Use Simpson's Rule With N = 6 To Estimate The Arc Length Of The Curve, L. Give Your Answer To Six Decimal

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Introduction to Arc Length Estimation

Estimating the length of a curve, also known as the arc length, is a fundamental problem in calculus with applications in physics, engineering, and computer graphics. When a function $y = f(x)$ defines a curve between two points, calculating the exact arc length involves integrating the square root of $1 + (f'(x))^2$ over the interval. However, for complex functions or when an analytical solution is difficult, numerical methods such as Simpson's Rule provide efficient approximations.

In this article, we focus on applying Simpson's Rule with $N = 6$ subdivisions to estimate the arc length L of a given curve, and we will present the answer rounded to six decimal places. This process involves understanding the formula for arc length, setting up the numerical approximation, and executing the calculations step by step.

Understanding the Arc Length Formula

The arc length L of a curve $y = f(x)$ from $x = a$ to $x = b$ is given by the integral:

$$
L = \int_{a}^{b} \sqrt{1 + (f'(x))^2} \, dx
$$

Where:
- $f'(x)$ is the derivative of the function $f(x)$.
- The integral computes the accumulated length along the curve.

To approximate L numerically, we discretize the interval $[a, b]$ into N subintervals, evaluate the integrand at specific points, and then apply a numerical method such as Simpson's Rule.

Setting Up the Numerical Estimation

Suppose we are given a specific function $y = f(x)$, with known derivative $f'(x)$, over the interval $[a, b]$. For the purpose of this example, let's consider the function:

\[

$$
f(x) = \sin(x)
$$

over the interval $[0, \pi/2]$. This is a common choice because its derivative $f'(x) = \cos(x)$ is straightforward to compute.

Step 1: Define the parameters
- $a = 0$
- $b = \pi/2$
- Number of subintervals $N = 6$

Step 2: Calculate the step size h

$$
h = \frac{b - a}{N} = \frac{\pi/2 - 0}{6} = \frac{\pi/2}{6} = \frac{\pi}{12} \approx 0.261799
$$

Step 3: Determine the points $x_0, x_1, ..., x_6$

$$
x_i = a + i \times h, \quad i = 0, 1, ..., 6
$$

which gives:

i	x_i	Approximate value
0	0	0.000000
1	$\pi/12$	0.261799
2	$\pi/6$	0.523599
3	$\pi/4$	0.785398
4	$3\pi/12$	1.047198
5	$\pi/2$	1.570796
6	$\pi/2$	1.570796

Note: For calculations, use the actual approximate decimal values.

Calculating the Integrand at Each Point

The integrand is:

$$
\sqrt{1 + (f'(x))^2}
$$

Since $f(x) = \sin(x)$, then $f'(x) = \cos(x)$.

At each x_i:

$$
\text{Integrand} = \sqrt{1 + \cos^2(x_i)}
$$

\]

Calculate these values:

i	\(x_i \)	\(\cos(x_i) \)	\((\cos(x_i))^2 \)	\(\sqrt{1 + (\cos x_i)^2} \)
0	0	1.000000	1.000000	\(\sqrt{2} \approx 1.414214 \)
1	\(\pi/12 \) (~0.262)	0.965926	0.933013	\(\sqrt{1 + 0.933013} \approx 1.379883 \)
2	\(\pi/6 \) (~0.524)	0.866025	0.75	\(\sqrt{1 + 0.75} \approx 1.322876 \)
3	\(\pi/4 \) (~0.785)	0.707106	0.5	\(\sqrt{1 + 0.5} \approx 1.224745 \)
4	\(3\pi/12 \) (~1.047)	0.5	0.25	\(\sqrt{1 + 0.25} \approx 1.118034 \)
5	\(\pi/2 \) (~1.571)	0	0	\(\sqrt{1 + 0} = 1.000000 \)
6	\(\pi/2 \) (~1.571)	0	0	Same as above, 1.000000

Note: Since \(x_6 \) is the same as \(x_5 \), the integrand at \(x_6 \) is also 1.000000.

Applying Simpson's Rule

Simpson's Rule approximates the integral as:

\[
L \approx \frac{h}{3} \left[f(x_0) + 4 \sum_{i=1,3,5} f(x_i) + 2 \sum_{i=2,4} f(x_i) + f(x_N) \right]
\]

Where:
- \(f(x_i) = \sqrt{1 + (f'(x_i))^2} \)
- The sums are over odd and even indices respectively.

Using the computed integrand values:

\[
\begin{aligned}
L &\approx \frac{h}{3} \times \big[f(x_0) + 4(f(x_1) + f(x_3) + f(x_5)) \\
&\quad + 2(f(x_2) + f(x_4)) + f(x_6) \big]
\end{aligned}
\]

Plugging in the values:

\[
\begin{aligned}
L &\approx \frac{0.261799}{3} \times \big[1.414214 + 4(1.379883 + 1.224745 + 1.000000) \\
&\quad + 2(1.322876 + 1.118034) + 1.000000 \big]
\end{aligned}
\]

Calculations:

- Sum of odd points:

$$
4 \times (1.379883 + 1.224745 + 1.000000) = 4 \times 3.604628 = 14.418512
$$

- Sum of even points:

$$
2 \times (1.322876 + 1.118034) = 2 \times 2.44091 = 4.88182
$$

Now, summing all:

$$
1.414214 + 14.418512 + 4.88182 + 1.000000 = 21.714546
$$

Finally, multiply by $h/3$:

$$
L \approx \frac{0.261799}{3} \times 21.714546 \approx 0.087266 \times 21.714546 \approx 1.896102
$$

Answer:

$$
\boxed{L \approx 1.896102}
$$

rounded to six decimal places.

Conclusion and Practical Considerations

Using Simpson's Rule with $N=6$ subdivisions provides an efficient and accurate way to approximate the arc length of a curve. While the example used the sine function over $[0, \pi/2]$, the procedure applies broadly to other functions, provided you can compute the derivative and evaluate the integrand at the specified points.

Key steps include:
- Determining the interval and subdivisions.
- Calculating h and the points x_i.
- Evaluating the integrand $\sqrt{1 + (f'(x_i))^2}$.
- Applying Simpson's Rule formula precisely

Frequently Asked Questions

What is Simpson's Rule and how is it used to estimate the arc length of a curve?

Simpson's Rule is a numerical method that approximates the integral of a function by dividing the interval into an even number of subintervals, applying quadratic polynomials to estimate the area under the curve. To estimate arc length, it is used to approximate the integral of the square root of 1 plus the derivative squared, over the given interval.

How do you set up Simpson's Rule with N=6 for calculating the arc length of a curve?

First, divide the interval of interest into 6 equal subintervals, compute the function values at each subdivision point, then apply Simpson’s Rule formula with N=6 to approximate the integral needed for arc length calculation.

What is the formula for estimating the arc length of a curve using Simpson's Rule with N=6?

The arc length L is estimated by: $L \approx (\Delta x/3) [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + 4f(x_5) + f(x_6)]$, where Δx is the width of each subinterval, and f(x) represents the integrand $\sqrt{(1 + (dy/dx)^2)}$.

What is the importance of choosing N=6 in Simpson's Rule for arc length calculation?

Choosing N=6 ensures an even number of subintervals, which is necessary for Simpson's Rule to be applicable, and provides a balance between computational effort and accuracy in approximating the integral for arc length.

How do you compute the derivative needed for the arc length integrand when using Simpson's Rule?

You differentiate the given curve equation y=f(x) to find dy/dx, then compute $\sqrt{(1 + (dy/dx)^2)}$ at each subdivision point for use in the Simpson's Rule approximation.

After applying Simpson's Rule with N=6, how do you convert the result to six decimal places?

Once the approximate arc length is computed, round the result to six decimal places using standard rounding rules for precision and presentation.

Can Simpson's Rule with N=6 provide an accurate estimate for arc length? Why or why not?

Yes, when the function is smooth and well-behaved over the interval, Simpson's Rule with N=6 can give a good approximation of the arc length, balancing computational efficiency and accuracy.

What are common errors to avoid when using Simpson's Rule for arc length estimation?

Common errors include incorrect computation of the derivatives, miscalculating function values at subdivision points, using an odd number of subintervals, and forgetting to apply the correct weights in Simpson's Rule formula.

Additional Resources

Using Simpson's Rule With N = 6 To Estimate The Arc Length Of The Curve, L: An In-Depth Analytical Approach

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Introduction

Estimating the length of a curve, known as the arc length, is a fundamental problem in calculus that finds applications across diverse disciplines such as physics, engineering, computer graphics, and even biology. When an exact solution is complex or impossible to derive analytically, numerical methods become invaluable. Among these methods, Simpson's Rule stands out for its balance of accuracy and computational efficiency. This article delves into the process of employing Simpson's Rule with N=6 subdivisions to approximate the arc length, L, of a given curve, providing a comprehensive understanding that combines theoretical foundations, practical execution, and analytical insights.

Theoretical Foundations of Arc Length Calculation

Understanding Arc Length

The arc length of a curve y=f(x), between points x=a and x=b, is mathematically expressed as:

$$
L = \int_a^b \sqrt{1 + [f'(x)]^2} \, dx
$$

This integral accounts for the infinitesimal segments along the curve, summing their lengths to obtain the total arc length. The integrand, $\sqrt{1 + [f'(x)]^2}$, represents the differential arc element, considering both horizontal and vertical changes.

Challenges in Exact Computation

Calculating the integral analytically often hinges on the specific form of $f(x)$. Many functions lack elementary antiderivatives, necessitating numerical approximation methods. Simpson's Rule is particularly effective because it leverages quadratic polynomials to approximate the integrand, providing higher accuracy compared to methods like the trapezoidal rule.

Simpson's Rule: An Overview

Principles of Simpson's Rule

Simpson's Rule approximates the definite integral $\int_a^b f(x) \, dx$ by partitioning the interval $[a, b]$ into an even number of subintervals, N, each of width $h = \frac{b - a}{N}$. The rule then fits quadratic polynomials through sets of three points, integrating these locally to approximate the total integral:

$$
\int_a^b f(x) \, dx \approx \frac{h}{3} \left[f(x_0) + 4 \sum_{i=1,3,5,\ldots}^{N-1} f(x_i) + 2 \sum_{i=2,4,6,\ldots}^{N-2} f(x_i) + f(x_N) \right]
$$

where $x_i = a + i h$.

Choice of N and Its Implications

Using an even number of subintervals, N, ensures the applicability of Simpson's Rule. Increasing N enhances the approximation's accuracy but at the cost of additional evaluations. In this context, N=6 strikes a balance, providing a sufficiently refined approximation without excessive computational complexity.

Applying Simpson's Rule with N=6 to Estimate Arc Length

Step 1: Define the Curve and Its Derivative

Suppose the curve is given by a function $f(x)$. To compute the arc length, we need the derivative $f'(x)$. For illustration, consider a specific function such as:

$$
f(x) = x^2
$$

which is simple yet representative for the purpose of this analysis.

The derivative:

$$

$$
f'(x) = 2x
$$

Step 2: Determine the Interval $[a, b]$

Assuming the arc length is to be calculated from $x=a=0$ to $x=b=3$, the integral becomes:

$$
L = \int_0^3 \sqrt{1 + (2x)^2} \, dx = \int_0^3 \sqrt{1 + 4x^2} \, dx
$$

Step 3: Partition the Interval

Set $N=6$:

$$
h = \frac{b - a}{N} = \frac{3 - 0}{6} = 0.5
$$

Calculate the sample points:

$$
x_i = a + i h, \quad i=0,1,2,3,4,5,6
$$
$$
x_0 = 0, \quad x_1=0.5, \quad x_2=1.0, \quad x_3=1.5, \quad x_4=2.0, \quad x_5=2.5, \quad x_6=3.0
$$

Step 4: Compute the Function Values

Calculate $f(x_i) = \sqrt{1 + 4x_i^2}$:

x_i	$f(x_i)$	Calculation
0.0	1.0000	$\sqrt{1 + 0} = 1$
0.5	1.1180	$\sqrt{1 + 4 \times 0.25} = \sqrt{1 + 1} = \sqrt{2} \approx 1.4142$ (Note: correction needed here)
1.0	2.2361	$\sqrt{1 + 4 \times 1} = \sqrt{1 + 4} = \sqrt{5} \approx 2.2361$
1.5	3.3541	$\sqrt{1 + 4 \times 2.25} = \sqrt{1 + 9} = \sqrt{10} \approx 3.1623$ (Note: check calculations)
2.0	4.4721	$\sqrt{1 + 4 \times 4} = \sqrt{1 + 16} = \sqrt{17} \approx 4.1231$
2.5	5.5902	$\sqrt{1 + 4 \times 6.25} = \sqrt{1 + 25} = \sqrt{26} \approx 5.0990$
3.0	6.7082	$\sqrt{1 + 4 \times 9} = \sqrt{1 + 36} = \sqrt{37} \approx 6.0828$

Note: Precise calculation yields:

- $f(0.5) = \sqrt{1 + 4 \times 0.25} = \sqrt{1 + 1} = \sqrt{2} \approx 1.4142$

- $f(1.5) = \sqrt{1 + 4 \times 2.25} = \sqrt{1 + 9} = \sqrt{10} \approx 3.1623$

- $f(2.0) = \sqrt{1 + 4 \times 4} = \sqrt{1 + 16} = \sqrt{17} \approx 4.1231$

- $f(2.5) = \sqrt{1 + 4 \times 6.25} = \sqrt{1 + 25} = \sqrt{26} \approx 5.0990$

- $f(3.0) = \sqrt{1 + 36} = \sqrt{37} \approx 6.0828$

Step 5: Apply Simpson's Rule

Using the formula:

$$
L \approx \frac{h}{3} \left[f(x_0) + 4 \sum_{\text{odd } i} f(x_i) + 2 \sum_{\text{even } i} f(x_i) + f(x_N) \right]
$$

Identify the terms:

- $f(x_0) = 1.0000$
- $f(x_1) = 1.4142$
- $f(x_2) = 2.2361$
- $f(x_3) = 3.1623$
- $f(x_4) = 4.1231$
- $f(x_5) = 5.0990$
- $f(x_6) = 6.0828$

Sum over odd indices $i=1,3,5$:

$$
4 \times (f(x_1) + f(x_3) + f(x_5)) = 4 \times (1.4142 + 3.1623 + 5.0990) = 4 \times 9.6755 = 38.7020
$$

Sum over even indices $i=2,4$:

$$
2 \times (f(x_2) + f(x_4)) = 2 \times (2.2361 + 4.1231) = 2 \times 6.3592 = 12.7184
$$

Now, sum all:

$$
S = f(x_0) + 38.7020 + 12.7184 + f(x_6) = 1.0000 + 38.7020 + 12.7184 + 6.0828 = 58.
$$

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