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Mathematics often presents intriguing puzzles that challenge our reasoning and problem-solving skills. One such fascinating problem involves two numbers with specific relationships, leading us through algebraic expressions to discover their values. This problem states that the sum of two numbers is 38, and additionally, when 8 is added to twice one of these numbers, the result equals five times the other number. Understanding and solving this problem requires translating words into algebraic equations and systematically solving for the unknowns. Let's explore this problem in detail, breaking down each step to ensure clarity and comprehension.

Understanding the Problem

The problem presents two key pieces of information:

- The sum of two numbers is 38.
- When 8 is added to twice one of the numbers, the resulting value equals five times the other number.

To solve this, we need to define variables for the two unknown numbers and then translate the given conditions into algebraic equations.

Defining Variables

Let's denote:

- x as the first number.
- y as the second number.

With these variables, the problem's statements can be expressed mathematically.

Formulating the Equations

From the problem statement, we derive two equations:

Equation 1: Sum of the Two Numbers

Since their sum is 38:

$$x + y = 38$$

Equation 2: Relationship Involving Twice One Number

The statement "when 8 is added to twice one of the numbers, the result is 5 times the other" can be interpreted in two ways:

- "Twice one of the numbers" refers to either $2x$ or $2y$.
- The phrase "when 8 is added to twice one of the numbers" indicates the addition of 8 to $2x$ or $2y$.

Assuming the problem means:

- Adding 8 to twice x , and this equals five times y , i.e.,

$$2x + 8 = 5y$$

Alternatively, if the problem intends to refer to y , then:

$$2y + 8 = 5x$$

To cover all bases, we will analyze both possibilities.

Solving the System of Equations

Case 1: $2x + 8 = 5y$

Using the first equation $x + y = 38$, express y in terms of x :

$$y = 38 - x$$

Substitute into the second equation:

$$2x + 8 = 5(38 - x)$$

$$2x + 8 = 190 - 5x$$

Bring all terms to one side:

$$2x + 8 + 5x = 190$$

$$7x + 8 = 190$$

Solve for x :

$$\backslash[7x = 182 \backslash]$$

$$\backslash[x = 26 \backslash]$$

Now, find $\backslash(y \backslash)$:

$$\backslash[y = 38 - 26 = 12 \backslash]$$

Solution for Case 1:

- First number $\backslash(x = 26 \backslash)$

- Second number $\backslash(y = 12 \backslash)$

Check the condition:

$$\backslash[2x + 8 = 2(26) + 8 = 52 + 8 = 60 \backslash]$$

$$\backslash[5y = 5(12) = 60 \backslash]$$

The condition holds true.

Case 2: $\backslash(2y + 8 = 5x \backslash)$

Again, express $\backslash(y \backslash)$ in terms of $\backslash(x \backslash)$:

$$\backslash[y = 38 - x \backslash]$$

Substitute into the second equation:

$$\backslash[2(38 - x) + 8 = 5x \backslash]$$

$$\backslash[76 - 2x + 8 = 5x \backslash]$$

$$\backslash[84 - 2x = 5x \backslash]$$

Bring all $\backslash(x \backslash)$ terms to one side:

$$\backslash[84 = 7x \backslash]$$

$$\backslash[x = 12 \backslash]$$

Find $\backslash(y \backslash)$:

$$\backslash[y = 38 - 12 = 26 \backslash]$$

Check the condition:

$$\backslash[2y + 8 = 2(26) + 8 = 52 + 8 = 60 \backslash]$$

$$\backslash[5x = 5(12) = 60 \backslash]$$

Again, the condition is satisfied.

Solution for Case 2:

- First number $\backslash(x = 12 \backslash)$

- Second number $\backslash(y = 26 \backslash)$

Summary of Solutions

The problem yields two valid solutions depending on which variable the condition refers to:

1. $(x, y) = (26, 12)$ with the relationship $2x + 8 = 5y$
2. $(x, y) = (12, 26)$ with the relationship $2y + 8 = 5x$

Both pairs satisfy the initial sum condition and the specified relationship involving addition and multiplication.

Interpreting the Results

The existence of two solutions highlights the importance of carefully interpreting word problems. The phrase "when 8 is added to twice one of the numbers" can be ambiguous unless explicitly clarified which number is being referred to.

In real-world scenarios or mathematical contexts, such ambiguity needs resolution for precise problem-solving. The solutions demonstrate that the same set of conditions can lead to multiple valid pairs of numbers, depending on the interpretation.

Applications and Importance of Algebra in Word Problems

This problem exemplifies the power of algebra in solving real-world-like puzzles. Translating words into algebraic formulas allows for systematic analysis and solution derivation. Such skills are vital in various fields including engineering, finance, science, and technology.

Key Skills Developed:

- Understanding and translating word problems into equations
- Systematic solving of simultaneous equations
- Interpreting solutions in context
- Critical thinking and attention to detail

Conclusion

Solving the problem about two numbers with a sum of 38 and a relationship involving addition and multiplication demonstrates the essential role of algebra in problem-solving. Whether the interpretation leads to the pair $(26, 12)$ or $(12, 26)$, the process underscores the importance of careful analysis and clear understanding of problem statements. Mastering such techniques enhances mathematical reasoning and prepares learners for more complex challenges in academics and real life.

Key Takeaways:

- Always define variables clearly.
- Translate words into algebraic equations carefully.
- Consider multiple interpretations if the problem statement is ambiguous.
- Verify solutions by substituting back into original conditions.

By practicing such problems, learners develop essential skills that form the foundation for advanced mathematics and analytical thinking.

Frequently Asked Questions

What are the two numbers whose sum is 38?

Let the numbers be x and y . The sum is $x + y = 38$.

How do you set up equations for the problem involving adding 8 to twice one number?

If we assume one number is x , then the other is y . The condition '8 added to twice one number' leads to an equation like $2x + 8 = 5y$, or vice versa, depending on which number is doubled.

What is the algebraic method to solve for the two numbers in this problem?

Use the two equations: $x + y = 38$ and $2x + 8 = 5y$ (or $2y + 8 = 5x$). Substitute one into the other and solve for the variables.

Can you find the specific values of the two numbers using substitution?

Yes. For example, from $x + y = 38$, express y as $38 - x$. Substitute into $2x + 8 = 5(38 - x)$ and solve for x , then find y .

What are the solutions to the problem after solving the equations?

The solutions are $x = 14$ and $y = 24$, or vice versa, depending on the setup. These satisfy both conditions.

How can this problem be generalized to similar problems involving sums and linear conditions?

Such problems can be generalized by setting up systems of linear equations based on the sum and additional conditions, then solving using substitution or elimination methods.

What real-world scenarios can be modeled by this kind of algebraic problem?

Situations like sharing resources, budgeting, or distributing items where total amounts and specific adjustments are involved can be modeled with similar algebraic problems.

Additional Resources

The Sum Of Two Numbers Is 38. When 8 Is Added To Twice One Of The Numbers, The Result Is 5 Times The Other

Mathematics often presents intriguing problems that challenge our logical reasoning and algebraic skills. Among these, word problems involving multiple conditions require a careful approach to decipher the relationships between variables. Today, we explore a classic algebraic problem: "The sum of two numbers is 38. When 8 is added to twice one of the numbers, the result is five times the other." This problem not only emphasizes the importance of setting up equations correctly but also demonstrates how systematic reasoning can lead us to the solution.

In this article, we will break down this problem step by step, transforming the words into mathematical expressions, solving the resulting equations, and interpreting the solution in context. Whether you're a student brushing up on algebra or a curious reader interested in problem-solving strategies, this detailed exploration aims to clarify the process and reinforce foundational concepts.

Understanding the Problem: Breaking Down the Conditions

Before diving into equations, it is crucial to understand what the problem is asking and identify the key pieces of information.

The problem states:

- The sum of two numbers is 38.
- When 8 is added to twice one of the numbers, the result equals five times the other number.

These statements introduce two main pieces of information that relate the two numbers, which we will call:

- Number 1: x
- Number 2: y

Our goal is to find the specific values of x and y that satisfy both conditions simultaneously.

Key points to note:

- The sum condition: $x + y = 38$
- The second condition involves an operation on one number (doubling and adding 8) and comparing it to five times the other number.

Understanding these allows us to translate the words into algebraic equations systematically.

Formulating the Mathematical Equations

The core step in solving algebraic word problems is translating verbal statements into precise mathematical expressions.

Step 1: Assigning Variables

Let:

- x be the first number.
- y be the second number.

Step 2: Writing the Equations from the Conditions

Condition 1: The sum of the numbers is 38

$$x + y = 38$$

\]

Condition 2: "When 8 is added to twice one of the numbers, the result is 5 times the other."

This statement is slightly ambiguous because it does not specify which number is doubled and added to 8. To precisely formulate the problem, we need to consider both possible interpretations:

- Interpretation A: The "twice one of the numbers" refers to (x) , and the added 8, resulting in a quantity equal to 5 times (y) :

$$\begin{aligned} & \left[\right. \\ & 2x + 8 = 5y \\ & \left. \right] \end{aligned}$$

- Interpretation B: The "twice one of the numbers" refers to (y) , and the added 8 results in 5 times (x) :

$$\begin{aligned} & \left[\right. \\ & 2y + 8 = 5x \\ & \left. \right] \end{aligned}$$

In most algebraic problems of this type, the intended meaning is that the "twice one of the numbers" refers to the same number in the context of the second equation. Given the symmetry, both interpretations are valid, but the problem likely aims for the first interpretation unless specified otherwise.

For clarity, we will proceed with Interpretation A:

$$\begin{aligned} & \left[\right. \\ & 2x + 8 = 5y \\ & \left. \right] \end{aligned}$$

This gives us a second equation to work with.

Solving the System of Equations

Having established the two equations:

$$\begin{aligned} & \left[\right. \\ & \begin{cases} \end{cases} \end{aligned}$$

$$x + y = 38 \quad (1)$$

$$2x + 8 = 5y \quad (2)$$

$\end{cases}$

$\]$

Our objective is to find values of x and y satisfying both.

Step 1: Express one variable in terms of the other

From equation (1):

$\[$

$$x = 38 - y$$

$\]$

Step 2: Substitute into the second equation

Substituting $x = 38 - y$ into equation (2):

$\[$

$$2(38 - y) + 8 = 5y$$

$\]$

Expanding:

$\[$

$$76 - 2y + 8 = 5y$$

$\]$

Simplify:

$\[$

$$84 - 2y = 5y$$

$\]$

Bring all terms involving y to one side:

$\[$

$$84 = 5y + 2y$$

$\]$

$\[$

$$84 = 7y$$

\]

Solve for y :

\[

$$y = \frac{84}{7} = 12$$

\]

Step 3: Find x

Using $x = 38 - y$:

\[

$$x = 38 - 12 = 26$$

\]

Step 4: Verify the solution

Plugging $x = 26$ and $y = 12$ into the second original equation:

\[

$$2x + 8 = 5y$$

\]

\[

$$2(26) + 8 = 5(12)$$

\]

\[

$$52 + 8 = 60$$

\]

\[

$$60 = 60$$

\]

The equality holds, confirming the solution.

Interpreting the Results and Alternative Scenarios

The derived solution indicates:

- Number 1: $(x = 26)$

- Number 2: $(y = 12)$

Given the initial conditions, these satisfy both the sum and the secondary condition when interpreted as above.

Alternative Interpretation: Swapping the roles

Suppose the "twice one of the numbers" referred to (y) , leading to the equation:

$$\begin{aligned} &[\\ 2y + 8 &= 5x \\ &] \end{aligned}$$

Repeating similar steps:

- From $(x + y = 38)$, express $(y = 38 - x)$.

- Substitute into $(2y + 8 = 5x)$:

$$\begin{aligned} &[\\ 2(38 - x) + 8 &= 5x \\ &] \end{aligned}$$

$$\begin{aligned} &[\\ 76 - 2x + 8 &= 5x \\ &] \end{aligned}$$

$$\begin{aligned} &[\\ 84 - 2x &= 5x \\ &] \end{aligned}$$

$$\begin{aligned} &[\\ 84 &= 7x \\ &] \end{aligned}$$

$$\begin{aligned} &[\\ x &= 12 \\ &] \end{aligned}$$

And:

$$\begin{aligned} &[\\ y &= 38 - 12 = 26 \end{aligned}$$

\]

This yields a complementary solution with the roles of the numbers reversed:

- \(\ x = 12 \backslash\)

- \(\ y = 26 \backslash\)

Both solutions satisfy the problem's initial conditions, provided the interpretation aligns with the problem's wording.

Key Takeaways and Practical Applications

This problem exemplifies core principles of algebra and problem-solving:

- Systematic translation: Converting words into equations is fundamental. Carefully interpreting the language prevents errors.
- Variable assignment: Assigning variables to unknown quantities simplifies complex word problems.
- Substitution method: Using substitution to reduce systems to single-variable equations streamlines solving.
- Verification: Always substituting solutions back into original equations ensures correctness.

These skills are widely applicable beyond classroom exercises, including in fields like engineering, finance, and data analysis, where modeling relationships between quantities is essential.

Conclusion: The Power of Algebraic Reasoning

The problem "The sum of two numbers is 38. When 8 is added to twice one of the numbers, the result is five times the other" offers a rich example of how algebraic methods help decode real-world scenarios. By carefully assigning variables, translating verbal statements into equations, and applying systematic solving techniques, we can uncover precise solutions that satisfy multiple conditions.

Whether the numbers are 26 and 12 or 12 and 26, the process remains the same: clarity in interpretation, logical formulation, and diligent verification are the keys to successful problem-solving. Such exercises not only sharpen mathematical skills but also cultivate analytical thinking that is valuable across countless disciplines.

In summary, mastering these problem-solving strategies enhances our ability to approach complex questions with confidence and precision, turning linguistic puzzles into clear, quantitative solutions.

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