Two Balls Are Picked At Random From A Jar That Contains Two Red And Ten White Balls. Find The Probability

Two Balls Are Picked At Random From A Jar That Contains Two Red And Ten White Balls. Find The Probability

Understanding probability is essential in various fields, from statistics to everyday decision-making. In this article, we explore the problem of determining the probability of drawing certain colored balls from a jar containing a mixture of red and white balls. Specifically, we examine the scenario where two balls are randomly selected from a jar with two red and ten white balls. We will analyze the problem step-by-step, providing a comprehensive explanation of how to calculate the probability of different outcomes.

Understanding the Problem

Before diving into the calculations, it's crucial to understand the problem's parameters clearly.

Details of the Jar

- Total number of balls: 12
- Red balls: 2
- White balls: 10

What We Need to Find

- The probability of various events, such as:
- Both balls drawn are red
- Both balls drawn are white
- One red and one white are drawn

Fundamentals of Probability

To solve this problem, we need to recall some basic concepts of probability:

Sample Space

- The set of all possible outcomes when drawing balls from the jar.

Event

- A subset of the sample space, such as drawing two red balls.

Probability of an Event

- Defined as the ratio of favorable outcomes to total possible outcomes:

$$
P(\text{event}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}
$$

Key Assumption

- Drawing is done without replacement, meaning once a ball is drawn, it is not put back into the jar before the second draw.

Calculating Total Possible Outcomes

The total number of ways to select any 2 balls from 12 is given by combinations:

$$
\text{Total outcomes} = \binom{12}{2} = \frac{12 \times 11}{2 \times 1} = 66
$$

This represents all possible pairs of two balls that can be drawn from the jar.

Calculating Favorable Outcomes

Let's analyze each event separately.

1. Both Balls Are Red

- Number of ways to choose 2 red balls out of 2:

$$
\binom{2}{2} = 1
$$

- Since there is only one way to pick both red balls, the favorable outcomes for this event are 1.

- Therefore, the probability:

$$
P(\text{both red}) = \frac{\binom{2}{2}}{\binom{12}{2}} = \frac{1}{66}
$$

2. Both Balls Are White

- Number of ways to choose 2 white balls out of 10:

$$
\binom{10}{2} = \frac{10 \times 9}{2} = 45
$$

- Favorable outcomes:

$$
45
$$

- Probability:

$$
P(\text{both white}) = \frac{45}{66} = \frac{15}{22}
$$

3. One Red and One White

- Number of ways to choose 1 red and 1 white:

- Ways to choose 1 red out of 2:

$$
\binom{2}{1} = 2
$$

- Ways to choose 1 white out of 10:

$$
\binom{10}{1} = 10
$$

- Total favorable outcomes:

$$
2 \times 10 = 20
$$

- Probability:

$$
P(\text{one red and one white}) = \frac{20}{66} = \frac{10}{33}
$$

Summary of Probabilities

Event	Favorable Outcomes	Probability
Both red	1	$\frac{1}{66}$
Both white	45	$\frac{15}{22}$
One red, one white	20	$\frac{10}{33}$

Additional Considerations

Understanding these calculations helps in grasping the fundamentals of probability in real-world scenarios. Some points to keep in mind:

Impact of Changing the Number of Balls

- If the number of red or white balls changes, the probabilities need to be recalculated accordingly.
- For example, adding more red balls increases the probability of drawing two red balls.

Without Replacement vs. With Replacement

- The above calculations assume drawing without replacement.
- If balls are replaced after each draw, probabilities would change because the composition of the jar remains constant.

Applications of These Concepts

- Quality control in manufacturing
- Card games and gambling
- Decision-making in uncertain situations
- Statistical sampling techniques

Practice Problems to Reinforce Learning

To solidify understanding, try solving these related problems:

1. What is the probability of drawing at least one red ball in two draws?

2. If three balls are drawn instead of two, what is the probability that exactly two are white?

3. Suppose the jar contains 4 red and 8 white balls. How does this change the probability of drawing two red balls?

Conclusion

Calculating the probability of drawing specific colored balls from a mixture involves understanding combinations and the total number of possible outcomes. In our problem, the probability that both balls are red is $\frac{1}{66}$, both white is $\frac{15}{22}$, and one red and one white is $\frac{10}{33}$. Mastery of these basic probability concepts provides a foundation for more complex statistical analysis and problem-solving in various fields.

By practicing similar problems and understanding the underlying principles, you can improve your ability to analyze probabilistic scenarios confidently and accurately.

Frequently Asked Questions

What is the probability of picking two red balls from the jar containing 2 red and 10 white balls?

The probability is 1/66. This is calculated as (2/12) (1/11) because after choosing one red ball, only one red remains out of 11 remaining balls.

How do you calculate the probability of selecting two white balls from the jar?

The probability is 45/66 or 15/22. This is found by computing (10/12) (9/11) since the first white ball is 10 out of 12, and the second white ball is 9 remaining out of 11.

What is the probability of picking one red and one white ball in any order?

The probability is 30/66 or 5/11. It includes two scenarios: red then white, and white then red, each calculated and summed up.

In the problem, do we consider the order of picking the balls when calculating probability?

No, since the balls are picked at random without regard to order, we consider combinations rather than permutations.

What is the total number of ways to pick any two balls from the jar?

The total number of combinations is 66, calculated as C(12, 2) = 1211/2 = 66.

How can we verify the probability of selecting two white balls?

By calculating the ratio of favorable outcomes (ways to pick two white balls) to total outcomes. There are C(10, 2) = 45 ways for white balls, so the probability is 45/66.

What is the probability of not picking any red balls (i.e., both white)?

The probability is 45/66 or 15/22, same as picking two white balls, since only white balls are involved.

Can the probability of picking two red balls be simplified further?

Yes, it simplifies to 1/66, which is already in its simplest form.

If one ball is picked and not replaced, how does that affect probability calculations for the second pick?

The probabilities change after the first pick because the total number of balls decreases, so calculations must account for the updated counts.

What are the key steps to solve probability problems involving drawing balls from a jar?

Identify total outcomes, determine favorable outcomes, calculate combinations if order doesn't matter, and simplify the resulting fractions to find the probability.

Additional Resources

Probability of Picking Two Balls at Random from a Jar Containing Red and White Balls

In the realm of probability theory, understanding the likelihood of specific events occurring within a given set of outcomes is fundamental. A classic problem that exemplifies this is the scenario where two balls are randomly drawn from a jar containing a mixture of differently colored balls. Such problems are not only academically engaging but also serve as foundational concepts in fields ranging from statistics and data science to gambling and decision-making processes. Today, we delve into a specific instance: a jar containing two red balls and ten white balls, and we aim to determine the probability of various outcomes when two balls are drawn at random without replacement.

Understanding the Scenario: The Jar's Contents

Before calculating probabilities, it’s essential to clearly comprehend the initial conditions and the problem’s parameters.

Contents of the Jar:

- Red balls: 2
- White balls: 10

Total number of balls: 2 + 10 = 12

Event of interest: Drawing two balls at random, without replacement.

The question posed is:

- What is the probability of different specific outcomes during this process? For example:
- Both balls are red
- Both balls are white
- One ball is red and the other is white

Fundamental Concepts in Probability

Understanding and calculating these probabilities involves key concepts:

2.1 Sample Space

The set of all possible outcomes of drawing two balls from the jar. Since the order in which we draw the balls may or may not matter, the problem can be approached in two ways:

- Ordered sampling: where the sequence of drawing matters
- Unordered sampling: where only the combination matters, regardless of order

In most cases involving drawing balls without replacement, the order is irrelevant unless specified.

2.2 Events

Specific outcomes or sets of outcomes, such as "both balls are red."

2.3 Probabilities

Expressed as ratios of favorable outcomes to total possible outcomes, assuming each outcome is equally likely (uniform probability).

Calculating Total Possible Outcomes

The first step is to determine the total number of ways to draw any two balls from the 12 in the jar.

2.1 Combinatorial Approach

Since the balls are identical within their color groups but distinguishable as individual balls, the total number of combinations of 12 balls taken 2 at a time is:

$$
\text{Total outcomes} = \binom{12}{2} = \frac{12 \times 11}{2 \times 1} = 66
$$

This represents all possible pairs of two balls that can be drawn from the jar.

Calculating Specific Probabilities

Now, let's analyze each specific event separately.

3.1 Probability Both Balls Are Red

Favorable outcomes: Both red balls are drawn.

Since there are 2 red balls, the number of ways to select both:

$$
\binom{2}{2} = 1
$$

Calculation:

$$
P(\text{both red}) = \frac{\text{Number of ways to select 2 red balls}}{\text{Total number of ways to select any 2 balls}} = \frac{1}{66}
$$

3.2 Probability Both Balls Are White

Favorable outcomes: Both white balls are drawn.

Number of ways to select 2 white balls:

$$
\binom{10}{2} = \frac{10 \times 9}{2} = 45
$$

Calculation:

$$
P(\text{both white}) = \frac{45}{66} = \frac{15}{22}
$$

3.3 Probability One Red and One White

Favorable outcomes: One red ball and one white ball.

Number of ways to select one red and one white:

$$
\binom{2}{1} \times \binom{10}{1} = 2 \times 10 = 20
$$

Calculation:

$$
P(\text{one red, one white}) = \frac{20}{66} = \frac{10}{33}
$$

Summary of Probabilities

Event	Number of Favorable Outcomes	Probability
Both balls are red	1	$\frac{1}{66}$
Both balls are white	45	$\frac{15}{22}$
One red and one white	20	$\frac{10}{33}$

The sum of these probabilities equals 1, confirming the completeness of the outcomes:

$$
\frac{1}{66} + \frac{15}{22} + \frac{10}{33} = 1
$$

Deeper Analytical Insights

4.1 Conditional Probabilities

Suppose we are interested in the probability of drawing a white ball first, then a red ball, or vice versa. These are conditional probabilities and can be computed as follows:

First event: Drawing a white ball first, then a red ball

- Probability of drawing a white ball first:

$$
P(\text{white first}) = \frac{10}{12}
$$

- After drawing one white ball, remaining balls:

$$
\text{Remaining balls} = 11
$$

\]

- Probability of drawing a red ball next:

$$
P(\text{red second} | \text{white first}) = \frac{2}{11}
$$

- Combined probability:

$$
P(\text{white first, red second}) = \frac{10}{12} \times \frac{2}{11} = \frac{10 \times 2}{12 \times 11} = \frac{20}{132} = \frac{5}{33}
$$

Similarly, for the reverse order:

First event: Drawing a red ball first, then a white ball

- Probability of drawing a red ball first:

$$
P(\text{red first}) = \frac{2}{12} = \frac{1}{6}
$$

- Remaining balls:

$$
11
$$

- Probability of drawing a white ball next:

$$
P(\text{white second} | \text{red first}) = \frac{10}{11}
$$

- Combined probability:

$$
\frac{1}{6} \times \frac{10}{11} = \frac{10}{66} = \frac{5}{33}
$$

Adding both scenarios:

$$
\frac{5}{33} + \frac{5}{33} = \frac{10}{33}
$$

which matches the earlier calculation for "one red, one white" in any order.

4.2 Expected Outcomes

Knowing these probabilities allows us to compute expected values—such as the expected number of red balls in multiple draws or the likelihood of specific sequences occurring over repeated trials.

Real-World Applications and Implications

Understanding such probabilities extends beyond simple theoretical exercises, impacting various domains:

- Quality Control: For example, if a manufacturer inspects randomly sampled items (red and white representing defective and non-defective items), the probabilities assist in estimating defect rates.
- Gambling and Games of Chance: Many casino games rely on understanding card probabilities or similar draw scenarios.
- Medical Testing: When analyzing test results for diseases, the chance of certain outcomes can be modeled similarly.
- Decision-Making Under Uncertainty: Probabilities guide strategic choices in business or policy under uncertainty.

Conclusion and Reflection

The problem of selecting two balls from a jar containing two red and ten white balls exemplifies fundamental probability principles rooted in combinatorics. Through systematic calculation—identifying total possible outcomes, enumerating favorable outcomes, and applying basic probability formulas—we derive meaningful insights into the likelihood of various scenarios. The analysis underscores the importance of understanding underlying assumptions, such as the randomness and fairness of the selection process, and highlights the versatility of probability theory in tackling diverse real-world problems.

By dissecting this classic problem, we observe the elegance and practical utility of combinatorial reasoning and probability calculations. Whether in academic pursuits, industrial applications, or everyday decision-making, mastering such foundational concepts empowers individuals and organizations to make informed, rational choices amid uncertainty.

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