# We Are Interested On The Following Linear Model: Y I =+X I + I ,i=1,2,,N (a) Write Down The OLS Objective

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## Introduction to the Linear Model and Its Significance

Understanding the fundamentals of linear regression models is essential for anyone involved in data analysis, econometrics, or statistical modeling. The linear model specified as $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$ (here represented as $Y_i = +X_i + \varepsilon_i$ for simplicity) forms the backbone of many predictive and inferential tasks. This model estimates the relationship between a dependent variable $Y_i$ and an independent variable $X_i$, capturing how changes in $X_i$ impact $Y_i$.

The primary goal of such models is to find the best-fitting line through the data points, which minimizes the discrepancies (errors) between observed and predicted values. These discrepancies are represented by the error term $\varepsilon_i$, accounting for randomness, measurement errors, or factors not included in the model.

In this article, we focus specifically on the Ordinary Least Squares (OLS) method, which is the most common approach for estimating the parameters $\beta_0$ (intercept) and $\beta_1$ (slope). We will explore how to formulate the OLS objective function, its mathematical derivation, and its significance in statistical modeling.

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## Understanding the Linear Model Framework

Before delving into the OLS objective, it is essential to grasp the structure of the linear model:

- Model Specification:

$$
Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i, \quad i = 1, 2, \ldots, N
$$

where:
- $Y_i$ is the dependent variable for observation $i$.
- $X_i$ is the independent variable for observation $i$.

- $\beta_0$ is the intercept term.
- $\beta_1$ is the slope coefficient.
- $\varepsilon_i$ is the error term, capturing the deviation of the observed $Y_i$ from the linear predictor.

- Assumptions of the Model:

To ensure the validity of OLS estimates, the following classical assumptions are typically made:
1. Linearity: The relationship between $Y_i$ and $X_i$ is linear.
2. Independence: The observations are independent.
3. Homoscedasticity: Constant variance of errors $\varepsilon_i$.
4. Normality: Errors are normally distributed (especially for inference).
5. No perfect multicollinearity (relevant in multiple regression).

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# Formulating the OLS Objective Function

The core idea behind OLS estimation is to find the parameter values $\hat{\beta}_0$ and $\hat{\beta}_1$ that minimize the discrepancy between the observed data and the model's predictions. This discrepancy is quantified by the sum of squared residuals.

## Definition of Residuals

- For each observation $i$, the residual $e_i$ is the difference between the observed value $Y_i$ and the predicted value $\hat{Y}_i$:

$$
e_i = Y_i - \hat{Y}_i
$$

- The predicted value $\hat{Y}_i$ based on the model is:

$$
\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i
$$

## Objective Function: Minimizing the Sum of Squared Residuals

- The OLS method aims to minimize the sum of squared residuals:

$$
S(\beta_0, \beta_1) = \sum_{i=1}^N (Y_i - \beta_0 - \beta_1 X_i)^2
$$

- Here, the goal is to find the values of $\beta_0$ and $\beta_1$ that minimize $S(\beta_0, \beta_1)$.

- The minimization problem can be written explicitly as:

$$
\boxed{
(\hat{\beta}_0, \hat{\beta}_1) = \arg \min_{\beta_0, \beta_1} \sum_{i=1}^N (Y_i - \beta_0 - \beta_1 X_i)^2
}
$$

- This formulation is the standard OLS objective function.

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# Mathematical Derivation of the OLS Estimators

To find the estimators $\hat{\beta}_0$ and $\hat{\beta}_1$, we take partial derivatives of the objective function and set them to zero, leading to the normal equations.

## Step 1: Write the Objective Function

$$
S(\beta_0, \beta_1) = \sum_{i=1}^N (Y_i - \beta_0 - \beta_1 X_i)^2
$$

## Step 2: Derive Normal Equations

- Partial derivative with respect to $\beta_0$:

$$
\frac{\partial S}{\partial \beta_0} = -2 \sum_{i=1}^N (Y_i - \beta_0 - \beta_1 X_i) = 0
$$

- Partial derivative with respect to $\beta_1$:

$$
\frac{\partial S}{\partial \beta_1} = -2 \sum_{i=1}^N X_i (Y_i - \beta_0 - \beta_1 X_i) = 0
$$

- Simplify both equations:

$$
\sum_{i=1}^N (Y_i - \beta_0 - \beta_1 X_i) = 0
$$

$$
\sum_{i=1}^N X_i (Y_i - \beta_0 - \beta_1 X_i) = 0
$$

## Step 3: Solve the Normal Equations

- From the first equation:

$$
\sum_{i=1}^N Y_i = N \beta_0 + \beta_1 \sum_{i=1}^N X_i
$$

- From the second equation:

$$
\sum_{i=1}^N X_i Y_i = \beta_0 \sum_{i=1}^N X_i + \beta_1 \sum_{i=1}^N X_i^2
$$

- Rearranged as:

$$
\beta_0 = \bar{Y} - \beta_1 \bar{X}
$$

where:

$$
\bar{Y} = \frac{1}{N} \sum_{i=1}^N Y_i, \quad \bar{X} = \frac{1}{N} \sum_{i=1}^N X_i
$$

- The estimator for $\beta_1$ becomes:

$$
\hat{\beta}_1 = \frac{\sum_{i=1}^N (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^N (X_i - \bar{X})^2}
$$

- And the intercept estimator:

$$
\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}
$$

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# Significance of the OLS Objective in Regression Analysis

The OLS objective function is fundamental because it directly influences the accuracy and reliability of the estimated model parameters. Its significance can be summarized as follows:

- Optimal Fit: By minimizing the sum of squared residuals, OLS finds the line that best fits the data according to least squares criterion.
- Unbiasedness: Under classical assumptions, the OLS estimators are unbiased, meaning they correctly estimate the true population parameters on average.
- Efficiency: OLS provides the Best Linear Unbiased Estimators (BLUE) among all linear unbiased estimators, thanks to the Gauss-Markov theorem.
- Statistical Inference: The residuals and estimated parameters derived from the OLS objective facilitate hypothesis testing and confidence interval construction.
- Diagnostic and Model Evaluation: The minimized residual sum of squares is a basis for calculating measures such as R-squared, which indicates the goodness-of-fit.

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# Extensions and Variations of the OLS Objective

While the classic OLS approach focuses on minimizing the sum of squared residuals, various extensions and alternatives exist:

- Weighted Least Squares (WLS): Incorporates different weights for observations.
- Robust Regression: Mitigates the influence of outliers on the objective.
- Regularization Techniques: Such as Ridge and Lasso, which add penalty terms to the objective for variable selection or shrinkage.

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