

THIS QUESTION IS ABOUT DISCRETE FOURIER TRANSFORM OF THE POINTSEQUENCEE=1F=2G=4H=5PLEASE HELP ME TO SOLVE

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UNDERSTANDING AND COMPUTING THE DISCRETE FOURIER TRANSFORM (DFT) OF A SEQUENCE IS FUNDAMENTAL IN MANY FIELDS SUCH AS SIGNAL PROCESSING, COMMUNICATIONS, AND DATA ANALYSIS. IF YOU'RE GRAPPLING WITH A SEQUENCE LIKE E=1, F=2, G=4, H=5 AND WANT TO UNDERSTAND HOW TO FIND ITS DFT, YOU'RE IN THE RIGHT PLACE. THIS ARTICLE WILL GUIDE YOU STEP-BY-STEP THROUGH THE PROCESS OF CALCULATING THE DFT OF THIS SEQUENCE, EXPLAINING KEY CONCEPTS AND PROVIDING PRACTICAL EXAMPLES TO HELP YOU MASTER THIS ESSENTIAL TECHNIQUE.

WHAT IS THE DISCRETE FOURIER TRANSFORM (DFT)?

THE DISCRETE FOURIER TRANSFORM IS A MATHEMATICAL TECHNIQUE USED TO ANALYZE THE FREQUENCY CONTENT OF A DISCRETE SIGNAL OR SEQUENCE. IT TRANSFORMS A SEQUENCE OF COMPLEX NUMBERS INTO ANOTHER SEQUENCE, REVEALING THE AMPLITUDE AND PHASE OF DIFFERENT FREQUENCY COMPONENTS WITHIN THE ORIGINAL DATA.

BASIC CONCEPTS OF DFT

- **SEQUENCE INPUT:** A SERIES OF DATA POINTS, OFTEN REPRESENTING SIGNAL AMPLITUDE OVER TIME.
- **FREQUENCY DOMAIN OUTPUT:** THE DFT PROVIDES INFORMATION ABOUT THE SIGNAL'S FREQUENCY COMPONENTS, INCLUDING THEIR MAGNITUDES AND PHASES.
- **MATHEMATICAL FORMULA:** FOR A SEQUENCE $\{x[n]\}$, THE DFT IS GIVEN BY:

$$X[k] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j 2\pi k n / N}$$

WHERE N IS THE TOTAL NUMBER OF POINTS, k IS THE FREQUENCY INDEX, AND j IS THE IMAGINARY UNIT.

STEP-BY-STEP CALCULATION OF DFT FOR SEQUENCE E=1, F=2, G=4, H=5

GIVEN THE SEQUENCE:

$$x[0] = 1, \quad x[1] = 2, \quad x[2] = 4, \quad x[3] = 5$$

LET'S PROCEED THROUGH THE CALCULATION SYSTEMATICALLY.

1. DEFINE THE SEQUENCE AND PARAMETERS

- NUMBER OF POINTS, $N = 4$
- SEQUENCE: $\{x[n]\}$ FOR $n = 0, 1, 2, 3$

2. COMPUTE THE DFT COEFFICIENTS $X[k]$ FOR $k = 0, 1, 2, 3$

FOR EACH k , COMPUTE:

$$X[k] = \sum_{n=0}^3 x[n] \cdot e^{-j 2\pi kn / 4}$$

CALCULATIONS FOR EACH k :

$k=0$

$$X[0] = x[0] + x[1] + x[2] + x[3] = 1 + 2 + 4 + 5 = 12$$

$k=1$

$$X[1] = 1 \cdot e^{-j 2\pi \cdot 0 \cdot 1/4} + 2 \cdot e^{-j 2\pi \cdot 1 \cdot 1/4} + 4 \cdot e^{-j 2\pi \cdot 2 \cdot 1/4} + 5 \cdot e^{-j 2\pi \cdot 3 \cdot 1/4}$$

SIMPLIFY EXPONENTS:

- $e^{-j 0} = 1$
- $e^{-j \pi/2} = -j$
- $e^{-j \pi} = -1$
- $e^{-j 3\pi/2} = j$

COMPUTE:

$$X[1] = 1 \cdot 1 + 2 \cdot (-j) + 4 \cdot (-1) + 5 \cdot j$$

$$X[1] = 1 - 2j - 4 + 5j = (1 - 4) + (-2j + 5j) = -3 + 3j$$

$k=2$

EXPONENTS:

- $e^{-j 0} = 1$
- $e^{-j \pi} = -1$
- $e^{-j 2\pi} = 1$
- $e^{-j 3\pi} = -1$

CALCULATE:

$$X[2] = 1 \cdot 1 + 2 \cdot (-1) + 4 \cdot 1 + 5 \cdot (-1) = 1 - 2 + 4 - 5 = -2$$

$k=3$

EXPONENTS:

- $e^{-j 0} = 1$
- $e^{-j 3\pi/2} = j$

$$- \left(e^{-j 3 \pi} \right) = -1$$

$$- \left(e^{-j 9 \pi / 2} \right) = -j$$

CALCULATE:

$$X[3] = 1 + 2j + 4(-1) + 5(-j) = 1 + 2j - 4 - 5j = (1 - 4) + (2j - 5j) = -3 - 3j$$

SUMMARY OF DFT COEFFICIENTS:

$$X[0] = 12$$

$$X[1] = -3 + 3j$$

$$X[2] = -2$$

$$X[3] = -3 - 3j$$

INTERPRETING THE DFT RESULTS

THE COMPUTED DFT COEFFICIENTS PROVIDE A COMPREHENSIVE PICTURE OF THE FREQUENCY CONTENT OF THE ORIGINAL SEQUENCE.

MAGNITUDE AND PHASE

TO BETTER UNDERSTAND THE FREQUENCY COMPONENTS, CONVERT THE COMPLEX COEFFICIENTS INTO MAGNITUDE AND PHASE:

$$|X[k]| = \sqrt{\text{Re}(X[k])^2 + \text{Im}(X[k])^2}$$

$$\angle \phi_k = \arctan \left(\frac{\text{Im}(X[k])}{\text{Re}(X[k])} \right)$$

CALCULATIONS:

- For $X[0] = 12$:

$$|X[0]| = 12, \angle \phi_0 = 0$$

- For $X[1] = -3 + 3j$:

$$|X[1]| = \sqrt{(-3)^2 + 3^2} = \sqrt{9 + 9} = \sqrt{18} \approx 4.24$$

$$\angle \phi_1 = \arctan \left(\frac{3}{-3} \right) = \arctan(-1) \approx -45^\circ \text{ (or } 135^\circ \text{ depending on quadrant)}$$

\\

- For $(X[2] = -2)$:

\\
 $|X[2]| = 2$, $\text{QUAD } \phi_2 = \pi$
 \\

- For $(X[3] = -3 - 3j)$:

\\
 $|X[3]| = \sqrt{(-3)^2 + (-3)^2} \approx 4.24$
 \\
 $\phi_3 = \arctan\left(\frac{-3}{-3}\right) = \arctan(1) = 45^\circ$ $\text{TEXT}\{, \text{ BUT IN THE THIRD QUADRANT } \}$
 $\rightarrow \text{PHASE} \approx -135^\circ$
 \\

THESE MAGNITUDE AND PHASE VALUES HELP IN RECONSTRUCTING THE ORIGINAL SIGNAL OR ANALYZING ITS SPECTRAL PROPERTIES.

PRACTICAL APPLICATIONS OF DFT

UNDERSTANDING HOW TO COMPUTE THE DFT OF A SEQUENCE LIKE $E=1, F=2, G=4, H=5$ HAS PRACTICAL SIGNIFICANCE IN VARIOUS DOMAINS.

SIGNAL PROCESSING

- EXTRACTING FREQUENCY COMPONENTS FROM TIME-DOMAIN SIGNALS.
- FILTERING SPECIFIC FREQUENCIES TO REMOVE NOISE OR ENHANCE SIGNALS.

DATA ANALYSIS

- IDENTIFYING PERIODIC PATTERNS WITHIN DATASETS.
- TRANSFORMING DATA FOR COMPRESSION ALGORITHMS.

COMMUNICATION SYSTEMS

- MODULATING AND DEMODULATING SIGNALS.
- SPECTRUM ANALYSIS FOR CHANNEL ALLOCATION.

TOOLS AND SOFTWARE FOR COMPUTING DFT

WHILE MANUAL CALCULATIONS ARE INSTRUCTIVE, REAL-WORLD APPLICATIONS TYPICALLY RELY ON SOFTWARE TOOLS:

- **MATLAB/OCTAVE:** USE THE 'FFT' FUNCTION FOR FAST COMPUTATION.
- **PYTHON:** UTILIZE LIBRARIES SUCH AS NUMPY WITH `numpy.fft.fft()`.
- **ONLINE CALCULATORS:** VARIOUS WEB-BASED TOOLS ARE AVAILABLE FOR QUICK DFT COMPUTATION.

EXAMPLE IN PYTHON:

```
'''PYTHON
IMPORT NUMPY AS NP

SEQUENCE = [1, 2, 4, 5]
DFT_RESULT = NP.FFT
```

FREQUENTLY ASKED QUESTIONS

WHAT IS THE DISCRETE FOURIER TRANSFORM (DFT) OF THE POINT SEQUENCE $E=1, F=2, G=4, H=5$?

THE DFT OF THE SEQUENCE $[1, 2, 4, 5]$ CAN BE CALCULATED BY APPLYING THE FORMULA $X[k] = \sum_{n=0}^{N-1} x[n] e^{-j(2\pi/N)kn}$, RESULTING IN FOUR COMPLEX FREQUENCY COMPONENTS THAT REPRESENT THE SIGNAL'S FREQUENCY DOMAIN.

HOW DO I COMPUTE THE DFT FOR THE SEQUENCE $E=1, F=2, G=4, H=5$?

TO COMPUTE THE DFT, ASSIGN THE SEQUENCE VALUES TO $x[0]=1, x[1]=2, x[2]=4, x[3]=5$, THEN APPLY THE DFT FORMULA FOR $k=0, 1, 2, 3$ TO FIND EACH FREQUENCY COMPONENT.

WHAT IS THE SIGNIFICANCE OF THE POINTS $E=1, F=2, G=4, H=5$ IN DFT COMPUTATION?

THESE POINTS REPRESENT THE DISCRETE TIME DOMAIN SAMPLES OF A SIGNAL, AND THEIR DFT PROVIDES INSIGHT INTO THE FREQUENCY CONTENT OF THAT SIGNAL.

CAN I USE A SOFTWARE TOOL TO CALCULATE THE DFT OF THE SEQUENCE 1, 2, 4, 5?

YES, SOFTWARE TOOLS LIKE MATLAB, PYTHON (WITH NUMPY), OR ONLINE DFT CALCULATORS CAN QUICKLY COMPUTE THE DFT FOR YOUR SEQUENCE, PROVIDING BOTH MAGNITUDE AND PHASE INFORMATION.

WHAT IS THE EXPECTED OUTPUT OF THE DFT FOR THE SEQUENCE 1, 2, 4, 5?

THE OUTPUT WILL BE A SET OF FOUR COMPLEX NUMBERS REPRESENTING THE FREQUENCY COMPONENTS, WHICH CAN BE CONVERTED TO MAGNITUDE AND PHASE FOR ANALYSIS.

HOW DOES THE SEQUENCE 1, 2, 4, 5 RELATE TO THE FREQUENCY DOMAIN AFTER DFT?

THE SEQUENCE'S DFT REVEALS THE AMPLITUDE AND PHASE OF THE VARIOUS FREQUENCY COMPONENTS THAT MAKE UP THE ORIGINAL SIGNAL IN THE TIME DOMAIN.

ARE THERE ANY SPECIAL PROPERTIES OF THE DFT FOR THE SEQUENCE 1, 2, 4, 5?

SINCE THE SEQUENCE IS REAL-VALUED, ITS DFT WILL HAVE CONJUGATE SYMMETRY, MEANING THE SECOND HALF OF THE SPECTRUM IS THE CONJUGATE MIRROR OF THE FIRST HALF.

WHAT ARE THE STEPS TO MANUALLY COMPUTE THE DFT OF THE SEQUENCE 1, 2, 4, 5?

STEP 1: WRITE DOWN THE SEQUENCE VALUES. STEP 2: USE THE DFT FORMULA TO COMPUTE EACH $X[k]$ FOR $k=0, 1, 2, 3$. STEP

3: SIMPLIFY THE SUMS TO FIND THE COMPLEX FREQUENCY COMPONENTS.

How can I interpret the magnitude and phase of the DFT results for the sequence 1, 2, 4, 5?

The magnitude indicates the strength of each frequency component, while the phase shows its phase shift relative to the start of the sequence, helping analyze the signal's frequency characteristics.

Additional Resources

This question is about Discrete Fourier Transform of the point sequence $x[n] = 1, 2, 4, 5$. Please help me to solve: An in-depth exploration

Introduction

The Discrete Fourier Transform (DFT) is one of the foundational tools in digital signal processing, enabling us to analyze the frequency components of discrete signals. When faced with a simple sequence of data points, understanding how to compute its DFT can unlock insights into its spectral content, periodicity, and other intrinsic properties. The question posed—focused on a specific point sequence with given values—serves as an excellent case study to explore the principles, calculations, and interpretations associated with the DFT.

In this comprehensive review, we will dissect the problem, clarify the methodology for calculating the DFT, and interpret the results within the context of signal analysis. By doing so, we aim to provide readers with a clear, detailed understanding of how the DFT operates on a simple sequence and how to approach similar problems systematically.

Understanding the Discrete Fourier Transform

What is the DFT?

The Discrete Fourier Transform (DFT) converts a finite sequence of equally spaced samples of a function or signal from the time (or spatial) domain into the frequency domain. It reveals the amplitude and phase of the sinusoidal components that make up the original sequence.

Mathematically, for a sequence $x[n]$ of length N , the DFT $X[k]$ is defined as:

$$X[k] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j 2 \pi \frac{k n}{N}}$$

Where:

- n is the index in the sequence,
- k is the frequency bin index,
- j is the imaginary unit ($j^2 = -1$),
- N is the total number of points.

The result $X[k]$ is a complex number, representing both the magnitude and phase of the frequency component at the bin k .

Why Use the DFT?

- To analyze the spectral content of signals.

- TO FILTER SIGNALS IN THE FREQUENCY DOMAIN.
- TO PERFORM CONVOLUTION EFFICIENTLY.
- TO DETECT PERIODICITY AND FREQUENCY CHARACTERISTICS.

CLARIFYING THE GIVEN DATA AND PROBLEM STATEMENT

THE QUESTION PROVIDES A SEQUENCE OF DATA POINTS, SEEMINGLY LABELED AS:

- E = 1
- F = 2
- G = 4
- H = 5

WHILE THE NOTATION IS SOMEWHAT INFORMAL OR POSSIBLY A TYPO, THE ESSENTIAL INFORMATION APPEARS TO BE A SEQUENCE OF FOUR DATA POINTS: [1, 2, 4, 5].

ASSUMED DATA SEQUENCE

FOR THE PURPOSE OF CALCULATING THE DFT, WE INTERPRET THE SEQUENCE AS:

$$\begin{aligned} & \backslash [\\ x[N] &= \{x[0], x[1], x[2], x[3]\} = \{1, 2, 4, 5\} \\ & \backslash] \end{aligned}$$

THIS IS A COMMON APPROACH WHEN THE INITIAL PROBLEM STATEMENT IS AMBIGUOUS.

OBJECTIVE

CALCULATE THE DFT OF THIS SEQUENCE, ANALYZE ITS SPECTRAL COMPONENTS, AND INTERPRET THE RESULTS.

STEP-BY-STEP CALCULATION OF THE DFT

STEP 1: DEFINE THE SEQUENCE

$$\begin{aligned} & \backslash [\\ x[0] &= 1 \backslash \backslash \\ x[1] &= 2 \backslash \backslash \\ x[2] &= 4 \backslash \backslash \\ x[3] &= 5 \\ & \backslash] \end{aligned}$$

THE SEQUENCE LENGTH:

$$\begin{aligned} & \backslash [\\ N &= 4 \\ & \backslash] \end{aligned}$$

STEP 2: GENERAL DFT FORMULA FOR N=4

$$\begin{aligned} & \backslash [\\ X[k] &= \sum_{n=0}^3 x[n] \cdot e^{-j 2 \pi \frac{k n}{4}} \\ & \backslash] \end{aligned}$$

FOR $(k = 0, 1, 2, 3)$.

STEP 3: COMPUTE EACH $\{X[k]\}$

RECALL: $\{e^{-j 2 \pi \frac{k N}{4}}\}$ CAN BE SIMPLIFIED USING KNOWN ROOTS OF UNITY:

$$\{W_N = e^{-j 2 \pi / N}\}$$

FOR $\{N=4\}$,

$$\{W_4 = e^{-j \pi / 2} = \cos(\pi/2) - j \sin(\pi/2) = 0 - j1 = -j\}$$

THE POWERS:

$$\begin{aligned} - \{W_4^0 &= 1\} \\ - \{W_4^1 &= -j\} \\ - \{W_4^2 &= (W_4)^2 = (-j)^2 = -1\} \\ - \{W_4^3 &= (-j)^3 = -j \times -j \times -j = -j \times (-1) = j\} \end{aligned}$$

NOW, COMPUTE $\{X[0]\}$:

$$\{X[0] = x[0] + x[1] + x[2] + x[3] = 1 + 2 + 4 + 5 = 12\}$$

COMPUTE $\{X[1]\}$:

$$\{X[1] = x[0] \cdot W_4^0 + x[1] \cdot W_4^1 + x[2] \cdot W_4^2 + x[3] \cdot W_4^3\}$$

$$\{X[1] = 1 \cdot 1 + 2 \cdot (-j) + 4 \cdot (-1) + 5 \cdot j\}$$

$$\{X[1] = 1 - 2j - 4 + 5j = (1 - 4) + (-2j + 5j) = -3 + 3j\}$$

COMPUTE $\{X[2]\}$:

$$\{X[2] = x[0] \cdot W_4^0 + x[1] \cdot W_4^2 + x[2] \cdot W_4^4 + x[3] \cdot W_4^6\}$$

BUT NOTE THAT:

$$\{W_4^2 = -1\}$$

$$\{W_4^4 = (W_4^2)^2 = (-1)^2 = 1 = W_4^0\}$$

$$\{W_4^6 = W_4^{6 \bmod 4} = W_4^2 = -1\}$$

Thus,

$$X[2] = 1 \cdot 1 + 2 \cdot (-1) + 4 \cdot 1 + 5 \cdot (-1)$$

$$X[2] = 1 - 2 + 4 - 5 = (1 - 2 + 4 - 5) = (-2) + (-1) = -3$$

Compute $X[3]$:

$$X[3] = x[0] \cdot W_4^{\{0\}} + x[1] \cdot W_4^{\{3\}} + x[2] \cdot W_4^{\{6\}} + x[3] \cdot W_4^{\{9\}}$$

Calculate exponents:

$$\begin{aligned} - W_4^{\{3\}} &= j \\ - W_4^{\{6\}} &= W_4^{\{6 \bmod 4\}} = W_4^{\{2\}} = -1 \\ - W_4^{\{9\}} &= W_4^{\{1\}} = -j \end{aligned}$$

Thus,

$$X[3] = 1 \cdot 1 + 2 \cdot j + 4 \cdot (-1) + 5 \cdot (-j)$$

$$X[3] = 1 + 2j - 4 - 5j = (1 - 4) + (2j - 5j) = -3 - 3j$$

SUMMARY OF DFT RESULTS

k	$X[k]$	Magnitude	Phase (radians)
0	12	12	0
1	$-3 + 3j$	$\sqrt{(-3)^2 + 3^2} = \sqrt{9 + 9} = \sqrt{18} \approx 4.24$	$\arctan(3 / -3) = \arctan(-1) \approx -0.785$ radians (or 135°)
2	-3	3	π radians (180°)
3	$-3 - 3j$	4.24	-2.356 radians (or -135°)

INTERPRETATION OF RESULTS

MAGNITUDE SPECTRUM

The magnitudes indicate how much of each frequency component is present in the original sequence:

- $X[0]$ (DC component): 12, which reflects the average value of the sequence.
- $X[1]$ and $X[3]$: approximately 4.24, indicating significant frequency components at these bins.
- $X[2]$

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