The Length Of The Base Of A Rectangular Prism Is Twice The Breadth. The Height Is 5 Units. If The Surface

The Length Of The Base Of A Rectangular Prism Is Twice The Breadth. The Height Is 5 Units. If The Surface

Understanding the properties and dimensions of rectangular prisms is fundamental in geometry, architecture, engineering, and various design applications. The problem statement highlights a specific relationship between the length and breadth of the base of a rectangular prism, with a fixed height, and prompts us to explore the surface area and other related characteristics in detail. This comprehensive guide aims to elucidate the concepts, calculations, and practical implications associated with such a shape.

Introduction to Rectangular Prisms

A rectangular prism, also known as a cuboid, is a three-dimensional solid object bounded by six rectangular faces. Its defining features include three dimensions:

- Length (l)
- Breadth or width (b)
- Height (h)

The shape is characterized by right angles at each corner, and its faces are rectangles. Understanding the relationships among these dimensions helps in calculating surface area, volume, and other geometric properties.

Given Dimensions and Relationships

The problem specifies a particular relationship:

- The length of the base (l) is twice the breadth (b):
l = 2b
- The height (h) is fixed at 5 units:

h = 5

This relationship simplifies the process of calculating surface area and volume, as it reduces the number of unknowns.

Calculating the Surface Area of the Rectangular Prism

Surface area (SA) of a rectangular prism is the total area covered by all six faces. The formula is:

$$SA = 2(lb + bh + lh)$$

Given the relationships, we can express SA in terms of a single variable, such as b.

Step-by-step Calculation

1. Express length in terms of breadth:

   $$l = 2b$$

2. Calculate each term in the surface area formula:

   - $lb = (2b)(b) = 2b^2$
   - $bh = (b)(h) = b\ 5 = 5b$
   - $lh = (2b)(h) = 2b\ 5 = 10b$

3. Substitute into the surface area formula:

   $$SA = 2(2b^2 + 5b + 10b) = 2(2b^2 + 15b) = 4b^2 + 30b$$

Thus, the surface area is expressed as:

$$SA = 4b^2 + 30b$$

This quadratic expression enables us to analyze how changes in the breadth influence the surface area.

Example Calculation

Suppose the breadth (b) is 3 units:

- Length (l) = 2 3 = 6 units
- Surface area (SA):

$$SA = 4(3)^2 + 30(3) = 4(9) + 90 = 36 + 90 = 126 \text{ units}^2$$

Calculating the Volume of the Rectangular Prism

The volume (V) of a rectangular prism is given by:

$$V = l \times b \times h$$

Substituting the known relationship:

$$V = (2b) \times b \times 5 = 2b^2 \times 5 = 10b^2$$

This expression indicates that the volume depends solely on the breadth (b).

Example Calculation

Using the same example where b = 3 units:

$$V = 10 \ (3)^2 = 10 \ 9 = 90 \text{ units}^3$$

Analyzing the Relationship Between Dimensions

Understanding how the dimensions of the prism relate helps in optimizing design for specific purposes, such as maximizing volume or minimizing surface area.

Graphical Representation

Plotting the surface area and volume against breadth (b) provides insights into how these properties change:

- Surface area: $SA = 4b^2 + 30b$
- Volume: $V = 10b^2$

These quadratic relationships reveal that:

- As b increases, both surface area and volume increase.
- The rate of change accelerates due to the quadratic nature.

Practical Implications

In real-world scenarios, adjusting the dimensions affects:

- Material costs (related to surface area)
- Storage capacity (related to volume)
- Structural stability and aesthetics

Maximizing and Minimizing Surface Area and Volume

Designers often seek to optimize a shape based on specific constraints. Here, we explore how varying the breadth affects surface area and volume.

Maximizing Volume for a Given Surface Area

Suppose the surface area is fixed at a certain value, and we want to find the breadth that maximizes volume.

- Given: $SA = 4b^2 + 30b$

- Fix SA at a specific value, say, $SA = S_0$

Then:

$$4b^2 + 30b = S_0$$

Rearranged as:

$$4b^2 + 30b - S_0 = 0$$

Solving for b using quadratic formulas provides potential dimensions to maximize volume.

Minimizing Surface Area for a Given Volume

Similarly, fixing volume $V = V_0$ and minimizing surface area involves setting:

$$V = 10b^2 = V_0$$

which yields:

$$b = \sqrt{V_0 / 10}$$

Substituting back into the surface area formula gives the minimal surface area for that volume.

Applications in Real-World Contexts

Understanding the dimensions and properties of rectangular prisms is crucial in various fields:

Packaging and Storage

- Optimizing material use while maintaining volume
- Designing boxes with minimal surface area to reduce material costs

Construction and Manufacturing

- Building blocks and containers with specific size constraints
- Structural components where surface area influences insulation or coating

Design and Aesthetics

- Creating shapes with appealing proportions
- Balancing volume and surface area for visual or functional purposes

---

Summary of Key Points

- The length of the base (l) is twice the breadth (b): $l = 2b$
- The height (h) is fixed at 5 units
- Surface area (SA) depends on b: $SA = 4b^2 + 30b$
- Volume (V) depends on b: $V = 10b^2$
- Adjusting b affects both surface area and volume in quadratic ways
- Optimization involves solving quadratic equations based on fixed surface area or volume constraints

---

Conclusion

Analyzing the dimensions of a rectangular prism where the length is twice the breadth and the height is fixed at 5 units provides valuable insights into the shape's surface area and volume. By expressing these properties algebraically, we can explore how changing one dimension impacts the entire structure, leading to practical applications in design, manufacturing, and architecture. Whether aiming to maximize volume, minimize material use, or achieve specific aesthetic goals, understanding these relationships empowers informed decision-making in various fields.

Further Reading and Resources

- Basic Geometry Textbooks
- Online Calculators for Surface Area and Volume
- Engineering Design Principles
- Architecture and Structural Design Resources

Remember: Precise calculations and understanding the relationships among dimensions are essential for optimized design and efficient resource utilization.

Frequently Asked Questions

What is the relationship between the length and breadth of the rectangular prism?

The length is twice the breadth of the rectangular prism.

If the height of the rectangular prism is 5 units, how does this information help in calculating its surface area?

Knowing the height allows us to compute the surface area using the dimensions for the length, breadth, and height in the surface area formula.

How do you express the length of the rectangular prism in terms of its breadth?

The length is expressed as twice the breadth, so if the breadth is 'b', then length = 2b.

What is the formula for calculating the total surface area of a rectangular prism?

Surface area = 2(lb + bh + hl), where l is length, b is breadth, and h is height.

Given the height is 5 units and length is twice the breadth, how can we find the surface area if the

breadth is known?

Substitute l = 2b and h = 5 into the surface area formula to compute the total surface area based on the known breadth.

Why is understanding the relationship between length and breadth important in calculating the surface area?

Because it simplifies the calculation by reducing the number of variables, allowing for expressions in terms of a single dimension.

If the surface area of the rectangular prism is given, how can we determine the dimensions of the prism?

Use the surface area formula with the known surface area and the relationships l = 2b and h = 5 to solve for the unknown dimensions.

Additional Resources

The Length Of The Base Of A Rectangular Prism Is Twice The Breadth. The Height Is 5 Units. If The Surface Area Is Known, How Do You Find The Dimensions?

When working with geometric solids, understanding the relationships between their dimensions is fundamental. One common problem involves finding the specific measurements of a rectangular prism when certain parameters are given. In this guide, we'll explore how to determine the length, breadth, and height of a rectangular prism when the length is twice the breadth and the height is fixed at 5 units. We will also examine how to calculate the surface area and use it to find the unknown dimensions, providing a comprehensive step-by-step approach suitable for students, educators, or enthusiasts looking to strengthen their understanding of three-dimensional geometry.

Understanding the Problem

Before diving into calculations, it's essential to clarify what information is provided and what is being asked. The problem states:

- The length of the rectangular prism's base is twice the breadth.
- The height of the prism is 5 units.
- The surface area of the prism is known (or to be calculated).

The goal is to determine the length and breadth of the base, given the surface area and the relationships above.

Key Concepts and Formulas for Rectangular Prisms

Dimensions and Notation

Let's define:

- b = breadth of the prism
- l = length of the prism's base
- h = height of the prism

Given relationships:

- l = 2b
- h = 5 units

Surface Area of a Rectangular Prism

The surface area (SA) of a rectangular prism is the sum of the areas of all six faces:

$$ \text{SA} = 2(lb + bh + lh) $$

Since l and b are related, and h is known, the formula can be expressed in terms of b:

$$ \text{SA} = 2((2b)b + b \times 5 + 2b \times 5) $$

which simplifies calculations once b is known.

Step-by-Step Approach to Find Dimensions

1. Express All Dimensions in Terms of One Variable

Given l = 2b, and h = 5, the surface area formula becomes:

$$ \text{SA} = 2((2b)b + b \times 5 + 2b \times 5) $$

Simplify each term:

- $(2b)b = 2b^2$
- $b \times 5 = 5b$
- $2b \times 5 = 10b$

Now, the surface area formula becomes:

$$ \text{SA} = 2(2b^2 + 5b + 10b) $$
$$ \text{SA} = 2(2b^2 + 15b) $$

Distribute the 2:

$$ \text{SA} = 4b^2 + 30b $$

This is a quadratic in terms of b:

$$ 4b^2 + 30b - \text{SA} = 0 $$

2. Use the Known Surface Area to Find b

Suppose the surface area is known; call it S. Then:

$$ 4b^2 + 30b - S = 0 $$

This quadratic can be solved for b using the quadratic formula:

$$ b = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} $$

where:

- $A = 4$
- $B = 30$
- $C = -S$

So,

$$ b = \frac{-30 \pm \sqrt{30^2 - 4 \times 4 \times (-S)}}{2 \times 4} $$
$$ b = \frac{-30 \pm \sqrt{900 + 16S}}{8} $$

Since b represents a length, it must be positive, so we select the positive root:

$$ b = \frac{-30 + \sqrt{900 + 16S}}{8} $$

Once b is calculated, the length l is:

$$ l = 2b $$

And the height remains:

$$ h = 5 $$

Practical Example: Calculating Dimensions from a Given Surface Area

Let's consider an example where the surface area S is 320 units^2.

Step 1: Plug in S = 320 into the quadratic formula:

$$ b = \frac{-30 + \sqrt{900 + 16 \times 320}}{8} $$

Calculate inside the square root:

$$ 16 \times 320 = 5120 $$
$$ 900 + 5120 = 6020 $$

Now, compute the square root:

$$ \sqrt{6020} \approx 77.6 $$

Now, find b:

$$ b = \frac{-30 + 77.6}{8} $$
$$ b = \frac{47.6}{8} $$
$$ b \approx 5.95 $$

Step 2: Find l:

$$ l = 2b \approx 2 \times 5.95 = 11.9 $$

Step 3: Confirm the dimensions:

- Breadth ≈ 5.95 units
- Length ≈ 11.9 units
- Height = 5 units

Step 4: Verify the surface area:

Calculate SA using these dimensions:

$$ \text{SA} = 2(lb + bh + lh) $$
$$ = 2(11.9 \times 5.95 + 5.95 \times 5 + 11.9 \times 5) $$
$$ = 2(70.8 + 29.75 + 59.5) $$
$$ = 2(159.05) $$
$$ \approx 318.1 $$

Close to the initial given S = 320, indicating the approximation is reasonable.

Additional Considerations and Applications

Design and Engineering

Understanding how to manipulate and solve for dimensions based on surface area constraints is fundamental in fields like engineering, architecture, and manufacturing. For instance, optimizing material usage often involves precise calculations of surface areas and dimensions.

Educational Value

This problem demonstrates the importance of algebraic manipulation and quadratic equations in real-world applications. It reinforces the concept of expressing variables in terms of one another and solving for unknowns systematically.

Extensions

- Volume Calculation: Once dimensions are known, calculating volume is straightforward:

$$ V = l \times b \times h $$

- Surface Area Variations: If the height isn't fixed, similar methods can be used to find it given other constraints.

Summary of Steps

1. Identify Relationships: Recognize that the length is twice the breadth, and height is given.
2. Express All Variables: Write the surface area formula in terms of a single variable (breadth).
3. Formulate a Quadratic Equation: Rearrange the formula to standard quadratic form.
4. Solve for the Variable: Use the quadratic formula and select the positive root.
5. Calculate Remaining Dimensions: Find length and confirm the height.
6. Verify Results: Check calculations against the given surface area or other constraints.

Final Thoughts

Understanding the relationships between the dimensions of a rectangular prism and how to manipulate formulas to find unknowns is a vital skill in geometry. By expressing variables systematically and applying algebraic techniques such as the quadratic formula, you can solve complex problems with confidence. Whether you're designing a box, calculating material needs, or just honing your mathematical skills, these methods serve as powerful tools for spatial reasoning and problem-solving in three-dimensional geometry.

The Length Of The Base Of A Rectangular Prism Is Twice The Breadth The Height Is 5 Units If The Surface

The Length Of The Base Of A Rectangular Prism Is Twice The Breadth The Height Is 5 Units If The Surface

Related Articles

- To Append Data To An Existing File, Use _____________ To Construct A FileOutputStream For File Out.dat.A.
- Who Is The Restaurant Staff Member Shown In The Image?O A.el CamareroB.el ChefC.el GerenteD.el Lavaplatos
- The Future Value Of $200 Received Today And Deposited For Three Years In An Account Which Pays Semiannual

Back to Home