

The Verticals Of A Triangle Are A(8, -24), B(8, -8), And C(16, -8). Of The Triangle Is Dilated By A Scale

The Verticals Of A Triangle Are A(8, -24), B(8, -8), And C(16, -8). Of The Triangle Is Dilated By A Scale This statement introduces a fascinating exploration into the properties of triangles, specifically focusing on the vertices and how they transform under dilation. Understanding how a triangle's vertices behave when scaled is fundamental in geometry, offering insights into similarity, congruence, and the preservation of shapes. In this article, we will analyze the given triangle with vertices A(8, -24), B(8, -8), and C(16, -8), and examine the effects of dilation on this triangle, including how to determine the scale factor, the resulting coordinates, and the implications for geometric properties.

Understanding the Triangle's Initial Coordinates

Vertices and Their Significance

The initial step in exploring the dilation of a triangle is to understand its original vertices:

- Vertex A (8, -24)
- Vertex B (8, -8)
- Vertex C (16, -8)

These coordinates are plotted on the Cartesian plane, where the x-values denote horizontal positions and the y-values denote vertical positions. Analyzing these points reveals the triangle's shape, size, and orientation.

Plotting the Triangle

Plotting the vertices:

- Point A is located at (8, -24), which is 8 units along the x-axis and 24 units down the y-axis.
- Point B is at (8, -8), vertically aligned with A but 16 units higher (since -8 is greater than -24).
- Point C is at (16, -8), horizontally 8 units to the right of B, at the same vertical level.

Connecting these points:

- Segment AB is vertical, connecting (8, -24) and (8, -8).
- Segment BC is horizontal, connecting (8, -8) and (16, -8).
- Segment AC is diagonal, connecting (8, -24) and (16, -8).

This configuration forms a right triangle with a vertical side (AB), a horizontal side (BC), and hypotenuse AC.

Calculating the Side Lengths and the Triangle's Properties

Distance Formula Application

To understand how the triangle will change under dilation, first compute the lengths of its sides using the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Applying this to each pair:

- AB:
$$\sqrt{(8 - 8)^2 + (-8 - (-24))^2} = \sqrt{0 + (16)^2} = 16$$
- BC:
$$\sqrt{(16 - 8)^2 + (-8 - (-8))^2} = \sqrt{8^2 + 0} = 8$$
- AC:
$$\sqrt{(16 - 8)^2 + (-8 - (-24))^2} = \sqrt{8^2 + 16^2} = \sqrt{64 + 256} = \sqrt{320} \approx 17.89$$

Summary of side lengths:

Side	Length
AB	16 units
BC	8 units
AC	approximately 17.89 units

Understanding Dilation in Geometry

What Is Dilation?

Dilation is a transformation that produces an image that is the same shape as the original but is a scaled version, either larger or smaller. It is characterized by:

- A center of dilation (a fixed point)
- A scale factor (k), which determines the degree of enlargement or reduction

The key properties of dilation include:

- Lines connecting the center and any point on the figure remain straight after dilation.
- The shape remains similar; corresponding angles are equal.
- Side lengths are multiplied by the scale factor.

Mathematical Representation of Dilation

Given a point $P(x, y)$, a center of dilation $C(x_c, y_c)$, and a scale factor k , the dilated point P' is calculated as:

$$P' = (x_c + k(x - x_c), y_c + k(y - y_c))$$

This formula ensures that the point moves along the line connecting it to the center, scaled by k .

Applying Dilation to the Triangle

Choosing the Center of Dilation

The problem statement does not specify the center of dilation, which is essential to proceed. Common choices include:

- The origin $(0,0)$
- The centroid of the triangle
- Any vertex of the triangle

For this analysis, we will consider the origin $(0,0)$ as the center of dilation, a common choice for simplicity.

Determining the Scale Factor

Suppose the triangle is dilated by a scale factor k . The value of k could be any positive number:

- $(k > 1)$: enlargement
- $(0 < k < 1)$: reduction
- $(k = 1)$: no change

In the absence of a specified (k) , we will analyze the general case and then provide examples with specific scale factors.

Calculating the Dilated Coordinates

Using the dilation formula for each vertex:

```
\[
\begin{aligned}
A' &= (0 + k(8 - 0), 0 + k(-24 - 0)) = (8k, -24k) \\
B' &= (0 + k(8 - 0), 0 + k(-8 - 0)) = (8k, -8k) \\
C' &= (0 + k(16 - 0), 0 + k(-8 - 0)) = (16k, -8k)
\end{aligned}
\]
```

This results in the new vertices being scaled versions of the original, relative to the origin.

Effect of Dilation on the Triangle's Properties

Side Lengths After Dilation

Side lengths after dilation are scaled by the same factor (k) :

- $(A'B' = 16k)$
- $(B'C' = 8k)$
- $(A'C' \approx 17.89k)$

This maintains the similarity of the triangle, as all sides are proportionally scaled.

Area of the Triangle

Original area:

```
\[
\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}
\]
```

Choosing AB as the base (length 16):

- Height is the horizontal distance from C to line AB. Since AB is vertical at $x=8$, and C is at $x=16$, the height is $(16 - 8 = 8)$.

$$\text{Area} = \frac{1}{2} \times 16 \times 8 = 64 \text{ square units}$$

After dilation, the new area:

$$\text{Area}' = k^2 \times 64$$

because area scales with the square of the scale factor.

Practical Applications and Examples

Example 1: Scale Factor $(k=2)$

If the triangle is dilated by a factor of 2 with the origin as the center:

- New vertices:

$$A' = (8 \times 2, -24 \times 2) = (16, -48)$$

$$B' = (8 \times 2, -8 \times 2) = (16, -16)$$

$$C' = (16 \times 2, -8 \times 2) = (32, -16)$$

- New side lengths:

$$A'B' = 32, \quad B'C' = 16, \quad A'C' \approx 35.78$$

- New area:

$$64 \times 2^2 = 64 \times 4 = 256 \text{ square units}$$

This demonstrates how the triangle enlarges proportionally.

Example 2: Scale Factor $(k=0.5)$

Dilating by a factor of 0.5:

- Vertices:

$$\begin{aligned} &A' = (4, -12), \quad B' = (4, -4), \quad C' = (8, -4) \end{aligned}$$

- Side lengths:

$$\begin{aligned} &A'B' = 8, \quad B'C' = 4, \quad A'C' \approx 8.94 \end{aligned}$$

- Area:

$$64 \times 0.5^2 = 64 \times 0.25 = 16 \text{ square units}$$

This shows the triangle shrinking while maintaining similarity.

Conclusion: Significance of Dilation in Geometry

Dilation is a powerful geometric transformation that

Frequently Asked Questions

What are the coordinates of the vertices of the given triangle?

The vertices are $A(8, -24)$, $B(8, -8)$, and $C(16, -8)$.

How do you find the length of side AB in the triangle?

You calculate the distance between points $A(8, -24)$ and $B(8, -8)$ using the distance formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

What is the length of side AB in the triangle?

Side AB has a length of 16 units, calculated as $\sqrt{[(8 - 8)^2 + (-8 + 24)^2]} = \sqrt{[0 + 16^2]} = 16$.

How do you determine the scale factor of the dilation of the triangle?

The scale factor is determined by comparing the lengths of corresponding sides before and after dilation; if only the original and dilated triangles are known, you divide the length of a side in the dilated triangle by the corresponding side in the original.

What effect does dilation have on the vertices of a triangle?

Dilation enlarges or reduces the triangle proportionally, moving each vertex away from or toward the center of dilation by the scale factor.

If the triangle is dilated by a scale factor of 2 from point A, what are the new coordinates of vertex B?

Assuming the center of dilation is at A(8, -24), the new B' is found by scaling the vector from A to B: $(8 - 8, -8 + 24) = (0, 16)$. Multiply by 2: $(0, 32)$. Add back to A: $(8 + 0, -24 + 32) = (8, 8)$.

How do you find the new coordinates of all vertices after a dilation with a specific scale factor?

Identify the center of dilation, then for each vertex, find the vector from the center to the vertex, multiply that vector by the scale factor, and add it to the center's coordinates to find the new vertex.

What is the importance of understanding the vertices and scale factors in triangle dilations?

Understanding these helps in accurately transforming triangles, calculating new side lengths and areas, and solving geometric problems involving similar figures.

Can you determine the area of the original triangle based on the given vertices?

Yes, using the coordinate geometry formula: $\text{Area} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$, substituting the given vertices to find the area.

Additional Resources

Triangles and Dilations: An Expert Exploration of Vertices and Transformations

Understanding the Vertices of the Triangle: Points A(8, -24), B(8, -8), and C(16, -8)

In the realm of coordinate geometry, the fundamental building block of many geometric studies is the triangle. The vertices of a triangle define its shape, size, and position within a coordinate plane. Here, we are presented with a triangle with vertices at points A(8, -24), B(8, -8), and C(16, -8). To fully grasp the implications of these points, we'll analyze their positions, the shape they form, and their geometric relationships.

Mapping the Points: Visualizing the Triangle in the Coordinate Plane

Let's first plot these points to visualize the triangle:

- Point A (8, -24): Located 8 units along the x-axis and 24 units down along the y-axis.
- Point B (8, -8): Directly above point A, sharing the same x-coordinate but 16 units higher.
- Point C (16, -8): To the right of point B, sharing the same y-coordinate but 8 units farther along the x-axis.

This positioning indicates:

- Vertical Alignment of A and B: Since both share $x=8$, they form a vertical line segment.
- Horizontal Alignment of B and C: Both have $y=-8$, forming a horizontal segment.
- Triangle Formation: Connecting these points creates a right-angled triangle with a vertical leg from A to B and a horizontal leg from B to C.

Calculating Lengths of Sides

To understand the triangle's properties, calculating the lengths of its sides is essential.

- AB (Vertical side):


```

\[
\text{AB} = |y_B - y_A| = |-8 - (-24)| = 16
\]
- BC (Horizontal side):
\[
\text{BC} = |x_C - x_B| = |16 - 8| = 8
\]
- AC (Hypotenuse):
Using the distance formula:
\[
AC = \sqrt{(x_C - x_A)^2 + (y_C - y_A)^2}
\]
\[
AC = \sqrt{(16 - 8)^2 + (-8 - (-24))^2} = \sqrt{8^2 + 16^2} = \sqrt{64 + 256}
= \sqrt{320} \approx 17.89
\]

```

Summary of side lengths:

Side	Length
AB	16 units
BC	8 units
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This confirms that the triangle is a right triangle with the right angle at point B (since AB is vertical and BC is horizontal).

Exploring the Dilation of the Triangle: The Scale Factor and Its Impact

The concept of dilation is central to geometric transformations. When a figure is dilated by a scale factor, all distances from a fixed point (usually the origin or a specific center) are multiplied by that factor, effectively resizing the figure while maintaining its shape and proportions.

What Is Dilation?

Dilation (or scaling) is a transformation that enlarges or reduces figures in the coordinate plane. It is characterized by:

- A center of dilation: the fixed point about which the figure is scaled.
- A scale factor (k): a positive real number indicating how much the figure is enlarged or reduced.
- If $(k > 1)$, the figure enlarges.

- If $(0 < k < 1)$, the figure reduces in size.
- If $(k < 0)$, the figure is reflected and scaled, which introduces a reflection component.

In our context, we are considering a dilation of the triangle with a certain scale factor (k) , centered at a particular point (which could be the origin, centroid, or any other point).

Choosing the Center of Dilation

The center of dilation significantly affects the resulting image. Common choices include:

- Origin $(0,0)$: Simplifies calculations; the most straightforward case.
- Centroid: The average of the vertices' coordinates; often used in geometric proofs.
- Any vertex of the triangle: For specific transformations or applications.

For clarity, we'll assume the dilation is centered at the origin unless otherwise specified.

Applying the Scale Factor

Suppose the scale factor (k) is given or chosen. The transformed points, (A', B', C') , can be calculated as follows:

```
\[
\text{For each point } (x, y): \quad (x', y') = (k \times x, k \times y)
\]
```

Example:

- For a scale factor $(k=2)$:

```
\[
A' = (2 \times 8, 2 \times -24) = (16, -48)
\]
\[
B' = (2 \times 8, 2 \times -8) = (16, -16)
\]
\[
C' = (2 \times 16, 2 \times -8) = (32, -16)
\]
```

This results in a similar triangle twice as large, maintaining all angles and side ratios.

Impact of Dilation on the Triangle's Properties

Understanding the effects of dilation extends beyond mere resizing; it impacts several properties of the original figure.

1. Preservation of Shape and Angles

Dilation is a similarity transformation. This means:

- The shape of the triangle remains unchanged.
- All angles are preserved.
- The proportions of the sides stay consistent.

In our example, the right-angled triangle maintains its right angle at B, and the side ratios remain constant after dilation.

2. Change in Side Lengths

Side lengths are scaled by the factor (k) . For a scale factor (k) :

$$\begin{aligned} & \text{New side length} = k \times \text{Original side length} \end{aligned}$$

This is crucial for applications where precise size adjustments are necessary, such as engineering or computer graphics.

3. Change in Area

While linear dimensions scale by (k) , areas scale by (k^2) . For example:

- If $(k=2)$, the area increases by a factor of (4) .
- If $(k=0.5)$, the area decreases to a quarter of the original.

Calculating the original area:

Using the base and height (AB and BC):

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 8 \times 16 = 64 \text{ square units} \end{aligned}$$

\]

Post-dilation area (for scale factor k):

\[

$\text{New Area} = k^2 \times 64$

\]

This quadratic relationship emphasizes the importance of the scale factor in size and space considerations.

Practical Applications and Implications

Understanding the vertices and dilation effects of triangles isn't purely academic; it has real-world implications across various fields.

1. Computer Graphics and Design

In digital design, resizing geometric figures while maintaining proportions is fundamental. Recognizing how vertices transform under dilation simplifies processes like:

- Scaling images or objects.
- Creating zoom effects.
- Generating similar shapes at different sizes.

2. Engineering and Architectural Planning

Precise transformations ensure structural elements are scaled correctly during modeling, especially in CAD software. Knowing how dilation affects dimensions aids in:

- Material estimation.
- Structural integrity checks.
- Replicating components at different scales.

3. Mathematical Education and Visualization

Teaching concepts like dilation with concrete examples, such as the given triangle, enhances comprehension. Visualizing how vertices move under scaling provides an intuitive grasp of similarity and proportionality.

Conclusion: From Vertices to Transformation Mastery

The vertices of the triangle at $A(8, -24)$, $B(8, -8)$, and $C(16, -8)$ provide a clear illustration of fundamental geometric principles—shape, size, and position within the coordinate plane. Their analysis reveals a right triangle with a specific orientation and dimensions. When subjected to dilation by a scale factor k , these points—and consequently the entire triangle—undergo a uniform transformation that preserves shape but alters size.

Understanding these transformations is essential for precise geometric modeling, whether in academic pursuits, design, engineering, or digital artistry. Recognizing how vertices translate under scaling not only deepens comprehension of coordinate geometry but also equips practitioners with the tools to manipulate shapes effectively, maintaining proportions and angles while adjusting for desired sizes.

In essence, mastering the principles of triangle vertices and dilation empowers individuals to navigate complex geometric transformations confidently, transforming simple points into versatile tools for innovation and problem-solving across disciplines.

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