

Two Conducting Cylindrical Wires Are Made Out Of The Same Material. Wire X Has Twice The Length And Twice

Two Conducting Cylindrical Wires Are Made Out Of The Same Material. Wire X Has Twice The Length And Twice the Radius of Wire Y. This scenario presents an interesting case for understanding electrical resistance, conductivity, and the relationships between physical dimensions and material properties. When analyzing such a problem, it's essential to explore how the dimensions of the wires influence their electrical characteristics, particularly resistance, and how these factors relate to real-world applications such as wiring in circuits, transmission lines, and electrical engineering design.

In this article, we will delve into the fundamental principles governing the electrical resistance of cylindrical wires, examine how changes in length and radius impact resistance, and explore the practical implications of these relationships. Understanding these concepts is pivotal for engineers and students alike, as it aids in selecting appropriate wire sizes for specific applications, optimizing electrical efficiency, and ensuring safety standards are met.

Understanding Electrical Resistance in Conducting Wires

What Is Electrical Resistance?

Electrical resistance is a measure of how much a material opposes the flow of electric current. It depends on both the material's intrinsic properties—its resistivity—and its physical dimensions, specifically length and cross-sectional area. The basic formula for resistance (R) of a uniform cylindrical wire is:

$$R = \rho \left(\frac{L}{A} \right)$$

where:

- ρ (rho) is the resistivity of the material,
- L is the length of the wire,
- A is the cross-sectional area of the wire.

Since both wires are made of the same material, their resistivity (ρ) values are equal, simplifying comparisons.

Role of Material Properties

The resistivity (ρ) is an intrinsic property indicating how strongly a material opposes electric current. Materials like copper and aluminum have

low resistivity, making them suitable for conductors. The resistivity remains constant regardless of the wire dimensions but is crucial when calculating resistance.

Analyzing the Dimensions of the Two Wires

Given the problem statement:

- Wire X is twice as long as Wire Y.
- Wire X has twice the radius of Wire Y.

Let's define:

- Length of Wire Y = L
- Radius of Wire Y = r
- Therefore, length of Wire X = $2L$
- Radius of Wire X = $2r$

Because the wires are cylindrical, their cross-sectional area (A) can be calculated using:

$$A = \pi r^2$$

Applying this:

- Cross-sectional area of Wire Y = πr^2
- Cross-sectional area of Wire X = $\pi (2r)^2 = 4\pi r^2$

This means Wire X has four times the cross-sectional area of Wire Y.

Calculating Resistance of Each Wire

Using the resistance formula $R = \rho (L / A)$, we can now find the resistance of each wire:

Resistance of Wire Y

$$R_Y = \rho (L / (\pi r^2))$$

Resistance of Wire X

$$R_X = \rho (2L / (4\pi r^2)) = \rho (2L / 4\pi r^2) = (1/2) \rho (L / \pi r^2) = (1/2) R_Y$$

This reveals a key insight: Wire X's resistance is half that of Wire Y.

Implications of Resistance Differences

This analysis illustrates that even though Wire X is longer and thicker, its resistance is significantly less than that of Wire Y. Specifically, doubling the length of the wire increases resistance linearly, but doubling the radius increases the cross-sectional area quadratically, which decreases resistance.

Summary of Resistance Relationship:

- Resistance of Wire X = $(1/2)$ Resistance of Wire Y.

This inverse proportionality between resistance and cross-sectional area underscores the importance of dimensions in electrical design. Thicker wires (larger radii) have lower resistance, which reduces energy losses during transmission.

Practical Applications and Considerations

Wire Selection for Electrical Circuits

Choosing the right wire size depends on balancing safety, efficiency, and cost. Understanding how dimensions affect resistance helps engineers select appropriate wire gauges to minimize power losses and prevent overheating.

Key considerations include:

- Current-carrying capacity (ampacity)
- Resistance and voltage drop
- Mechanical strength and flexibility
- Cost and availability

Transmission Lines and Power Distribution

In power transmission, reducing resistance is vital to minimize energy losses over long distances. Thicker and shorter conductors are preferred, but material costs and physical constraints often limit these choices. The relationship shown in the analysis guides engineers in designing efficient transmission systems.

Material Efficiency and Cost Optimization

Using the same material, increasing cross-sectional area (radius) effectively reduces resistance, but at increased material and manufacturing costs. Engineers must optimize dimensions to balance performance with expenses.

Additional Factors Influencing Resistance and

Performance

While the basic calculation provides a foundational understanding, real-world scenarios involve additional factors:

Temperature Effects

Resistivity (ρ) varies with temperature. Generally, resistance increases with temperature, so wire performance can change under load or in high-temperature environments.

Skin Effect

At high frequencies, alternating current tends to flow near the surface of conductors, effectively reducing the cross-sectional area and increasing resistance.

Material Purity and Quality

Impurities and manufacturing quality influence resistivity and, consequently, resistance.

Summary and Key Takeaways

- When two cylindrical wires are made of the same material, their resistance depends on their length and cross-sectional area.
- Increasing the length of a wire increases its resistance linearly.
- Increasing the radius of a wire increases its cross-sectional area quadratically, significantly decreasing resistance.
- In the given scenario, doubling the length and radius results in the new wire (Wire X) having half the resistance of the original (Wire Y).
- Understanding these relationships is crucial for designing efficient electrical systems, optimizing material usage, and ensuring safety.

Final thought: The interplay between physical dimensions and electrical properties is fundamental to electrical engineering. Mastery of these relationships enables the creation of safer, more efficient, and cost-effective electrical infrastructure.

Keywords: electrical resistance, cylindrical wires, resistivity, wire dimensions, conductors, resistance calculation, electrical engineering, wire sizing, power transmission, material properties

Frequently Asked Questions

What is the relationship between the length and resistance of wire X compared to wire Y if wire X has twice the length and diameter?

Since wire X has twice the length and twice the diameter, its resistance increases due to the longer length but decreases due to the larger diameter. Overall, the resistance of wire X is four times that of wire Y.

How does the resistance of wire X compare to wire Y if wire X has twice the length and twice the diameter?

The resistance of wire X is four times greater than that of wire Y because resistance is directly proportional to length and inversely proportional to the cross-sectional area, which increases with the square of the diameter.

If both wires are made of the same material and wire X has twice the length and diameter of wire Y, how does their conductivity compare?

Conductivity, being an intrinsic property of the material, remains the same for both wires; however, their resistance differs due to geometric differences.

How does increasing both the length and diameter of a wire affect its electrical resistance?

Increasing the length increases the resistance proportionally, while increasing the diameter decreases resistance proportionally to the area. If both are increased, the net effect depends on the relative amounts of change; in the case of twice the length and diameter, the resistance increases overall.

What is the formula to calculate the resistance of a cylindrical wire, and how does it apply here?

The resistance $R = (\rho L) / A$, where ρ is resistivity, L is length, and A is cross-sectional area. For wire X, with twice the length and diameter, the resistance increases by a factor of 4 compared to wire Y.

Why does increasing the diameter of a wire reduce its resistance?

Because resistance is inversely proportional to the cross-sectional area, and increasing the diameter increases the area, thereby reducing the resistance.

If wire X has twice the length and diameter of wire Y, what are the implications for their use in electrical circuits?

Wire X will have higher resistance due to its longer length, which may lead

to greater voltage drops and power loss. The larger diameter can help compensate somewhat, but overall, wire X's higher resistance affects its efficiency in circuits.

Additional Resources

Two Conducting Cylindrical Wires Are Made Out Of The Same Material. Wire X Has Twice The Length And Twice The Diameter.

Understanding the electrical properties of conductors is fundamental in designing and optimizing electrical systems. When examining two cylindrical wires made from the same material but differing in their physical dimensions, it becomes essential to analyze how these differences influence their electrical resistance, conductivity, and overall performance. This article delves into the physics governing such wires, exploring the implications of their dimensions, and providing insights relevant to engineers, physicists, and students alike.

Introduction to Electrical Conductivity and Resistance

Before analyzing the specific case of the two wires, it is crucial to understand the fundamental concepts of electrical conductivity and resistance.

Electrical Conductivity (σ)

Electrical conductivity (σ) is a material property that measures how easily electrons can flow through a material. Metals typically have high conductivity, which makes them suitable for conducting electrical current. Since both wires are made of the same material, they share the same conductivity value.

Electrical Resistance (R)

Resistance quantifies how much a material opposes the flow of electric current. It is influenced by the material's properties, its dimensions, and temperature. The resistance R of a uniform cylindrical wire is given by:

$$R = \frac{\rho L}{A}$$

where:

- ρ (ρ) = resistivity of the material (inverse of conductivity, $\rho = 1/\sigma$),
- L = length of the wire,
- A = cross-sectional area.

This formula indicates that resistance increases with length and resistivity but decreases with larger cross-sectional area.

Dimensional Specifications of the Wires

The scenario involves two wires made from the same material but differing in dimensions:

- Wire X:
 - Length: (L_X)
 - Diameter: (D_X)
- Wire Y:
 - Length: (L_Y)
 - Diameter: (D_Y)

Given in the problem:

- $(L_X = 2L_Y)$ (Wire X is twice as long as Wire Y)
- $(D_X = 2D_Y)$ (Wire X has twice the diameter of Wire Y)

From these, we can derive other properties such as their cross-sectional areas and resistances, which will be critical in understanding their electrical behavior.

Analyzing the Physical Dimensions and Cross-Sectional Areas

Since both wires are cylindrical, their cross-sectional area (A) is given by:

$$[A = \pi \left(\frac{D}{2} \right)^2 = \frac{\pi D^2}{4}]$$

Applying this to each wire:

- Wire X:

$$[A_X = \frac{\pi D_X^2}{4}]$$

- Wire Y:

$$[A_Y = \frac{\pi D_Y^2}{4}]$$

Given $(D_X = 2 D_Y)$:

$$[A_X = \frac{\pi (2 D_Y)^2}{4} = \frac{\pi 4 D_Y^2}{4} = \pi D_Y^2]$$

Similarly,

$$[A_Y = \frac{\pi D_Y^2}{4}]$$

Key insight:

The cross-sectional area of Wire X is four times that of Wire Y:

$$[A_X = 4 A_Y]$$

This relationship will significantly influence the resistance calculations, as resistance is inversely proportional to cross-sectional area.

Calculating Resistance of Each Wire

Using the resistance formula:

$$[R = \frac{\rho L}{A}]$$

and considering the dimensions:

- Wire X:

$$[R_X = \frac{\rho L_X}{A_X}]$$

- Wire Y:

$$[R_Y = \frac{\rho L_Y}{A_Y}]$$

Given the relationships:

$$[L_X = 2L_Y]$$

$$[A_X = 4A_Y]$$

we substitute these into the resistance formulas:

$$[R_X = \frac{\rho \times 2 L_Y}{4 A_Y} = \frac{\rho L_Y}{2 A_Y}]$$

$$[R_Y = \frac{\rho L_Y}{A_Y}]$$

Comparing these:

$$[R_X = \frac{1}{2} R_Y]$$

This indicates that Wire X has half the resistance of Wire Y, despite being twice as long. The increased diameter (and thus larger cross-sectional area) of Wire X more than compensates for its increased length, resulting in a lower overall resistance.

Implications for Electrical Conductance and Current Flow

Electrical conductance (G), the reciprocal of resistance, reflects a wire's ability to conduct current:

$$[G = \frac{1}{R}]$$

- Wire X:

$$[G_X = \frac{1}{R_X} = \frac{2}{R_Y}]$$

- Wire Y:

$$\left[G_Y = \frac{1}{R_Y} \right]$$

Therefore,

$$\left[G_X = 2 G_Y \right]$$

Interpretation:

Wire X, despite being longer, is more conductive due to its larger diameter, leading to a greater ability to carry current compared to Wire Y.

Electrical Power Dissipation and Heat Generation

Resistance influences how much heat is generated when a current flows through a wire, following Joule's law:

$$\left[P = I^2 R \right]$$

where:

- P = power dissipated as heat,
- I = current flowing through the wire.

Given Wire X's lower resistance, for the same current, it dissipates less heat than Wire Y. Conversely, if both wires carry identical currents, Wire Y, with its higher resistance, would generate more heat, possibly affecting its temperature stability and longevity.

Voltage and Current Distribution in the Wires

Suppose both wires are connected across the same voltage source V:

$$\left[I = \frac{V}{R} \right]$$

- Wire X:

$$\left[I_X = \frac{V}{R_X} = \frac{V}{(1/2) R_Y} = 2 \frac{V}{R_Y} \right]$$

- Wire Y:

$$\left[I_Y = \frac{V}{R_Y} \right]$$

Result:

Wire X will conduct twice the current of Wire Y under the same applied voltage, again illustrating how dimensions influence electrical behavior.

Energy Considerations and Material Efficiency

From an energy efficiency perspective, a wire with lower resistance is preferable because it minimizes power loss during transmission. The analysis indicates that increasing diameter significantly reduces resistance, which translates into more efficient energy transmission over longer distances.

Trade-offs include:

- Mechanical strength vs. weight: Larger diameter wires are stronger but heavier.
- Cost implications: Larger wires require more material, increasing costs.
- Application-specific choices: For high-current applications, thicker wires like Wire X are advantageous.

Material Properties and their Role

Since both wires are made of the same material, their resistivity (ρ) remains constant. However, the physical dimensions play a crucial role in their resistance and conductance. If materials with different resistivities are used, the resistance calculations would need adjustment accordingly.

The resistivity influences how much the dimensions matter; materials with very low resistivity (like copper or silver) are preferred for electrical conductors because they naturally have low resistance. The analysis here assumes a typical metallic conductor.

Practical Applications and Design Considerations

Understanding the interplay between length, diameter, and resistance is vital in various engineering fields:

- Power Transmission:

Thicker cables are used to reduce resistance and power losses over long distances.

- Electronic Components:

Conductors within circuits are designed with specific dimensions to regulate current flow and heat dissipation.

- Structural Engineering:

Mechanical strength considerations often influence wire dimensions alongside electrical properties.

- Cost and Material Efficiency:

Optimizing dimensions to balance performance, cost, and weight is essential in large-scale projects.

Conclusion: Synthesis of Dimensional Effects on Conducting Wires

The analysis of two cylindrical wires made from the same material, with one being twice as long and twice as thick, reveals nuanced relationships between physical dimensions and electrical properties. Despite its longer length, the thicker wire (Wire X) exhibits significantly lower resistance due to its larger cross-sectional area. This results in higher conductance and allows greater current flow under the same voltage, highlighting the importance of geometry in electrical design.

In practical terms, the findings underscore that increasing the diameter of a wire has a more pronounced effect on reducing resistance than simply increasing its length. For engineers and designers, this means that optimizing wire dimensions is crucial for efficiency, safety, and cost-effectiveness in electrical systems.

Ultimately, this case exemplifies fundamental principles in electromagnetism and materials science, illustrating how physical dimensions govern electrical behavior. Such insights are instrumental in advancing technologies and ensuring the efficient transmission and utilization of electrical energy across diverse applications.

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