What Is Limit Of StartFraction StartRoot X + 4 EndRoot Minus 3 Over X Minus 5 EndFraction As X Approaches

What Is Limit Of \(\displaystyle \frac{\sqrt{x + 4} - 3}{x - 5}\) As \(x\) Approaches

Understanding the concept of limits is fundamental in calculus, especially when analyzing the behavior of functions as variables approach specific points. The expression \(\displaystyle \frac{\sqrt{x + 4} - 3}{x - 5}\) is a classic example where direct substitution yields an indeterminate form, prompting a need for algebraic manipulation or advanced limit techniques. This article aims to thoroughly explain what the limit of this function is as \(x\) approaches a particular value, and how to evaluate it systematically.

Introduction to Limits in Calculus

What Is a Limit?

A limit describes the value that a function approaches as the input approaches a specific point. Formally, for a function \(f(x)\), the limit as \(x\) approaches \(a\) is denoted as:
\[
\lim_{x \to a} f(x) = L
\]
if \(f(x)\) gets arbitrarily close to \(L\) as \(x\) gets closer to \(a\), from both sides if applicable.

Why Are Limits Important?

- Foundation of Derivatives: Limits are used to define derivatives.
- Understanding Continuity: Limits determine whether functions are continuous at a point.
- Evaluating Indeterminate Forms: Many functions produce forms like \(0/0\), which require limit techniques to resolve.

Analyzing the Given Expression

The Expression in Question

The function under investigation is:

$$
f(x) = \frac{\sqrt{x + 4} - 3}{x - 5}
$$

Our goal is to evaluate:

$$
\lim_{x \to a} \frac{\sqrt{x + 4} - 3}{x - 5}
$$

for a specific value of a.

Step 1: Identify the point a that x approaches.

Determining the Limit: Approaching $x \to 5$

Why Choose $x \to 5$?

In the given function, substituting $x = 5$ directly results in:

$$
\frac{\sqrt{5 + 4} - 3}{5 - 5} = \frac{\sqrt{9} - 3}{0} = \frac{3 - 3}{0} = \frac{0}{0}
$$

which is an indeterminate form. This suggests that the limit as x approaches 5 is not immediately apparent and requires further analysis.

Evaluating $\lim_{x \to 5}$

Since direct substitution yields $0/0$, we need to manipulate the expression to evaluate the limit.

Methods to Evaluate the Limit

1. Algebraic Rationalization

This approach involves multiplying numerator and denominator by the conjugate of the numerator to eliminate the square root.

2. Applying L'Hôpital's Rule

When encountering indeterminate forms $0/0$, L'Hôpital's rule states that:

$$
\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}
$$

provided the derivatives exist and the limit exists.

3. Recognizing the Limit as a Derivative

Sometimes, the limit resembles the definition of a derivative, enabling an alternative approach.

Step-by-Step Evaluation of the Limit

Step 1: Recognize the Indeterminate Form

As shown, directly substituting $x=5$ gives $0/0$. Therefore, algebraic manipulation or L'Hôpital's rule is necessary.

Step 2: Rationalize the Numerator

Multiply numerator and denominator by the conjugate of the numerator:

$$
\frac{\sqrt{x + 4} - 3}{x - 5} \times \frac{\sqrt{x + 4} + 3}{\sqrt{x + 4} + 3} = \frac{(\sqrt{x + 4} - 3)(\sqrt{x + 4} + 3)}{(x - 5)(\sqrt{x + 4} + 3)}
$$

Recall the difference of squares:

$$
(\sqrt{x + 4})^2 - 3^2 = (x + 4) - 9 = x - 5
$$

So, numerator simplifies to:

$$
x - 5
$$

Thus, the expression becomes:

$$
\frac{x - 5}{(x - 5)(\sqrt{x + 4} + 3)} = \frac{1}{\sqrt{x + 4} + 3}
$$

$$
\lim_{x \to 5} ...
$$

for all $x \neq 5$.

Step 3: Simplify and Find the Limit

Now, the original limit simplifies to:

$$
\lim_{x \to 5} \frac{1}{\sqrt{x + 4} + 3}
$$

Substitute $x = 5$:

$$
\frac{1}{\sqrt{5 + 4} + 3} = \frac{1}{\sqrt{9} + 3} = \frac{1}{3 + 3} = \frac{1}{6}
$$

Therefore:

$$
\boxed{\lim_{x \to 5} \frac{\sqrt{x + 4} - 3}{x - 5} = \frac{1}{6}}
$$

Interpretation of the Result

The limit evaluates to $\frac{1}{6}$, which indicates that as x approaches 5, the value of the function $\frac{\sqrt{x + 4} - 3}{x - 5}$ approaches $\frac{1}{6}$.

This result can be viewed as the instantaneous rate of change or derivative of the function $\sqrt{x + 4}$ at $x=5$. In fact, the process of rationalizing the numerator aligns with the derivative calculation of $\sqrt{x + 4}$.

Connection to Derivatives and Differentiability

Derivative of $f(x) = \sqrt{x + 4}$

Using the power rule:

$$
f(x) = (x + 4)^{1/2}
$$

$$
f'(x) = \frac{1}{2}(x + 4)^{-1/2} = \frac{1}{2\sqrt{x + 4}}
$$

\]

At \(x=5\):
\[
f'(5) = \frac{1}{2 \sqrt{9}} = \frac{1}{2 \times 3} = \frac{1}{6}
\]

This matches our limit, confirming the interpretation that the limit is the derivative of \(\sqrt{x + 4}\) at \(x=5\).

Broader Implications and Applications

Understanding the Limit in Context

- Calculus Fundamentals: Limits provide the foundation for defining derivatives and integrals.
- Physics: Rate of change calculations, such as velocity or acceleration.
- Engineering: Analyzing system behaviors near equilibrium points.
- Economics: Marginal analysis involves limits.

Common Techniques in Limit Evaluation

- Direct Substitution: The first step, but not always sufficient.
- Factoring and Rationalizing: To eliminate indeterminate forms.
- L'Hôpital's Rule: When faced with \(0/0\) or \(\infty/\infty\) forms.
- Recognizing Derivatives: Using the limit definition of derivatives for functions.

Summary and Key Takeaways

- The limit of \(\frac{\sqrt{x + 4} - 3}{x - 5}\) as \(x \to 5\) is \(\frac{1}{6}\).
- Direct substitution yields an indeterminate form \(0/0\), which can be resolved via rationalization.
- Rationalizing the numerator simplifies the expression and reveals the limit as the derivative of \(\sqrt{x + 4}\) at \(x=5\).
- The process demonstrates the interconnectedness of limits, derivatives, and function behaviors.

Additional Resources for Learning Limits

- Textbooks: "Calculus" by James Stewart.
- Online Platforms: Khan Academy’s calculus section.
- Practice Problems: Websites offering limit evaluation exercises.
- Tutoring: Seek help from calculus tutors for personalized guidance.

Conclusion

Evaluating the limit of \(\displaystyle \frac{\sqrt{x + 4} - 3}{x - 5}\) as \(x\) approaches 5 exemplifies essential calculus techniques, including algebraic manipulation and understanding the derivative's definition. Recognizing the indeterminate form and applying rationalization not only simplifies the problem but also deep

Frequently Asked Questions

What is the limit of (√(x + 4) - 3) / (x - 5) as x approaches 5?

The limit is 1/4. To find it, you can rationalize the numerator or use L'Hôpital's Rule since direct substitution results in 0/0 form.

How do I evaluate the limit of (√(x + 4) - 3) / (x - 5) as x approaches 5?

You can multiply numerator and denominator by the conjugate √(x + 4) + 3 to simplify, then substitute x = 5 to find the limit.

Why does the expression (√(x + 4) - 3) / (x - 5) require rationalization when finding its limit?

Because direct substitution yields an indeterminate form 0/0, rationalization helps simplify the expression to evaluate the limit accurately.

Can L'Hôpital's Rule be used to find the limit of (√(x + 4) - 3) / (x - 5) as x approaches 5?

Yes, since substituting x = 5 results in 0/0, L'Hôpital's Rule can be applied by differentiating numerator and denominator separately.

What is the significance of the conjugate in calculating the limit of (√(x + 4) - 3) / (x - 5)?

Using the conjugate allows for rationalizing the numerator, eliminating the radical and simplifying the limit calculation.

What is the final value of the limit of (√(x + 4) - 3) / (x - 5) as x approaches 5?

The limit is 1/4, obtained by rationalizing and simplifying the expression as x approaches 5.

Additional Resources

What Is Limit Of (√(x + 4) - 3) / (x - 5) As x Approaches — A Comprehensive Guide

When delving into the fascinating world of calculus, one of the fundamental concepts students and professionals encounter is the idea of a limit. Specifically, understanding what is limit of (√(x + 4) - 3) / (x - 5) as x approaches a particular value is crucial for grasping the behavior of functions near points of interest. This guide aims to demystify this limit, breaking down the steps, techniques, and reasoning involved to help you develop a clear understanding of the process.

Understanding the Limit in Context

The expression under consideration is:

$$
\lim_{x \to a} \frac{\sqrt{x + 4} - 3}{x - 5}
$$

While the specific value of a isn't explicitly given in the prompt, the structure suggests that x is approaching a value near 5, because the denominator $x - 5$ approaches zero as $x \to 5$. To analyze this limit meaningfully, we typically consider the behavior as x approaches 5.

Why Is This Limit Important?

This type of limit appears frequently in calculus, especially when dealing with derivatives, which measure the rate of change of functions. Recognizing how to evaluate such limits helps in understanding the slopes of tangent lines, rates of change, and the continuity of functions.

Step-by-Step Breakdown of the Limit

Step 1: Recognize the Indeterminate Form

First, substitute x with 5 directly:

\[
\frac{\sqrt{5 + 4} - 3}{5 - 5} = \frac{\sqrt{9} - 3}{0} = \frac{3 - 3}{0} = \frac{0}{0}
\]

This is an indeterminate form, which indicates that direct substitution won't yield the limit directly. Instead, we need to apply algebraic techniques to resolve this.

Step 2: Rationalize the Numerator

When facing a limit involving a square root difference, rationalization is a common and effective strategy. Multiply numerator and denominator by the conjugate of the numerator:

\[
\frac{\sqrt{x + 4} - 3}{x - 5} \times \frac{\sqrt{x + 4} + 3}{\sqrt{x + 4} + 3}
\]

This gives:

\[
\frac{(\sqrt{x + 4} - 3)(\sqrt{x + 4} + 3)}{(x - 5)(\sqrt{x + 4} + 3)} = \frac{(x + 4) - 9}{(x - 5)(\sqrt{x + 4} + 3)} = \frac{x - 5}{(x - 5)(\sqrt{x + 4} + 3)}
\]

Observe that the numerator simplifies to \(x - 5\), which cancels with the denominator:

\[
\frac{x - 5}{(x - 5)(\sqrt{x + 4} + 3)} = \frac{1}{\sqrt{x + 4} + 3}
\]

Now, the limit reduces to:

\[
\lim_{x \to 5} \frac{1}{\sqrt{x + 4} + 3}
\]

Evaluating the Simplified Limit

Since the expression no longer has an indeterminate form, we can now substitute \(x = 5\):

\[
\frac{1}{\sqrt{5 + 4} + 3} = \frac{1}{\sqrt{9} + 3} = \frac{1}{3 + 3} = \frac{1}{6}
\]

Therefore,

\[
\boxed{
\lim_{x \to 5} \frac{\sqrt{x + 4} - 3}{x - 5} = \frac{1}{6}
}

\]

Interpreting the Result: What Does This Limit Tell Us?

This limit calculation essentially finds the derivative of the function $f(x) = \sqrt{x + 4}$ at $x = 5$. In calculus, the derivative at a point a can be defined as:

$$
f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}
$$

In our case:

- $f(5) = \sqrt{5 + 4} = 3$
- The difference quotient becomes:

$$
\frac{\sqrt{x + 4} - 3}{x - 5}
$$

which is exactly the limit we're evaluating.

Thus, the limit equals the derivative of $\sqrt{x + 4}$ at $x = 5$, which is $\frac{1}{6}$.

Why Is This Important?

Understanding this connection bridges the concept of limits with derivatives, emphasizing the fundamental role of limits in calculus. It also demonstrates how algebraic manipulation, such as rationalization, can simplify complex expressions to evaluate their limits.

General Techniques for Evaluating Limits Like This

While the specific example is manageable through rationalization, many limits require different approaches. Here’s a list of common techniques:

1. Direct Substitution

- Try substituting the approaching value directly.
- Works if the function is continuous at that point.

2. Rationalization

- Multiply numerator and denominator by the conjugate to eliminate roots.
- Useful for limits involving square roots or other radicals.

3. Factoring

- Factor numerator and denominator to cancel common terms.
- Especially helpful when the limit produces an indeterminate form like $0/0$.

4. Simplification

- Simplify the expression algebraically to resolve indeterminate forms.

5. L'Hôpital's Rule

- When direct substitution yields $0/0$ or ∞/∞, differentiate numerator and denominator separately and then take the limit.
- Useful for more complex limits.

6. Series Expansion (Advanced)

- Use Taylor or Maclaurin series to analyze the behavior near the point of interest.

Visualizing the Limit

Understanding the limit geometrically can provide additional insights:

- The function $f(x) = \sqrt{x + 4}$ is continuous and smooth.
- As x approaches 5, the slope of the tangent line is precisely the derivative—the value of our limit.
- The limit $\frac{1}{6}$ indicates that near $x=5$, the function increases at a rate of approximately $\frac{1}{6}$ per unit increase in x.

Summary: Key Takeaways

- The limit $\lim_{x \to 5} \frac{\sqrt{x + 4} - 3}{x - 5}$ evaluates to $\frac{1}{6}$.
- The process involves recognizing the indeterminate form and rationalizing the numerator to simplify.
- The result corresponds to the derivative of $\sqrt{x + 4}$ at $x=5$.
- Techniques such as rationalization, factoring, and L'Hôpital's Rule are essential tools for evaluating limits involving roots and indeterminate forms.
- Understanding these concepts is foundational for advanced calculus topics like derivatives, integrals, and continuity.

Final Thoughts

Mastering the evaluation of limits like what is limit of (√(x + 4) - 3) / (x - 5) as x approaches is an essential step in developing a solid calculus foundation. Recognizing patterns, employing appropriate algebraic manipulations, and understanding the underlying concepts empower you to tackle a wide range of limit problems confidently. Remember, limits reveal the behavior of functions at specific points, and their mastery opens doors to exploring the dynamic world of calculus with clarity and precision.

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