

Three Vertices Of A Rectangle Are Shown On The Grid.What Are The Coordinates Of The Missing Vertex? Enter

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Understanding the properties of rectangles and how to determine the coordinates of missing vertices is a fundamental aspect of coordinate geometry. Whether you're a student learning geometry, a teacher preparing lesson plans, or a math enthusiast solving a challenging problem, grasping how to find the missing vertex of a rectangle based on known points is an essential skill. In this comprehensive guide, we will explore the methods, formulas, and step-by-step processes involved in determining the coordinates of the missing vertex when three vertices of a rectangle are provided on a coordinate grid.

Introduction to Rectangles on the Coordinate Plane

A rectangle is a four-sided polygon (quadrilateral) with opposite sides equal and parallel, and four right angles (each measuring 90 degrees). When represented on a coordinate plane, the vertices of a rectangle can be identified by their (x, y) coordinates.

Key properties of a rectangle on the coordinate plane:

- Opposite sides are parallel and equal in length.
- The diagonals bisect each other and are equal in length.
- Adjacent sides are perpendicular.

Knowing these properties allows us to use coordinate geometry techniques to find missing vertices when three vertices are known.

Understanding the Problem: Finding the Missing Vertex

Suppose you are given three vertices of a rectangle on a grid:

- Point A: (x_1, y_1)
- Point B: (x_2, y_2)
- Point C: (x_3, y_3)

Your task is to find the coordinates of the fourth vertex, D: (x_4, y_4) .

This problem involves identifying which points are adjacent and which are diagonally opposite, then applying geometric or algebraic methods to determine the missing point.

Methods to Find the Missing Vertex of a Rectangle

There are several techniques to find the missing vertex, depending on how the known points are arranged and what information is given. The most common methods include:

1. Using the Properties of Diagonals
2. Vector Addition Method
3. Midpoint Formula and Symmetry
4. Coordinate Geometry Approach

Let's explore each method in detail.

1. Using the Properties of Diagonals

In a rectangle, the diagonals are equal in length and bisect each other. This means that the midpoint of both diagonals is the same.

Approach:

- Identify which points are diagonally opposite.
- Use the midpoint formula to find the missing vertex.

Steps:

- If two points are diagonally opposite, their midpoint is the same as the midpoint of the other diagonal.
- The midpoint formula is:

$$\text{Midpoint} = \left(\frac{x_a + x_b}{2}, \frac{y_a + y_b}{2} \right)$$

- Find the midpoint of the known diagonal and use it to determine the coordinates of the missing vertex.

Example:

Suppose points A and C are diagonally opposite, and B is adjacent to A.

- Find the midpoint of AC:

$$M_{AC} = \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2} \right)$$

- The missing vertex D must satisfy that its midpoint with B is also M_{AC} .
- Therefore, the coordinates of D are:

$$x_4 = 2 \times x_{mid} - x_2$$

$$y_4 = 2 \times y_{mid} - y_2$$

2. Vector Addition Method

This method involves using vector operations to find the missing vertex.

Concept:

- In a rectangle, the vector from one vertex to the adjacent vertex combined with the vector to the third vertex helps determine the position of the fourth vertex.

Steps:

- Assign points such that the known points are A, B, and C.
- The missing point D can be found using the vector addition:

$$\vec{D} = \vec{A} + \vec{C} - \vec{B}$$

or

$$\vec{D} = \vec{A} + \vec{B} - \vec{C}$$

depending on the arrangement.

Example:

If A, B, and C are given, and assuming A and C are diagonally opposite, then:

$$\begin{aligned} x_4 &= x_1 + x_3 - x_2 \\ y_4 &= y_1 + y_3 - y_2 \end{aligned}$$

This formula is derived from the properties of rectangles and the parallelogram law.

3. Midpoint and Symmetry Approach

Because the diagonals bisect each other, the midpoint of the rectangle's diagonals is the same point. Using this, you can:

- Calculate the midpoints of known pairs.
- Use symmetry to find the missing vertex.

Steps:

1. Calculate the midpoints of the pairs of known points.
2. Use the symmetry of the rectangle to find the missing vertex based on these midpoints.

Step-by-Step Example

Let's work through a concrete example to illustrate the process of finding the missing vertex.

Given:

- Point A: (2, 3)
- Point B: (6, 7)
- Point C: (2, 7)

Question:

Find the coordinates of point D.

Step 1: Visualize and Identify Known Points

Plotting the points:

- A: (2, 3)
- B: (6, 7)
- C: (2, 7)

Observing these points, we notice:

- A and C are vertically aligned (same x-coordinate).
- B is diagonally positioned relative to A and C.

Step 2: Determine Diagonally Opposite Vertices

Since A and C share the same x-coordinate, they are likely aligned vertically, and B may be adjacent to either A or C.

To confirm the arrangement, calculate the midpoints:

- Midpoint of A and C:

$$M_{AC} = \left(\frac{2 + 2}{2}, \frac{3 + 7}{2} \right) = (2, 5)$$

- Midpoint of B and D (unknown D):

$$M_{BD} = \left(\frac{6 + x_4}{2}, \frac{7 + y_4}{2} \right)$$

Since the diagonals bisect each other:

$$M_{AC} = M_{BD}$$

Set equal:

$$(2, 5) = \left(\frac{6 + x_4}{2}, \frac{7 + y_4}{2} \right)$$

Solve for (x_4) and (y_4) :

$$\sqrt{\frac{6 + x_4}{2}} = 2 \rightarrow 6 + x_4 = 4 \rightarrow x_4 = -2$$

$$\sqrt{\frac{7 + y_4}{2}} = 5 \rightarrow 7 + y_4 = 10 \rightarrow y_4 = 3$$

Answer: The missing vertex D has coordinates $(-2, 3)$.

Additional Tips for Solving Coordinate Geometry Problems

- Plot the points on graph paper or using digital graphing tools for better visualization.
- Identify the arrangement of the known points: which are vertices, diagonally opposite, or adjacent.
- Use properties like midpoints, slopes, and distances to verify configurations.
- Remember the key properties: diagonals bisect each other, and opposite sides are parallel and equal.
- Practice with various configurations to become comfortable with different problem types.

Common Mistakes to Avoid

- Confusing which points are diagonally opposite versus adjacent.
- Forgetting that diagonals bisect each other, leading to incorrect midpoint calculations.
- Assuming the arrangement without verification, which can lead to incorrect results.
- Ignoring the coordinate plane's quadrants and signs of coordinates, especially when dealing with negative values.

Practice Problems for Mastery

1. Given points A(1, 2), B(4, 6), and C(1, 6), find point D.
2. Points P(3, 5), Q(7, 5), and R(3, 9) are vertices of a rectangle. Find the missing vertex.

Conclusion

Finding the missing vertex of a rectangle on the coordinate grid involves understanding the fundamental properties of rectangles, especially the behavior of diagonals that bisect each other and the parallel nature of sides. By applying concepts such as the midpoint formula, vector addition, and symmetry, you can confidently determine the coordinates of the unknown point. Practice with various configurations and problems to strengthen your skills in coordinate geometry, which is an essential part of math education and real-world applications involving spatial analysis.

Final Thoughts

Mastering the process of identifying missing vertices in rectangles not only enhances your geometric intuition but also improves your problem-solving skills in

Frequently Asked Questions

Given three vertices of a rectangle on the grid at points A(2, 3), B(5, 3), and C(2, 7), what are the coordinates of the missing fourth vertex?

The missing vertex is at (5, 7).

If two vertices of a rectangle are at (1, 2) and (1, 5), and the third vertex is at (4, 2), how do you find the fourth vertex?

The fourth vertex is at (4, 5).

Three vertices of a rectangle are shown at points (3, 4), (3, 8), and (6, 4). What are the coordinates of the missing vertex?

The missing vertex is at (6, 8).

On a grid, three vertices of a rectangle are at (0, 0), (0, 4), and (4, 0). How can you determine the fourth

vertex?

The fourth vertex is at (4, 4).

If the vertices of a rectangle are at (2, 2), (5, 2), and (2, 6), what is the coordinate of the remaining vertex?

The missing vertex is at (5, 6).

Given three vertices (1, 1), (4, 1), and (1, 5), how do you find the missing vertex of the rectangle?

The missing vertex is at (4, 5).

Three vertices of a rectangle are located at (7, 3), (7, 8), and (10, 3). What are the coordinates of the missing vertex?

The missing vertex is at (10, 8).

On a coordinate grid, three vertices are given as (3, 5), (3, 9), and (7, 5). How do you find the last vertex?

The last vertex is at (7, 9).

Additional Resources

Three Vertices Of A Rectangle Are Shown On The Grid. What Are The Coordinates Of The Missing Vertex? Enter

In the realm of coordinate geometry, the problem of determining a missing vertex of a rectangle given three vertices is both a classic and illustrative exercise. It challenges students and enthusiasts to apply fundamental geometric principles, vector operations, and algebraic techniques to uncover the unknown point. This problem not only reinforces the understanding of rectangles' properties but also enhances spatial reasoning and analytical skills. In this comprehensive review, we will explore the problem in detail, analyze the strategies for solving it, and elucidate the underlying mathematical concepts involved.

Understanding the Problem: The Basics of Coordinate Geometry and Rectangles

The Coordinate Plane and Vertices

The problem is set on a coordinate plane, a two-dimensional space where points are represented by ordered pairs (x, y) . When three vertices of a rectangle are known, the task is to find the fourth vertex's coordinates such that the four points form a perfect rectangle.

For example, suppose the given vertices are:

- Point A: (x_1, y_1)
- Point B: (x_2, y_2)
- Point C: (x_3, y_3)

The goal is to find point D: (x_4, y_4) , which completes the rectangle.

Properties of a Rectangle Relevant to the Problem

To solve this problem effectively, understanding key properties of rectangles in coordinate geometry is essential:

1. Opposite Sides Are Parallel and Equal in Length: In a rectangle, each pair of opposite sides are both parallel and equal in length.
2. Diagonals Are Equal and Bisect Each Other: The diagonals of a rectangle are equal in length and intersect at their midpoints.
3. Angles Are Right Angles: All interior angles are 90 degrees, which influences the slopes of adjacent sides.
4. Vertices Arrangement: The vertices can be arranged in various orders, but typically, the problem provides three consecutive vertices or three vertices forming a partial shape.

Approaches to Find the Missing Vertex

Various methods exist to determine the unknown vertex, each leveraging different properties of rectangles. The choice of approach depends on the known points' arrangement and the information provided.

Method 1: Using Midpoint and Diagonal Properties

Principle: The diagonals of a rectangle bisect each other; thus, the midpoints of the diagonals coincide. Given three vertices, the missing vertex can be found by exploiting this property.

Steps:

1. Identify the three known points, say A, B, and C.
2. Determine which points are adjacent and which form diagonals.
3. Use the property that the midpoint of the diagonal connecting two vertices is the same for both diagonals.

Example:

Suppose the known points are:

- A (x_1, y_1)
- B (x_2, y_2)
- C (x_3, y_3)

If A and C are opposite vertices, then the missing vertex D is obtained by:

$$D = B + A - C$$

or, more generally,

$$x_4 = x_2 + x_1 - x_3$$

$$y_4 = y_2 + y_1 - y_3$$

If B and C are opposite vertices:

$$D = A + C - B$$

Similarly, the formula adapts based on which points are opposite.

Advantages:

- Straightforward calculations.
- No need for slopes or angle calculations.

Limitations:

- Requires correct identification of opposite vertices.

Method 2: Using Vector Addition and Parallelogram Law

Principle: A rectangle's vertices form a parallelogram with right angles, and the sum of vectors around the shape satisfies specific relationships.

Steps:

1. Recognize that the three known points form three vertices of a parallelogram.
2. Use vector addition: the missing vertex is the one that satisfies the parallelogram law.

Example:

- If points A, B, C are known, then the missing vertex D can be found via:

$$D = A + C - B$$

or other variants depending on the known points' arrangement.

Advantages:

- Flexible for various configurations.
- Uses vector principles familiar in physics and geometry.

Limitations:

- Must correctly identify the configuration of the vertices.

Method 3: Using Slopes and Right Angles

Principle: Since rectangles have right angles, the slopes of adjacent sides are negative reciprocals.

Steps:

1. Calculate slopes of known sides.
2. Confirm whether two sides are perpendicular.
3. Use perpendicularity conditions to determine the position of the missing vertex.

Example:

Suppose points A and B are known, and the segment AB forms one side. To find D, analyze the slopes and construct the perpendicular side from the known points.

Advantages:

- Good for visualizing the geometric structure.
- Useful when the points are given in a sequence forming sides.

Limitations:

- More algebraically intensive.
- Slopes can be undefined or zero, requiring special handling.

Applying the Strategies: A Step-by-Step Example

Let's consider a concrete example with specific points to illustrate the process.

Given Points:

- A (2, 3)
- B (6, 3)
- C (6, 7)

Question:

Find the coordinates of the missing vertex D.

Step 1: Analyze the Known Points

- Points A and B share the same y-coordinate (3), indicating they could form a horizontal side.
- Points B and C share the same x-coordinate (6), indicating a vertical side.

Observation: The points suggest a shape with sides aligned along axes, characteristic of rectangles aligned with the grid axes.

Step 2: Determine Which Points Are Opposite Vertices

- If A and C are diagonally opposite, then the missing vertex D can be found using the midpoint property.

Calculating D:

Using the vector addition method:

$$D = B + C - A$$

- x-coordinate: $6 + 6 - 2 = 10$
- y-coordinate: $3 + 7 - 3 = 7$

Thus, D is at (10, 7).

Step 3: Verify the Solution

- Check whether the points form a rectangle:

Vertices:

- A (2, 3)
- B (6, 3)
- C (6, 7)
- D (10, 7)

Side lengths:

- AB: $|(6-2), (3-3)| = (4, 0)$ — length 4
- BC: $|(6-6), (7-3)| = (0, 4)$ — length 4
- CD: $|(10-6), (7-7)| = (4, 0)$ — length 4
- DA: $|(2-10), (3-7)| = (-8, -4)$

Calculate lengths:

- AB: $\sqrt{4^2 + 0^2} = 4$
- BC: $\sqrt{0^2 + 4^2} = 4$
- CD: $\sqrt{4^2 + 0^2} = 4$
- DA: $\sqrt{(-8)^2 + (-4)^2} = \sqrt{64 + 16} = \sqrt{80} \approx 8.94$

Since DA is longer, it indicates that the shape is not a perfect rectangle with all sides equal unless the points are arranged differently. But the key property is that the diagonals should bisect each other and be equal.

Check midpoints:

- Diagonal AC midpoint: $((2+6)/2, (3+7)/2) = (4, 5)$
- Diagonal BD midpoint: $((6+10)/2, (3+7)/2) = (8, 5)$

The midpoints differ, so the points are not forming a rectangle with these assignments.

Alternative assumption:

If we consider A and B as adjacent vertices:

- The missing vertex D would then be at:

$$D = A + C - B$$

Calculating:

- x: $2 + 6 - 6 = 2$
- y: $3 + 7 - 3 = 7$

So, D at (2, 7).

Now, check midpoints:

- Diagonal AC: midpoint $((2+6)/2, (3+7)/2) = (4, 5)$
- Diagonal BD: midpoint $((6+2)/2, (3+7)/2) = (4, 5)$

Midpoints coincide, confirming that points A, B, C, and D form a rectangle with diagonals bisecting at (4, 5).

Conclusion:

The missing vertex D is at (2, 7).

Mathematical Principles Underpinning the Solution

The process exemplifies several foundational principles:

1. Midpoint Formula: Used to verify the intersection of diagonals and to find missing vertices via vector addition.
2. Opposite Vertices and Diagonals: Recognizing that in a rectangle, the diagonals bisect each other and are equal in length.
3. Vector Addition: Applying the parallelogram law to determine

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