

Algebraically Determine The Solution(s) To The Equation Below And Verify Your Solution(s). (3 Marks Total)

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Introduction to Solving Algebraic Equations

In mathematics, solving algebraic equations is a fundamental skill that underpins many advanced topics and real-world applications. Whether you're tackling simple linear equations or complex quadratic expressions, mastering the process of algebraic solution and verification is essential. This article provides a detailed, step-by-step guide on how to algebraically determine the solutions to a given equation and verify those solutions to ensure accuracy. By understanding these procedures, students and enthusiasts can confidently approach similar problems and enhance their problem-solving skills.

Understanding the Importance of Solution Verification

Before diving into solving equations, it's crucial to recognize why verifying solutions is necessary. When solving algebraic equations, it's possible to arrive at extraneous solutions—values that satisfy the manipulated form of the equation but not the original. Verification helps confirm the validity of solutions, thus ensuring correctness and avoiding potential errors that could impact subsequent calculations or interpretations.

Step 1: Analyzing the Given Equation

The first step involves carefully examining the equation. For illustration, consider the following example:

Equation:

$$\backslash[2x + 5 = 11 \backslash]$$

This is a simple linear equation, but the principles apply broadly to more complex equations. When presented with a specific equation in a problem, identify the type of equation (linear, quadratic, rational, etc.) and determine the most suitable solving method.

Step 2: Algebraic Methods to Solve the Equation

Depending on the form of the equation, different algebraic techniques are employed. Below are common methods:

2.1 Solving Linear Equations

Linear equations are equations of the first degree, typically written as $\backslash(ax + b = 0 \backslash)$. To solve:

1. Isolate the variable term (e.g., $\backslash(2x \backslash)$)
2. Simplify the equation
3. Divide or multiply both sides to find the value of $\backslash(x \backslash)$

Example:

Given

$$\backslash[2x + 5 = 11 \backslash]$$

Solution:

- Subtract 5 from both sides:

$$\backslash[2x = 11 - 5 \backslash]$$

$$\backslash[2x = 6 \backslash]$$

- Divide both sides by 2:

$$\backslash[x = \frac{6}{2} \backslash]$$

$$\backslash[x = 3 \backslash]$$

2.2 Solving Quadratic Equations

Quadratic equations are second-degree and generally take the form $\backslash(ax^2 + bx + c = 0 \backslash)$. Methods include:

- Factoring
- Completing the square
- Quadratic formula

Example:

$$\text{Solve } \backslash(x^2 - 5x + 6 = 0 \backslash)$$

Solution:

- Factoring:

$$\backslash[(x - 2)(x - 3) = 0 \backslash]$$

- Set each factor equal to zero:

$$\backslash[x - 2 = 0 \rightarrow x = 2 \backslash]$$

$$\backslash[x - 3 = 0 \rightarrow x = 3 \backslash]$$

2.3 Solving Rational Equations

Rational equations involve fractions. To solve:

- Find a common denominator
- Multiply through by the LCD to clear fractions
- Solve the resulting equation
- Check for extraneous solutions

Example:

$$\text{Solve } \backslash(\frac{1}{x} + 2 = 3 \backslash)$$

Solution:

- Subtract 2:

$$\backslash[\frac{1}{x} = 1 \backslash]$$

- Cross-multiplied:

$$\backslash[1 = x \backslash]$$

- Verify that $\backslash(x \neq 0 \backslash)$ (since division by zero is undefined).

Solution: $\backslash(x = 1 \backslash)$

Step 3: Verifying Your Solution(s)

Verification is a crucial step to confirm that the solutions obtained satisfy the original equation. Follow these steps:

3.1 Substitute the Solution into the Original Equation

Replace the variable with each solution and check if the equation holds true.

Example:

For $\backslash(x = 3 \backslash)$ in the earlier linear equation:

$$\backslash[2(3) + 5 = 11 \backslash]$$

$$\backslash[6 + 5 = 11 \backslash]$$

$$\backslash[11 = 11 \backslash]$$

Since both sides are equal, $\backslash(x = 3 \backslash)$ is a valid solution.

3.2 Check All Solutions (Especially for Quadratic and Rational Equations)

Quadratic equations can have two solutions; verify each:

- For $\backslash(x = 2 \backslash)$:

$$\backslash[(2)^2 - 5(2) + 6 = 0 \backslash]$$

$$\backslash[4 - 10 + 6 = 0 \backslash]$$

$$\backslash[0 = 0 \backslash] \text{ (valid)}$$

- For $\backslash(x = 3 \backslash)$:

$$\backslash[9 - 15 + 6 = 0 \backslash]$$

$$\backslash[0 = 0 \backslash] \text{ (valid)}$$

Note: For rational equations, ensure the solutions do not make any denominators zero.

Step 4: Handling Common Errors and Special Cases

While solving equations, be mindful of potential pitfalls:

- Extraneous solutions: Arise from operations like squaring both sides or clearing fractions. Always verify.
- Division by zero: Solutions that make denominators zero are invalid.
- Multiple solutions: Quadratic and higher-degree equations may have multiple solutions; verify each.
- No solution cases: Some equations may have no solutions; verify by attempting to isolate variables.

Step 5: Practice Problems for Mastery

To solidify your understanding, try solving and verifying the following equations:

1. Solve $(3x - 4 = 2x + 1)$ and verify.
2. Solve $(x^2 + 4x + 4 = 0)$ and verify.
3. Solve $(\frac{2}{x} = 5)$ and verify.
4. Solve $(2x^2 - 8x = 0)$ and verify.

Conclusion

Algebraically determining the solution(s) to an equation and verifying those solutions are essential skills in mathematics that deepen understanding and ensure correctness. The process involves identifying the type of equation, applying suitable algebraic techniques, and confirming the solutions by substitution. Remember, verification helps catch extraneous solutions and confirms that the solutions satisfy the original problem. Practice regularly with various types of equations to build confidence and proficiency. Mastery of these skills not only prepares you for academic assessments worth 3 marks or more but also lays a solid foundation for advanced mathematical concepts and real-world problem-solving.

Keywords for SEO Optimization

- Algebraic solutions
- Solving equations step-by-step
- Verify solutions in algebra
- Linear equations solutions
- Quadratic equations solving methods
- Rational equations solving
- Extraneous solutions
- Algebra practice problems
- How to verify solutions
- Mathematical problem-solving techniques

Frequently Asked Questions

What is the first step to algebraically determine the solution to an equation?

The first step is to simplify both sides of the equation and isolate the variable by using inverse operations such as addition, subtraction, multiplication, or division.

How do you verify the solutions obtained from solving an algebraic equation?

You verify the solutions by substituting each solution back into the original

equation to check if both sides are equal.

What should you do if, after substitution, the equation does not hold true?

If the equation does not hold true upon substitution, then the solution is incorrect or extraneous, and you should re-express and re-solve the equation.

Can equations have more than one solution? How do you find all solutions?

Yes, equations can have multiple solutions. To find all solutions, you should solve the equation completely and consider all possible values that satisfy the equation.

What is an example of solving a simple linear equation algebraically?

For example, to solve $2x + 3 = 7$, subtract 3 from both sides to get $2x = 4$, then divide both sides by 2 to find $x = 2$.

Why is it important to verify your solutions in algebra?

Verifying solutions ensures that no extraneous solutions were introduced during solving and confirms that the solutions are valid for the original equation.

What tools or methods can help in solving algebraic equations efficiently?

Using inverse operations, simplifying expressions, and checking solutions by substitution are key methods. Additionally, tools like algebraic calculators or software can assist in solving complex equations.

Additional Resources

Algebraically Determine The Solution(s) To The Equation Below And Verify Your Solution(s)

Introduction

In the realm of algebra, solving equations accurately and efficiently is a fundamental skill that underpins more advanced mathematical concepts. The task of algebraically determining solutions to an equation—along with verifying their validity—is essential not only in academic settings but also in real-world applications such as engineering, computer science, and economics. This article aims to explore the meticulous process involved in solving an algebraic equation, emphasizing the importance of systematic problem-solving, verification of solutions, and clear presentation of findings. By dissecting a specific example, we will demonstrate the step-by-

step approach necessary for accurate solutions and the critical role of verification in ensuring these solutions are correct.

Understanding the Problem

Suppose we are given the following algebraic equation:

Equation:

$$\backslash[2x^2 - 5x = 3 \backslash]$$

Our goal is to:

- Algebraically determine all solutions to this equation.
- Verify each solution to confirm its correctness.
- Discuss the implications of the solution process and the importance of verification.

This particular quadratic equation serves as an excellent example because it involves multiple steps, including rearrangement, factoring or applying the quadratic formula, and verification.

Step 1: Rearrangement of the Equation

The first step in solving any algebraic equation is to write it in a standard, manageable form. The given equation is:

$$\backslash[2x^2 - 5x = 3 \backslash]$$

Subtract 3 from both sides to set the equation to zero:

$$\backslash[2x^2 - 5x - 3 = 0 \backslash]$$

This standard quadratic form is:

$$\backslash[ax^2 + bx + c = 0 \backslash]$$

where $\backslash(a = 2 \backslash)$, $\backslash(b = -5 \backslash)$, and $\backslash(c = -3 \backslash)$.

Step 2: Choosing the Solution Method

Quadratic equations can be solved via:

- Factoring (if factorable)
- Completing the square
- Quadratic formula

Given the coefficients, the quadratic formula is often the most straightforward and universally applicable method. The quadratic formula is:

$$\backslash[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \backslash]$$

Step 3: Applying the Quadratic Formula

Plugging in the coefficients:

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 2 \times (-3)}}{2 \times 2}$$

Simplify numerator and denominator:

$$x = \frac{5 \pm \sqrt{25 - 4 \times 2 \times (-3)}}{4}$$

Calculate discriminant:

$$25 - 4 \times 2 \times (-3) = 25 + 24 = 49$$

Since the discriminant is positive, there are two real solutions.

Calculate the square root:

$$\sqrt{49} = 7$$

Now, find the two solutions:

$$x = \frac{5 \pm 7}{4}$$

Solution 1:

$$x = \frac{5 + 7}{4} = \frac{12}{4} = 3$$

Solution 2:

$$x = \frac{5 - 7}{4} = \frac{-2}{4} = -\frac{1}{2}$$

Thus, the solutions are:

$$x = 3 \quad \text{and} \quad x = -\frac{1}{2}$$

Step 4: Verifying the Solutions

Verification is a critical step to confirm the solutions satisfy the original equation, especially since algebraic manipulations can sometimes lead to extraneous solutions (particularly when dealing with square roots or denominators).

Verify $(x = 3)$:

Substitute into the original equation:

$$2(3)^2 - 5(3) = 3$$

$$2 \times 9 - 15 = 3$$

$$18 - 15 = 3$$

$$3 = 3$$

Result: Valid solution.

Verify $(x = -\frac{1}{2})$:

Substitute into the original equation:

$$2 \left(-\frac{1}{2}\right)^2 - 5 \left(-\frac{1}{2}\right) = 3$$

Calculate each term:

```
\[ 2 \times \frac{1}{4} + \frac{5}{2} = 3 \]
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```
\[ \frac{2}{4} + \frac{5}{2} = 3 \]
```

```
\[ \frac{1}{2} + \frac{5}{2} = 3 \]
```

```
\[ \frac{1 + 5}{2} = 3 \]
```

```
\[ \frac{6}{2} = 3 \]
```

```
\[ 3 = 3 \]
```

Result: Valid solution.

Both solutions satisfy the original equation, confirming their correctness.

Deep Dive: The Significance of Verification in Algebra

Verification is more than a procedural step; it is an essential safeguard in algebraic problem-solving. It ensures that solutions derived through algebraic manipulations are genuine and not extraneous. For more complex equations, especially those involving radicals, absolute values, or rational expressions, solutions can sometimes arise that do not satisfy the original conditions—highlighting the importance of checking each candidate solution within the context of the initial problem.

In our case, the quadratic formula method is reliable, but verification still confirms the solutions' validity, reinforcing the importance of diligence.

Broader Implications and Real-World Applications

Solving quadratic equations algebraically is foundational in many disciplines:

- Physics: Calculating projectile trajectories involves quadratic equations.
- Economics: Finding maximum profit or minimum cost often reduces to solving quadratics.
- Engineering: Structural analysis and design frequently depend on solving quadratic relations.
- Computer Science: Algorithms involving quadratic functions are used in data analysis and graphics.

Moreover, the process of verification ensures the robustness of solutions in scenarios where errors in initial assumptions or calculations could lead to flawed conclusions.

Conclusion

The algebraic determination of solutions to equations like $(2x^2 - 5x = 3)$ exemplifies the systematic approach necessary for accurate mathematical problem-solving. By transforming the equation into standard form, applying the quadratic formula, and rigorously verifying each solution, we ensure the integrity of the results. This process underscores the importance of thoroughness and critical verification in mathematics, which are essential skills across scientific and engineering disciplines. As demonstrated, solutions such as $(x = 3)$ and $(x = -\frac{1}{2})$ stand validated,

exemplifying effective problem-solving practices rooted in algebraic principles.

References

- Stewart, J. (2015). Precalculus: Mathematics for Calculus. Cengage Learning.
- Larson, R., & Edwards, B. H. (2017). Precalculus with Limits: A Graphing Approach. Cengage Learning.
- Art of Problem Solving. (n.d.). Quadratic Equations. Retrieved from <https://artofproblemsolving.com>

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