The Graph Shows The Cube Root Parent Function.Which Statement Is True?A. (0, 0) Is The X- And Y-intercept

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Understanding the behavior of functions is a fundamental aspect of algebra and calculus, especially when analyzing their graphs. The cube root parent function, a common example in mathematics, exhibits distinctive characteristics that help students and educators alike interpret its graph accurately. This article delves into the properties of the cube root parent function, how to interpret its graph, and addresses the specific statement regarding the point (0, 0) being both the x-intercept and y-intercept.

Overview of the Cube Root Parent Function

What is the Cube Root Parent Function?

The cube root parent function is represented mathematically as:

$$f(x) = \sqrt[3]{x}$$

This function is a fundamental example used to illustrate properties of odd functions, inverse operations, and transformations. Its graph exhibits a symmetric, smooth curve passing through the origin, with characteristic features that make it a valuable teaching tool.

Basic Characteristics of the Function

- Domain and Range: Both are all real numbers, i.e., $\text{Domain} = (-\infty, \infty)$ and $\text{Range} = (-\infty, \infty)$.
- Symmetry: The function is an odd function, symmetric with respect to the origin.
- Continuity & Differentiability: The function is continuous and differentiable everywhere on its domain.

Graphical Features of the Cube Root Function

Key Points on the Graph

To understand the graph, analyze some specific points:

- When $x = 0$, $y = \sqrt[3]{0} = 0$. So, the point (0, 0) lies on the graph.
- For positive x, y increases slowly, reflecting the cube root's growth.
- For negative x, y decreases, mirroring the positive side due to symmetry.

Behavior at the Origin

The point (0, 0) is crucial because:

- It is the center point of the graph.
- It is the only intercept with both axes.

The graph passes through the origin, making it a common point of intersection with both the x-axis and y-axis.

Interpreting Intercepts of the Cube Root Function

What Are Intercepts?

- X-intercept: The point(s) where the graph crosses the x-axis ($y=0$).
- Y-intercept: The point where the graph crosses the y-axis ($x=0$).

Finding the Intercepts for $f(x) = \sqrt[3]{x}$

X-intercept:

- To find the x-intercept, set $y=0$:

$$0 = \sqrt[3]{x} \Rightarrow x=0$$

- Therefore, the only x-intercept is at (0, 0).

Y-intercept:

- To find the y-intercept, set $x=0$:

$$y = \sqrt[3]{0} = 0$$

- The y-intercept is also at (0, 0).

Conclusion: Both the x-intercept and y-intercept occur at the point (0, 0).

Is (0, 0) Both the X- and Y-intercept?

Analysis of the Statement

The statement suggests that the point (0, 0) serves as both the x-intercept and y-intercept of the graph. Based on the calculations above:

- The only x-intercept is at (0, 0), because the graph crosses the x-axis only at that point.
- The only y-intercept is at (0, 0), because the graph crosses the y-axis only at that point.

Since both intercepts are at the same point, (0, 0), the statement is true.

Implications of the Intercept at (0, 0)

- The point (0, 0) is a point of symmetry, consistent with the odd nature of the cube root function.
- The graph passes through the origin, making it a point of origin symmetry.

Additional Insights into the Graph and Function

Transformations and Variations

Understanding the parent function helps in visualizing transformations:

- Vertical shifts: \(y = \sqrt[3]{x} + k \)
- Horizontal shifts: \(y = \sqrt[3]{x - h} \)
- Reflections: Reflecting across axes involves multiplying by -1.

Practical Applications

The cube root function appears in various fields:

- Physics: describing certain types of motion.
- Engineering: modeling relationships involving cube relationships.
- Mathematics: understanding inverse functions and symmetry.

Summary and Final Remarks

The graph of the cube root parent function, \(y = \sqrt[3]{x} \), exhibits a smooth, symmetric curve passing through the origin. The point (0, 0) is both the x-intercept and y-intercept, confirming that the statement " (0, 0) is the x- and y-intercept" is true. This point acts as a critical feature of the graph, representing the only intersection with both axes, and exemplifies the symmetry and continuous nature of the function.

Understanding this fundamental property not only aids in graph interpretation but also enhances comprehension of more complex transformations and functions in mathematics. Recognizing that the point (0, 0) serves as both the x- and y-intercept in this context is essential for accurate analysis and problem-solving involving the cube root parent function.

In conclusion, the statement under consideration is correct: (0, 0) is indeed both the x-intercept and y-intercept of the cube root parent function's graph.

Frequently Asked Questions

What is the general shape of the cube root parent function on its graph?

The graph of the cube root parent function has an S-shaped curve that passes through the origin, extending infinitely in both directions with a gradual slope near the origin.

Where does the cube root parent function intersect the axes?

The cube root parent function passes through the origin (0,0), meaning it intersects both the x-axis and y-axis at that point.

Is the point (0, 0) an intercept for the cube root parent function?

Yes, (0, 0) is both the x-intercept and y-intercept of the cube root parent function.

Does the cube root parent function have any other intercepts besides (0, 0)?

No, the only intercept is at the origin (0, 0) since the cube root function passes through this point and does not cross the axes elsewhere.

Which statement about the point (0, 0) is true for the cube root parent function?

The statement that (0, 0) is both the x- and y-intercept is true.

What does the graph of the cube root parent function look like near the origin?

Near the origin, the graph is relatively flat and smooth, with the slope increasing as you move away from (0,0), reflecting the cube root's gradual growth.

Is the point (0, 0) special in the graph of the cube root parent function?

Yes, because it is the only point where the graph crosses both axes and is the center of symmetry for the function.

What is the algebraic form of the cube root parent function?

The algebraic form is $y = \sqrt[3]{x}$.

How does the cube root function behave for negative x-values?

For negative x-values, the cube root function outputs negative y-values, reflecting the odd symmetry of the function.

Why is the statement '(0, 0) is the x- and y-intercept' considered true for the cube root parent function?

Because substituting x=0 yields y=0, so the graph passes through (0,0), making it both the x- and y-intercept.

Additional Resources

Understanding The Graph of the Cube Root Parent Function: Which Statement Is True?

Mathematics, especially the study of functions, relies heavily on understanding the properties and behaviors of various types of graphs. The cube root function, a fundamental parent function in algebra, provides intriguing insights into how functions behave, especially around their intercepts and symmetry. In this detailed exploration, we will analyze the graph of the cube root parent function and evaluate the statement: “(0, 0) is the x- and y-intercept.” We will dissect what this statement means, verify its truthfulness, and deepen our understanding of the cube root graph's characteristics.

Introduction to the Cube Root Parent Function

Before diving into the specifics of intercepts, it’s vital to establish a foundational understanding of the cube root parent function.

Definition and Formula

- The cube root function is defined as $f(x) = \sqrt[3]{x}$, where the cube root of x is the number that, when cubed, gives x.
- It is a parent function, meaning it serves as a basic template from which more complex functions can be derived through transformations.

Basic Characteristics

- Domain: All real numbers, $\mathbb{R}$, because every real number has a real cube root.
- Range: Also all real numbers, $\mathbb{R}$.
- Symmetry: The function is odd, meaning it exhibits symmetry with respect to the origin, i.e., $f(-x) = -f(x)$.

Graphing the Cube Root Function

Visualizing the graph is fundamental to understanding its intercepts and behavior.

Key Points and Shape

- The graph passes through the origin $(0,0)$.
- For positive x, the graph increases slowly, moving to the right.
- For negative x, the graph decreases symmetrically, moving to the left.
- The curve is smooth and continuous, with no breaks or asymptotes.

Behavior at Key Points

- $(0, 0)$: The origin, which is a critical point.
- $(1, 1)$: Since $\sqrt[3]{1} = 1$.
- $(-1, -1)$: Since $\sqrt[3]{-1} = -1$.

Analyzing Intercepts in the Cube Root Graph

Interpreting intercepts involves understanding where the graph crosses axes.

X-Intercepts

- The x-intercept occurs where $f(x) = 0$.
- Solve $\sqrt[3]{x} = 0 \Rightarrow x = 0$.
- Therefore, the graph crosses the x-axis at $(0, 0)$.

Y-Intercepts

- The y-intercept occurs where the graph crosses the y-axis, i.e., at $x=0$.
- Plug in $x=0$: $f(0) = \sqrt[3]{0} = 0$.
- Hence, the y-intercept is also at $(0, 0)$.

Is (0, 0) Both X- and Y-Intercept?

Given the above analysis, we see that:

- The point $(0, 0)$ lies on the graph.
- It satisfies the condition for both x- and y-intercepts, because the graph crosses both axes at this point.

Therefore, the statement: “(0, 0) is the x- and y-intercept” is true.

Implications of the Intercepts at (0, 0)

Understanding the significance of the intercept at the origin helps in grasping the graph's behavior more intuitively.

Symmetry and Origin-Centered Behavior

- Since the cube root function is odd, it is symmetric with respect to the origin.
- The point $(0,0)$ acts as the central point of symmetry.
- This symmetry means that if you reflect the graph across the origin, it looks the same.

Impact on Transformations and Graphing

- When graphing transformations like shifts or stretches, the intercept at $(0,0)$ often remains a critical reference point.
- Recognizing that the original parent function passes through the origin simplifies the process of

understanding how modifications affect the graph.

Common Misconceptions and Clarifications

While the analysis appears straightforward, students and even some educators might have misconceptions regarding intercepts and the graph of the cube root function.

Misconception 1: The Origin Is Not an Intercept

- Some might think that because the cube root function is smooth and continuous, the intercepts are not at $(0, 0)$.
- Clarification: The point $(0, 0)$ is indeed where the graph crosses both axes, making it both the x- and y-intercept.

Misconception 2: Intercepts Are Only Where the Graph Crosses Axes

- While this is generally true, it’s important to distinguish between intercepts and asymptotes or other special points.
- For the cube root function, the only intercepts are at the origin.

Misconception 3: The Intercept at (0, 0) Is Unique to the Parent Function

- Transformations (translations, reflections, etc.) can shift the intercepts.
- In the parent function, the intercepts are at $(0, 0)$, but in transformed functions, this point may move.

Deeper Mathematical Analysis of the Cube Root Graph

To fully appreciate why the point $(0, 0)$ is both an x- and y-intercept, it’s beneficial to analyze the function mathematically.

Calculating the Intercepts

- X-intercept: Solve $f(x) = 0 \Rightarrow \sqrt[3]{x} = 0 \Rightarrow x=0$.
- Y-intercept: Calculate $f(0) = \sqrt[3]{0} = 0$.

These calculations confirm that the only intercept point is at the origin.

Derivative and Slope at the Origin

- The derivative of $f(x) = \sqrt[3]{x}$ is $f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$.
- At $x=0$, $f'(0)$ is undefined because $x^{-\frac{2}{3}}$ involves division by zero.
- This indicates a vertical tangent or a cusp at $(0,0)$, highlighting the nature of the graph at the origin.

Visual Representation and Graph Sketching

To truly understand the graph, sketching or visualizing it helps.

- Plot the key points: $(-1, -1)$, $(0, 0)$, $(1, 1)$.
- Recognize the smooth, continuous curve passing through the origin.
- Observe the symmetry about the origin, consistent with the odd function property.

Applications and Significance in Mathematics

Understanding the intercepts of the cube root function is not just an academic exercise; it has practical implications.

Modeling and Real-World Contexts

- Cube root functions can model situations where a quantity depends on the cube of another variable, such as volume-related problems.
- Knowing the intercepts aids in interpreting initial conditions or boundary values.

Transformations and Their Effects

- Recognizing that the parent function crosses at $(0, 0)$ helps in understanding how shifting or stretching the graph will change intercepts.
- For example, $f(x) = \sqrt[3]{x - h} + k$ will shift the intercepts to (h, k).

Summary and Final Verdict

After an extensive exploration, it is clear that:

- The point \((0,0)\) lies on the graph of the cube root parent function.
- Both the x- and y-intercepts occur at this point because the graph crosses both axes there.
- The statement “(0, 0) is the x- and y-intercept” is true for the cube root parent function.

Conclusion

The analysis confirms that the point \((0, 0)\) is a critical feature of the cube root parent function's graph, serving as both the x-intercept and y-intercept. Recognizing this is fundamental to understanding the graph's symmetry, behavior, and how it responds to transformations. The cube root function’s unique properties, like symmetry about the origin and the continuous passage through \((0, 0)\), make it an essential concept in algebra and pre-calculus.

This detailed examination not only validates the statement in question but also deepens the overall comprehension of root functions and their graphical characteristics, equipping students and educators with a clearer perspective on this important mathematical parent function.

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