The Points $(1,-2)$ And $(-4,10)$ Are Adjacent Vertices Of A Square. What Is The Perimeter Of The Square?

The Points $(1,-2)$ And $(-4,10)$ Are Adjacent Vertices Of A Square. What Is The Perimeter Of The Square?

Understanding the geometry of squares and their properties is fundamental in solving various mathematical problems. Among these, determining the perimeter of a square when given specific vertices is a common and intriguing challenge. In this article, we will explore how to find the perimeter of a square when two adjacent vertices are known—specifically, the points $(1, -2)$ and $(-4, 10)$. We will delve into the concepts of distance, vectors, and geometric transformations to arrive at the solution, providing a comprehensive guide suitable for students, educators, and math enthusiasts alike.

Introduction to Coordinates and Squares

Before dissecting the problem, it is essential to revisit some fundamental concepts related to coordinate geometry and properties of squares.

Coordinate Geometry Basics

- Coordinates: Points in the plane are represented as ordered pairs (x, y).
- Distance Formula: The distance between two points (x_1, y_1) and (x_2, y_2) is given by:

$$
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

- Vectors: A vector from point $A(x_1, y_1)$ to point $B(x_2, y_2)$ is represented as $\vec{AB} = (x_2 - x_1, y_2 - y_1)$.

Properties of a Square

- All four sides are equal in length.
- Adjacent sides are perpendicular (form a right angle).
- The diagonals are equal in length and bisect each other at right angles.

Given Data and Objective

The problem provides two points:
- $A(1, -2)$

- $B(-4, 10)$

These points are adjacent vertices of the square. The goal is to determine the perimeter of the square.

Step-by-Step Solution Approach

To find the perimeter, we need to determine the length of one side of the square, which requires understanding the position of points A and B, the orientation of the square, and the possible locations of the other vertices.

Step 1: Find the Length of the Side AB

Using the distance formula:
$$
AB = \sqrt{(-4 - 1)^2 + (10 - (-2))^2} = \sqrt{(-5)^2 + (12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13
$$

So, the length of the side AB is 13 units.

Step 2: Recognize the Geometric Implication

Since A and B are adjacent vertices of a square, the side length is 13 units. To find the full perimeter, we need the length of one side (which is known now) and confirm the orientation to identify the other vertices.

Finding the Other Vertices of the Square

The key challenge is that the square's orientation isn't specified. The side AB could be aligned in any direction, and the square could be rotated arbitrarily. To account for this, we analyze possible positions of the other vertices.

Step 3: Determine the Direction and Perpendicular Vectors

- The vector from A to B:

$$
\vec{AB} = (x_2 - x_1, y_2 - y_1) = (-4 - 1, 10 - (-2)) = (-5, 12)
$$

- The magnitude of $\vec{AB}$ is 13, as previously calculated.

- To find the other vertices, we need to find vectors perpendicular to $\vec{AB}$ with the same length (13 units). These vectors will help locate the remaining two vertices of the square.

Perpendicular vectors to $\vec{AB}$:
- Rotate $\vec{AB}$ by 90 degrees:

$$
\vec{v}_1 = (-12, -5)
$$

$$
\vec{v}_2 = (12, 5)
$$

- These vectors are perpendicular to $\vec{AB}$, since their dot product is zero:

$$
(-5)(-12) + (12)(-5) = 60 - 60 = 0
$$

- The length of these vectors:

$$
|\vec{v}_1| = \sqrt{(-12)^2 + (-5)^2} = \sqrt{144 + 25} = \sqrt{169} = 13
$$

$$
|\vec{v}_2| = 13
$$

So, these vectors are suitable for constructing the other vertices.

Step 4: Construct the Remaining Vertices

- Using the perpendicular vectors, the two potential positions for the other vertices are obtained by adding or subtracting these vectors from the known points.

Option 1:

$$
C = B + \vec{v}_1 = (-4, 10) + (-12, -5) = (-16, 5)
$$

$$
D = A + \vec{v}_1 = (1, -2) + (-12, -5) = (-11, -7)
$$

Option 2:

$$
C' = B + \vec{v}_2 = (-4, 10) + (12, 5) = (8, 15)
$$

\[

D' = A + \vec{v}_2 = (1, -2) + (12, 5) = (13, 3)
\]

Both options are valid, representing the two possible orientations of the square.

Calculating the Perimeter

Since the side length is confirmed as 13 units, the perimeter P of the square is:

\[
P = 4 \times \text{side length} = 4 \times 13 = 52
\]

Final Answer and Summary

The perimeter of the square is 52 units.

This problem illustrates key geometric concepts:
- Calculating the distance between two points.
- Understanding the orientation of a square via perpendicular vectors.
- Constructing potential vertices based on known points and vectors.
- Recognizing that the perimeter depends solely on the side length, which can be derived from the initial points.

Additional Insights and Tips

Key Points When Solving Similar Problems:

- Always start with the distance formula to find the length of known sides.
- Use vector operations to determine possible positions of other vertices, especially when the square can be rotated.
- Remember that the perpendicular vectors to a given side have the same length and are essential in constructing the square.
- The perimeter of a square depends only on the side length; the orientation or position in the plane does not affect the perimeter.

Practical Applications:

Understanding how to determine the perimeter of a square given points has applications in fields such as:
- Computer graphics and design.
- Engineering and architectural planning.
- Navigation and geospatial analysis.

Conclusion

Determining the perimeter of a square from two adjacent vertices involves a blend of coordinate geometry, vector analysis, and geometric reasoning. By calculating the side length through the distance formula and understanding the perpendicular directions, we establish that the perimeter of the square with vertices at $(1, -2)$ and $(-4, 10)$ is 52 units. This problem highlights the importance of foundational geometric principles and showcases how coordinate methods can be employed to solve complex spatial problems efficiently.

Frequently Asked Questions

Given the points (1, −2) and (−4, 10) are adjacent vertices of a square, what is the length of the side of the square?

The length of the side is the distance between the two points, which is $\sqrt{[(-4-1)^2 + (10-(-2))^2]} = \sqrt{[(-5)^2 + 12^2]} = \sqrt{[25 + 144]} = \sqrt{169} = 13$.

How do you find the perimeter of the square with vertices at (1, −2) and (−4, 10)?

First, find the side length (which is 13 units), then multiply by 4 (the number of sides): Perimeter = $4 \times 13 = 52$ units.

Are the points (1, −2) and (−4, 10) adjacent vertices of the square along a side or a diagonal?

Since the problem states they are adjacent vertices, these points lie along a side of the square, not a diagonal.

Can the distance between the points (1, −2) and (−4, 10) be used to find the perimeter of the square directly?

Yes, because the distance between adjacent vertices is the side length of the square. Multiplying this length by 4 gives the perimeter.

If the points are adjacent vertices of a square, what is the formula to find the perimeter given their coordinates?

Calculate the distance between the points using the distance formula $\sqrt{[(x_2-x_1)^2 + (y_2-y_1)^2]}$, then multiply by 4 to find the perimeter.

Additional Resources

The Points $(1,-2)$ And $(-4,10)$ Are Adjacent Vertices Of A Square. What Is The Perimeter Of The Square?

Understanding geometric figures often involves analyzing the positions of their vertices and the distances between them. When given specific points on a coordinate plane, such as $(1,-2)$ and $(-4,10)$, and told these points are adjacent vertices of a square, a natural question arises: what is the perimeter of that square? This problem combines fundamental concepts of coordinate geometry, including distance formulas, vector operations, and properties of squares, to arrive at a precise numerical answer.

In this article, we will systematically explore the solution, starting from basic principles, moving through detailed calculations, and finally arriving at the perimeter of the square.

Understanding the Problem and Key Concepts

Before diving into calculations, it's essential to clarify what the problem entails.

- What does it mean for two points to be adjacent vertices of a square?
Adjacent vertices are neighboring corners connected directly by one side of the square. Therefore, the line segment connecting these points forms one side of the square.

- Given points:
- Point A: $(1, -2)$
- Point B: $(-4, 10)$

- Objective:
Find the perimeter of the square that has these two points as adjacent vertices.

To solve this, we need to determine two key pieces of information:

1. The length of the side of the square, which is the distance between points A and B.
2. The orientation of the square, specifically the direction of the sides perpendicular to AB to locate the other vertices, and confirm the length of all sides.

Once the side length is known, the perimeter is simply four times that length, since all sides of a square are equal.

Calculating the Distance Between the Two Given Points

The first step is to compute the length of the side of the square between points A and B. The distance formula in the coordinate plane is derived from the Pythagorean theorem:

\[

$$
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

Applying this to points:

- $x_1 = 1$, $y_1 = -2$
- $x_2 = -4$, $y_2 = 10$

Calculations:

$$
x_2 - x_1 = -4 - 1 = -5
$$

$$
y_2 - y_1 = 10 - (-2) = 12
$$

Therefore,

$$
d = \sqrt{(-5)^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13
$$

This means the side length of the square is 13 units.

Key point: The length of the side of the square is 13 units.

Determining the Orientation of the Square

Knowing the side length is essential, but to find the entire perimeter, we need to understand how the square is oriented in the plane. Specifically, we need to identify the directions of the other sides.

Since the points $(1, -2)$ and $(-4, 10)$ are adjacent vertices, the side connecting them has a certain vector representation:

$$
\vec{AB} = (x_2 - x_1, y_2 - y_1) = (-5, 12)
$$

This vector points from point A to point B. The length of this vector matches the side length, which we have confirmed as 13.

To find the other sides, which are perpendicular to this vector, we need to find vectors perpendicular to $\vec{AB}$ of the same length.

Finding Perpendicular Vectors

In the coordinate plane, if $\vec{v} = (a, b)$, then any vector perpendicular to $\vec{v}$ can be obtained by swapping the components and changing the sign of one:

$$
\text{Perpendicular vectors:} \quad \pm (-b, a) \quad \text{or} \quad \pm (b, -a)

\]

Applying this to our vector:

\[
\vec{AB} = (-5, 12)
\]

Perpendicular vectors:

\[
\vec{p}_1 = (-12, -5)
\]
\[
\vec{p}_2 = (12, 5)
\]

These are perpendicular to $\vec{AB}$ because their dot product with $\vec{AB}$ is zero:

\[
(-5)(-12) + 12(-5) = 60 - 60 = 0
\]
\[
(-5)(12) + 12(5) = -60 + 60 = 0
\]

Now, we need to normalize these vectors to the side length of 13 to get the correct side segments.

Calculate the length of these perpendicular vectors:

\[
|\vec{p}_1| = \sqrt{(-12)^2 + (-5)^2} = \sqrt{144 + 25} = \sqrt{169} = 13
\]
\[
|\vec{p}_2| = 13
\]

Since these vectors are already of length 13, we can directly use them as the displacement vectors to locate the other vertices.

Locating the Remaining Vertices

- The square has vertices:

- A: $(1, -2)$
- B: $(-4, 10)$
- Let's call the remaining vertices C and D.

- The sides from A and B will be connected via these perpendicular vectors.

- To find vertex C, which is adjacent to A and B in the square, we can add the perpendicular vector to point A:

\[
C = A + \vec{p}

\]

- Similarly, D can be obtained from B:

$$
D = B + \vec{p}
$$

Alternatively, the other vertices are:

$$
C = A + \vec{p}_1 \quad \text{or} \quad A + \vec{p}_2
$$
$$
D = B + \vec{p}_1 \quad \text{or} \quad B + \vec{p}_2
$$

Choosing one:

$$
C = (1, -2) + (-12, -5) = (1 - 12, -2 - 5) = (-11, -7)
$$
$$
D = (-4, 10) + (-12, -5) = (-4 - 12, 10 - 5) = (-16, 5)
$$

Alternatively, using the other perpendicular vector:

$$
C' = (1, -2) + (12, 5) = (13, 3)
$$
$$
D' = (-4, 10) + (12, 5) = (8, 15)
$$

Either set of points is valid, and both define a square with side length 13.

Verifying the Square's Validity and Confirming the Perimeter

To confirm the shape is indeed a square, we need to verify:

- All sides are equal to 13 units.
- Adjacent sides are perpendicular (their dot product should be zero).

Let’s verify the side lengths:

- Side AB: length is 13, as previously calculated.
- Side AC (from A to C):

Using the first set:

$$
A = (1, -2), \quad C = (-11, -7)
$$
\[

AC = \sqrt{(-11 - 1)^2 + (-7 + 2)^2} = \sqrt{(-12)^2 + (-5)^2} = \sqrt{144 + 25} = 13
\]

- Side AD (from A to D):

\[
A = (1, -2), \quad D = (-16, 5)
\]
\[
AD = \sqrt{(-16 - 1)^2 + (5 + 2)^2} = \sqrt{(-17)^2 + 7^2} = \sqrt{289 + 49} = \sqrt{338} \neq 13
\]

This indicates that the square's vertices are more accurately located by consistent use of the perpendicular vectors, and the configuration with the points:

\[
A = (1, -2)
\]
\[
B = (-4, 10)
\]
\[
C = (-11, -7)
\]
\[
D = (-16, 5)
\]

forms a square with all sides of length 13.

Perimeter calculation:

Since all sides are length 13, the perimeter \(P\) is:

\[
P = 4 \times 13 = 52
\]

Thus, the perimeter of the square is 52 units.

Summary and Final Remarks

This problem exemplifies how coordinate geometry can be wielded to solve geometric puzzles efficiently. Starting with the given points, we calculated the side length via the distance formula, identified the direction of the sides and their perpendicular counterparts using vector operations, and verified the properties of a square through side lengths and perpendicularity.

The key steps summarized:

- Calculated the distance between the given vertices to find the side length.
- Derived perpendicular vectors to determine the orientation.

The Points 1 2 And 4 10 Are Adjacent Vertices Of A Square What Is The Perimeter Of The Square

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