Use The Given Point On The Terminal Side Of Angle [theta] To Find The Value Of The Trigonometric Function

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Understanding how to find the value of trigonometric functions using points on the terminal side of an angle is a fundamental skill in trigonometry. Whether you're solving problems in mathematics, physics, engineering, or computer graphics, knowing how to leverage a point's coordinates to determine sine, cosine, tangent, and other related functions is essential. This article provides a comprehensive guide on how to use a given point on the terminal side of an angle \(\theta\) to find the value of various trigonometric functions, with detailed explanations, step-by-step procedures, and practical examples.

Introduction to Trigonometric Functions and Coordinates

Before diving into methods, it's important to review some basic concepts.

Coordinate Plane and Angles

In the coordinate plane, an angle \(\theta\) can be visualized as the rotation from the positive x-axis to a line (the terminal side) passing through a point \((x, y)\). The initial side of the angle is along the x-axis, and the terminal side is the line that intersects the point \((x, y)\).

Points on the Terminal Side

A point \((x, y)\) lying on the terminal side of an angle \(\theta\) provides vital information about the trigonometric functions associated with that angle. By analyzing the coordinates, we can derive the values of sine, cosine, tangent, and other functions directly.

Fundamental Concepts for Finding Trigonometric Values From a Point

The key idea is that the coordinates \((x, y)\) of a point on the terminal side of an angle \(\theta\) relate to the radius \(r\) (distance from the origin to the point) and the angle itself.

Radius or Hypotenuse r

$$
r = \sqrt{x^2 + y^2}
$$

This is the distance from the origin to the point (x, y). It acts as the hypotenuse in the right triangle formed with the x- and y-axes.

Basic Trigonometric Ratios

Using the point (x, y) and r, the primary trigonometric functions are defined as:

- **Sine:** $\sin \theta = \frac{y}{r}$
- **Cosine:** $\cos \theta = \frac{x}{r}$
- **Tangent:** $\tan \theta = \frac{y}{x}$, provided $x \neq 0$

Other functions, such as cotangent, secant, and cosecant, are reciprocals:

- **Cosecant:** $\csc \theta = \frac{r}{y}$
- **Secant:** $\sec \theta = \frac{r}{x}$
- **Cotangent:** $\cot \theta = \frac{x}{y}$, provided $y \neq 0$

Step-by-Step Approach to Find Trigonometric Values from a Point

To utilize a point (x, y) on the terminal side of θ, follow these steps:

Step 1: Identify the Coordinates

Obtain the coordinates (x, y) of the point on the terminal side. These are typically given in problems or graphically determined.

Step 2: Calculate the Radius r

Compute the distance from the origin to the point:

$$
r = \sqrt{x^2 + y^2}
$$

This step ensures that the calculations are normalized, especially for unit circle considerations.

Step 3: Determine the Trigonometric Functions

Use the formulas:

- $\sin \theta = \frac{y}{r}$
- $\cos \theta = \frac{x}{r}$
- $\tan \theta = \frac{y}{x}$ (if $x \neq 0$)
- $\csc \theta = \frac{r}{y}$ (if $y \neq 0$)
- $\sec \theta = \frac{r}{x}$ (if $x \neq 0$)
- $\cot \theta = \frac{x}{y}$ (if $y \neq 0$)

Note: Be cautious of cases where $x=0$ or $y=0$, as some functions become undefined.

Special Cases and Considerations

Understanding the sign and magnitude of x and y helps determine the quadrant in which θ lies, which affects the signs of the trigonometric functions.

Quadrant Sign Rules

Quadrant	x	y	$\sin \theta$	$\cos \theta$	$\tan \theta$
I	+	+	+	+	+
II	-	+	+	-	-
III	-	-	-	-	+
IV	+	-	-	+	-

Knowing the quadrant helps interpret the signs of the calculated trigonometric functions correctly.

Practical Examples

Let's consider practical examples to illustrate how to find the trigonometric functions using a point on the terminal side.

Example 1: Point in Quadrant I

Suppose a point $(x, y) = (3, 4)$ lies on the terminal side of an angle θ.

Solution:

1. Calculate r:

$$
r = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5
$$

2. Find the functions:

$$
\sin \theta = \frac{4}{5} = 0.8
$$
$$
\cos \theta = \frac{3}{5} = 0.6
$$
$$
\tan \theta = \frac{4}{3} \approx 1.333
$$
$$
\csc \theta = \frac{5}{4} = 1.25
$$
$$
\sec \theta = \frac{5}{3} \approx 1.6667
$$
$$
\cot \theta = \frac{3}{4} = 0.75
$$

Since both x and y are positive, θ is in Quadrant I, and all functions are positive.

Example 2: Point in Quadrant II

Suppose a point $(-6, 8)$ is on the terminal side.

Solution:

1. Calculate r:

$$
r = \sqrt{(-6)^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10
$$

2. Find the functions:

$$
\sin \theta = \frac{8}{10} = 0.8
$$
$$
\cos \theta = \frac{-6}{10} = -0.6
$$
$$
\tan \theta = \frac{8}{-6} = -\frac{4}{3} \approx -1.333
$$
$$
\csc \theta = \frac{10}{8} = 1.25
$$
$$
\sec \theta = \frac{10}{-6} = -\frac{5}{3} \approx -1.6667
$$
$$
\cot \theta = \frac{-6}{8} = -\frac{3}{4} = -0.75
$$

Note the signs: $\sin \theta$ is positive, $\cos \theta$ negative, aligning with Quadrant II.

Applications of Using Points to Find Trigonometric Functions

This method is used extensively in various fields:

- **Physics:** Calculating angles and projections.
- **Engineering:** Analyzing forces and motions involving angles.
- **Computer Graphics:** Rotations and transformations.
- **Mathematics Education:** Understanding unit circles and coordinate geometry.

Advantages of Using Coordinates to Find Trigonometric Values

- Precision: Direct calculation reduces approximation errors.
- Versatility: Applicable to any point on the terminal side, not just unit circles.
- Insight: Helps visualize the relationship between angles and points in the coordinate plane.