$$

Dividing by the standard deviation:

$$Z = \frac{5}{2.8} \approx 1.7857$$

Result: The Z-score is approximately +1.79.

Step 3: Interpret the Z-Score

A Z-score of approximately +1.79 indicates that this man's height is about 1.79 standard deviations above the mean. In the context of a normal distribution, this is relatively high but not extremely rare.

Practical Applications of Z-Scores in Height Data

Understanding Z-scores for adult male heights has numerous real-world applications, such as:

1. Identifying Outliers

- Heights with Z-scores beyond ± 2 or ± 3 are often considered outliers.
- For instance, a man with a Z-score of $+3.0$ (roughly 3 standard deviations above the mean) would be significantly taller than the average, possibly qualifying as an outlier or exceptionally tall individual.

2. Medical and Health Assessments

- Z-scores help determine growth patterns or anomalies.
- For example, a height Z-score below -2 might indicate growth retardation, while above $+2$ could suggest unusually tall stature due to genetic or medical factors.

3. Population Comparisons

- Z-scores allow comparing heights across different populations or demographic groups, where means and standard deviations might differ.
- This standardization is crucial in cross-cultural or international health studies.

4. Probabilistic Analysis

- Using standard normal distribution tables or software, one can calculate the probability of observing a height less than or greater than a specific value.
- For example, what proportion of adult men are taller than 74 inches? Using the Z-score, we can find this probability.

Calculating Probabilities Using Z-Scores

Once a Z-score is known, it can be translated into a probability. The standard normal distribution provides the cumulative probability associated with a Z-score.

Example: Find the probability that an adult man is taller than 74 inches.

Step 1: Calculate the Z-score (already done):

$$Z \approx 1.79$$

Step 2: Use a Z-table or software to find $P(Z < 1.79)$:

- From standard normal tables, $P(Z < 1.79) \approx 0.9633$

Step 3: Find the probability of being taller than 74 inches:

$$P(X > 74) = 1 - P(Z < 1.79) = 1 - 0.9633 = 0.0367$$

Interpretation: Approximately 3.67% of adult men are taller than 74 inches.

Implications of Z-Score Calculations in Research and Policy

Understanding the distribution and Z-scores of heights can influence several domains:

- Public Health: Establishing growth standards and identifying health issues related to stature.
- Clothing and Apparel Industry: Designing sizes based on statistical height data.
- Ergonomics and Architecture: Ensuring spaces accommodate the height distribution of the population.
- Sports and Talent Identification: Recognizing individuals with exceptional height for specific sports or activities.

Limitations and Considerations

While Z-scores are powerful, they come with certain caveats:

- Assumption of Normality: Z-score calculations assume the data follows a normal distribution. Deviations from normality can lead to inaccurate interpretations.
- Population Variability: The parameters (mean and standard deviation) must be accurate and representative.
- Outliers: Extreme outliers can skew the mean and standard deviation, affecting Z-score calculations.
- Contextual Factors: Heights can vary due to genetics, nutrition, health, and ethnicity, which might not be captured solely through Z-scores.

Practical Example: Finding the Z-Score for a Specific Height

Suppose an individual reports a height of 72 inches. Let's compute the Z-score.

$$Z = \frac{72 - 69.0}{2.8} = \frac{3}{2.8} \approx 1.07$$

Interpretation:

- The height is approximately 1.07 standard deviations above the mean.
- From the standard normal table, $P(Z < 1.07) \approx 0.8577$.

This indicates that about 85.77% of adult men are shorter than 72 inches, and roughly 14.23% are taller.

Broader Perspectives and Statistical Significance

Understanding Z-scores extends beyond individual comparisons; it underpins hypothesis testing and confidence interval estimation.

Hypothesis Testing Example:

- Null Hypothesis (H_0): A man's height is typical, i.e., within ± 2 standard deviations.
- Alternative Hypothesis (H_1): The man's height is significantly different from the average.

If a Z-score exceeds ± 2 , it suggests the observation is statistically unusual at approximately the 95% confidence level.

Conclusion

The calculation and interpretation of Z-scores in the context of adult male heights provide a window into the population's distribution and individual standing within it. With a mean height of 69.0 inches and a standard deviation of 2.8 inches, Z-scores serve as a standardized metric to compare, analyze, and understand height data meaningfully.

Whether assessing individual health, conducting research, or designing inclusive products, Z-scores are indispensable in translating raw data into actionable insights. Their importance lies in their simplicity, universality, and the powerful probabilistic framework they offer, making them a cornerstone of statistical analysis in height and beyond.

References

- Moore, D. S., McCabe, G. P., & Craig, B. A. (2012). Introduction to the Practice of Statistics. W. H. Freeman.
- Ott, R. L., & Longnecker, M. (2010). An Introduction to Statistical Methods and Data Analysis. Cengage Learning.
- National Health and Nutrition Examination Survey (NHANES) Data on Adult Male Heights.
- Standard Normal Distribution Tables and Z-Score Resources.

Note: The calculations here are illustrative; actual probabilities and interpretations should be based on current and specific population data.

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