What Is The Equation Of The Line That Is Perpendicular To The Line Y = 2x + 5 Andpasses Through The Point

What Is The Equation Of The Line That Is Perpendicular To The Line Y = 2x + 5 Andpasses Through The Point is a common question in coordinate geometry that involves understanding the relationship between lines, their slopes, and how to derive equations based on given conditions. This problem requires knowledge of the slope-intercept form, the concept of perpendicular lines, and how to utilize a point to determine a specific line's equation. Whether you're a student learning algebra or someone working on a math problem, grasping how to find such a line is an essential skill that combines multiple concepts in geometry and algebra.

In this article, we will explore the step-by-step process to find the equation of a line perpendicular to a given line and passing through a specific point. We will cover fundamental concepts, including slopes and their relationships, the point-slope form of a line, and how to interpret and apply these ideas to solve real-world problems.

Understanding the Given Line: Y = 2x + 5

Before diving into the solution, it's important to understand the properties of the given line, which is expressed in the slope-intercept form:

Y = 2x + 5

This form reveals two critical pieces of information:

- Slope (m): The coefficient of x, which is 2 in this case.
- Y-intercept (b): The constant term, which is 5.

The slope indicates that for every unit increase in x, y increases by 2 units. The y-intercept is the point where the line crosses the y-axis, at (0, 5).

Concept of Perpendicular Lines and Their Slopes

Perpendicular lines are lines that intersect at right angles (90 degrees).

One key property of such lines is that their slopes are negative reciprocals of each other, provided neither line is vertical or horizontal.

Negative Reciprocal of a Slope

If a line has a slope of m, then a line perpendicular to it will have a slope of:

m_perpendicular = -1/m

Applying this to our given line:

- Since the slope of the original line is 2,
- The slope of the perpendicular line is:

m_perpendicular = -1/2

This relationship is fundamental when constructing the equation of the perpendicular line.

Finding the Equation of the Perpendicular Line

The process involves two main steps:

1. Identify the slope of the perpendicular line.
2. Use the point through which the line passes to find its equation.

Suppose the line passes through a specific point, say (x_1, y_1). The general approach is:

- Use the point-slope form of a line:

$y - y_1 = m(x - x_1)$

- Plug in the slope of the perpendicular line ($m = -1/2$) and the given point.

Example: Passing Through a Point (x_1, y_1)

Let's assume the point given is (x_1, y_1). To find the equation:

- Substitute into the point-slope form:

$y - y_1 = -1/2 (x - x_1)$

- Simplify to slope-intercept form or any preferred form.

Step-by-Step Solution with a Specific Point

Let's illustrate with an example. Suppose the line passes through the point (3, 4). How do we find its equation?

Step 1: Identify the slope of the perpendicular line

- Slope of original line: m = 2
- Slope of perpendicular line: m_perpendicular = -1/2

Step 2: Use the point-slope form

- Plug in the point (3, 4):

y - 4 = -1/2 (x - 3)

Step 3: Simplify to slope-intercept form

- Expand the right side:

y - 4 = -1/2 x + 3/2

- Add 4 to both sides:

y = -1/2 x + 3/2 + 4

- Convert 4 to a fraction with denominator 2:

y = -1/2 x + 3/2 + 8/2

- Combine the constants:

y = -1/2 x + (3/2 + 8/2) = -1/2 x + 11/2

The equation of the line perpendicular to y = 2x + 5 passing through (3, 4) is:

y = -1/2 x + 11/2

General Formula for Any Point

If you are given a point (x_1, y_1) through which the perpendicular line passes, and the original line $y = 2x + 5$, the general process is:

1. Find the slope of the original line:

$m = 2$

2. Find the perpendicular slope:

$m_{perpendicular} = -1/2$

3. Use the point-slope form:

$y - y_1 = -1/2 (x - x_1)$

4. Simplify to slope-intercept form as needed.

Special Cases to Consider

While the above method works in most cases, there are some special situations to be aware of:

- Vertical and Horizontal Lines: If the original line is horizontal (slope 0), the perpendicular line is vertical, which has an undefined slope. The equation of a vertical line passing through a point (x_1, y_1) is simply $x = x_1$.

- Original line is vertical: If the original line is vertical (say, $x = c$), then the perpendicular line is horizontal: $y = y_1$.

In our case, since the original line $y = 2x + 5$ has a slope of 2, these special cases are not applicable.

Conclusion and Summary

Finding the equation of a line perpendicular to a given line and passing through a specific point involves understanding the relationship between slopes of perpendicular lines and applying the point-slope form. The key points to remember are:

- The slope of the original line y = 2x + 5 is 2.
- The slope of the perpendicular line is the negative reciprocal, -1/2.
- Use the point-slope form with the given point and the perpendicular slope to derive the equation.
- Convert the equation to the preferred form, such as slope-intercept form, for clarity.

This method is widely applicable in coordinate geometry, whether you are solving homework problems or tackling real-world scenarios involving perpendicular lines and points.

Additional Tips for Solving Similar Problems

- Always identify the slope of the original line first.
- Remember the reciprocal relationship for perpendicular slopes.
- Use the point-slope form for straightforward calculation.
- Check for special cases like vertical or horizontal lines.
- Practice with different points to gain confidence.

By mastering these concepts, you'll be well-equipped to handle a variety of problems involving perpendicular lines and their equations, enhancing your understanding of the fundamental principles of coordinate geometry.

Note: If you have a specific point in mind for the line to pass through, simply substitute it into the steps above to find the exact equation.

Frequently Asked Questions

What is the slope of the line perpendicular to y = 2x + 5?

The slope of the given line is 2. The slope of a line perpendicular to it is the negative reciprocal, which is -1/2.

How do I find the equation of a line perpendicular to y = 2x + 5 passing through a specific point?

First, determine the perpendicular slope (-1/2), then use the point-slope form with the given point to find the equation.

What is the point-slope form of the equation for a line perpendicular to y = 2x + 5 passing through (x_1, y_1)?

The point-slope form is: $y - y_1 = -1/2 (x - x_1)$.

Can you give an example of finding the equation of a perpendicular line passing through (3, 4)?

Yes. Using slope -1/2 and point (3, 4): $y - 4 = -1/2 (x - 3)$. Simplify to get $y = -1/2 x + 5.5$.

How do I convert the point-slope form to slope-intercept form?

Distribute and solve for y to write the equation in $y = mx + b$ form.

What is the general process to find the perpendicular line's equation through a given point?

Identify the original line's slope, find its negative reciprocal, then use point-slope form with the given point to write the new line's equation.

Why is the slope of the perpendicular line -1/2 in this case?

Because perpendicular lines have slopes that are negative reciprocals of each other; since the original slope is 2, its negative reciprocal is -1/2.

How do I verify that my line is perpendicular to y = 2x + 5?

Check that the slopes are negative reciprocals. If the original slope is 2, and your line's slope is -1/2, then the lines are perpendicular.

What is the final equation of the line perpendicular to y = 2x + 5 passing through point (x_1, y_1)?

Using the point (x_1, y_1), the equation is $y - y_1 = -1/2 (x - x_1)$.

Additional Resources

Understanding the Equation of a Perpendicular Line to $y = 2x + 5$ Passing Through a Given Point

When working with linear equations in algebra, one common problem involves finding the equation of a line that is perpendicular to a given line and passes through a specific point. In this detailed exploration, we will analyze how to find such a line when the given line is $y = 2x + 5$, and the line must pass through a particular point.

This guide aims to provide a comprehensive understanding of the concepts, steps, and formulas involved, ensuring clarity for learners at various levels.

Fundamentals of Linear Equations and Their Graphs

Before diving into the specific problem, it is essential to understand the basic components of linear equations:

- Standard form of a line: $y = mx + b$, where:
- m is the slope of the line.
- b is the y-intercept, the point where the line crosses the y-axis.
- Graphing a line: Plot the y-intercept and use the slope to find additional points.

In the given equation:

$$
y = 2x + 5
$$

- The slope m is 2.
- The y-intercept is 5.

Understanding Perpendicular Lines and Their Slopes

One of the fundamental principles in coordinate geometry concerns the slopes of perpendicular lines:

- Perpendicular lines are lines that intersect at a right angle (90 degrees).
- The slopes of perpendicular lines are negative reciprocals of each other, provided neither slope is zero.

Mathematically,

$$
\text{If} \quad m_1 \quad \text{and} \quad m_2 \quad \text{are slopes of perpendicular lines, then} \quad m_1 \times m_2 = -1
$$

Example:

- Given $m_1 = 2$,
- The slope of a line perpendicular to it, m_2, is:

$$
m_2 = -\frac{1}{m_1} = -\frac{1}{2}
$$

This reciprocal relationship is crucial in our problem because it allows us to determine the slope of the perpendicular line once we know the original line's slope.

Step-by-Step Approach to Find the Equation of the Perpendicular Line Passing Through a Given Point

Suppose we are asked to find the equation of the line that:

- Is perpendicular to $y = 2x + 5$,
- Passes through a specific point, say (x_0, y_0).

The general procedure involves the following steps:

1. Identify the slope of the original line.

From the given equation:

$$
y = 2x + 5
$$

the slope m_1 is 2.

2. Determine the slope of the perpendicular line.

Using the negative reciprocal property:

$$
m_2 = -\frac{1}{2}
$$

This is the slope of the line we're trying to find.

3. Use the point-slope form of a line's equation.

The point-slope form is:

$$
y - y_1 = m (x - x_1)
$$

where (x_1, y_1) is a point the line passes through, and m is the slope.

Suppose the point given is (x_0, y_0). Substituting into the formula:

$$
y - y_0 = -\frac{1}{2} (x - x_0)
$$

4. Simplify the equation to slope-intercept form.

Distribute the slope:

$$
y - y_0 = -\frac{1}{2} x + \frac{1}{2} x_0
$$

Add y_0 to both sides:

$$
y = -\frac{1}{2} x + \frac{1}{2} x_0 + y_0
$$

This is the slope-intercept form $y = mx + b$ of the perpendicular line passing through (x_0, y_0).

Worked Example: Find the Equation of the Perpendicular Line Passing Through a Specific Point

Let's work through an example with concrete numbers for clarity.

Given:

- Original line: $y = 2x + 5$
- Point: $(3, 4)$

Step 1: Identify the original slope:

$$
m_1 = 2
$$

Step 2: Find the perpendicular slope:

$$
m_2 = -\frac{1}{2}
$$

Step 3: Use point-slope form:

$$
y - 4 = -\frac{1}{2} (x - 3)
$$

Step 4: Simplify:

$$
y - 4 = -\frac{1}{2} x + \frac{3}{2}
$$

Add 4 to both sides:

$$
y = -\frac{1}{2} x + \frac{3}{2} + 4
$$

Express 4 as $\frac{8}{2}$:

$$
y = -\frac{1}{2} x + \frac{3}{2} + \frac{8}{2} = -\frac{1}{2} x + \frac{11}{2}
$$

Final Equation:

$$
y = -\frac{1}{2} x + \frac{11}{2}
$$

Or, in decimal form:

$$
y = -0.5 x + 5.5
$$

This line is perpendicular to $y = 2x + 5$ and passes through $(3, 4)$.

Generalization for Any Point and Line

The above steps can be generalized for any given line and point:

- Step 1: Extract the slope from the given line.
- Step 2: Calculate the negative reciprocal for the perpendicular slope.
- Step 3: Use the point-slope form with the given point.
- Step 4: Simplify to slope-intercept or standard form as needed.

Additional Considerations and Variations

While the basic method is straightforward, several variations and considerations can arise:

1. When the given line is vertical or horizontal.

- If the original line is vertical (e.g., $x = c$), its slope is undefined.
- Perpendicular to a vertical line is a horizontal line, which has a slope of zero.
- To find the perpendicular line passing through a point (x_0, y_0):

$$
y = y_0
$$

- Conversely, if the original line is horizontal ($y = c$), the perpendicular line is vertical:

$$
x = x_0
$$

2. When the point coincides with the original line.

- The process remains the same; the only difference is that the point lies on the original line.

3. Graphical interpretation and verification.

- After deriving the equation, plot both the original line and the perpendicular line passing through the point to verify perpendicularity visually.
- Confirm that the slopes are negative reciprocals.

4. Using other forms of line equations

- Standard form: $Ax + By + C = 0$
- Given the original line in standard form, find its slope as $-A/B$ and proceed similarly.

Conclusion: Mastering the Concept of Perpendicular Lines

Understanding how to find the equation of a line perpendicular to a given line and passing through a particular point is a fundamental skill in algebra and coordinate geometry. It involves key concepts such as:

- Extracting the slope from the original line
- Applying the negative reciprocal rule for perpendicularity
- Utilizing the point-slope form to incorporate the given point
- Simplifying equations into preferred forms for clarity

This process not only aids in solving specific problems but also deepens comprehension of geometric relationships and the coordinate plane.

Practical Applications:

- Designing geometric figures

- Solving optimization problems
- Analyzing relationships between lines in various fields such as engineering, architecture, and computer graphics

Summary of the Main Steps:

1. Find the slope of the given line.
2. Determine the perpendicular slope via negative reciprocal.
3. Use the given point with the perpendicular slope in the point-slope formula.
4. Simplify to obtain the final line equation.

By mastering these steps and concepts, students and professionals can confidently handle problems involving perpendicular lines, enhancing their analytical and problem-solving skills.

Remember: The key to success in these problems lies in understanding the relationship between slopes and applying the algebraic formulas correctly. With practice, deriving the equation of a perpendicular line passing through any point becomes an intuitive and straightforward process.

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