

Which Set Of Numbers Can Represent The Side Lengths In Inches Of A Right Triangle?A. 8,12,15B. 10,24,26C.

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Understanding whether a set of three numbers can serve as the side lengths of a right triangle is a fundamental concept in geometry. This involves a process called the Pythagorean Theorem, which states that in a right triangle, the square of the hypotenuse (the longest side) equals the sum of the squares of the other two sides. In this article, we will analyze the two given sets of numbers—A. 8, 12, 15 and B. 10, 24, 26—to determine which can represent the side lengths of a right triangle. Through detailed examination, including applying the Pythagorean Theorem, we will clarify how to identify valid right triangles based on side lengths.

Understanding the Pythagorean Theorem

The Pythagorean Theorem is a cornerstone of right triangle geometry. It mathematically expresses the relationship between the three sides of a right triangle:

$$a^2 + b^2 = c^2$$

Where:

- a and b are the legs (the shorter sides),
- c is the hypotenuse (the longest side).

To determine if a set of three numbers can be the sides of a right triangle, the process involves:

1. Identifying the longest side (candidate for hypotenuse).
2. Checking whether the sum of the squares of the other two sides equals the square of the longest side.

When this condition holds true, the set forms a right triangle.

Analyzing Set A: 8, 12, 15

Let's evaluate whether the numbers 8, 12, and 15 can be the sides of a right triangle.

Step 1: Identify the Hypotenuse Candidate

The largest number among the three is 15, so we consider it as the hypotenuse candidate.

Step 2: Apply the Pythagorean Theorem

Calculate the squares:

$$- 8^2 = 64$$

$$- 12^2 = 144$$

$$- 15^2 = 225$$

Now, sum the squares of the two shorter sides:

$$64 + 144 = 208$$

Compare this sum to the square of the hypotenuse:

$$225$$

Since $208 \neq 225$, the set does not satisfy the Pythagorean theorem.

Conclusion for Set A

Because the sum of the squares of 8 and 12 does not equal the square of 15, set A cannot represent the side lengths of a right triangle.

Analyzing Set B: 10, 24, 26

Next, we evaluate the second set: 10, 24, and 26.

Step 1: Identify the Hypotenuse Candidate

The largest number is 26, so we consider it as the hypotenuse.

Step 2: Apply the Pythagorean Theorem

Calculate the squares:

$$- 10^2 = 100$$

$$- 24^2 = 576$$

$$- 26^2 = 676$$

Sum the squares of the two shorter sides:

$$100 + 576 = 676$$

Compare to the square of the hypotenuse:

676

Since both sides are equal, the set satisfies the Pythagorean theorem.

Conclusion for Set B

Because the sum of the squares of 10 and 24 equals the square of 26, set B can represent the side lengths of a right triangle.

Additional Considerations: Validity and Scaling

While our analysis confirms that set B forms a right triangle, it's also important to understand the concept of scaled and primitive Pythagorean triples.

Primitive vs. Non-Primitive Triples

- Primitive Pythagorean triples are sets of three integers that satisfy the Pythagorean theorem and have no common divisors other than 1. For example, 3-4-5, 5-12-13, 8-15-17.
- Non-primitive triples are multiples of primitive triples. For example, 6-8-10 is a scaled version of 3-4-5.

In our case, 10, 24, 26 is a scaled version of 5, 12, 13 multiplied by 2.

Implication for Practical Use

Knowing that a set of side lengths forms a right triangle is useful in construction, design, and navigation. Recognizing scaled triples can simplify calculations, especially in real-world measurements where exact integers are rare.

Summary of Key Findings

Set	Largest Side	Pythagorean Check	Is It a Right Triangle?
8, 12, 15	15	$8^2 + 12^2 = 64 + 144 = 208$; $15^2 = 225$	No
10, 24, 26	26	$10^2 + 24^2 = 100 + 576 = 676$; $26^2 = 676$	Yes

Therefore, only set B (10, 24, 26) can represent the side lengths of a right triangle.

Practical Applications and Related Concepts

Understanding which sets of numbers can form right triangles has broad applications beyond pure mathematics, including:

- Construction and Engineering: Ensuring structures are properly aligned using Pythagorean triples.
- Navigation and Surveying: Calculating distances and angles accurately.
- Design and Art: Creating geometrically accurate layouts and patterns.

Furthermore, this knowledge supports the development of Pythagorean triples generation algorithms, essential in various computational applications.

Conclusion

Determining whether a set of numbers can represent the side lengths of a right triangle hinges on applying the Pythagorean Theorem. Our detailed analysis shows that set A (8, 12, 15) does not satisfy the theorem's condition, whereas set B (10, 24, 26) does. Consequently, the correct answer is that only set B can be the side lengths of a right triangle. Recognizing such patterns and applying the Pythagorean theorem is foundational in geometry, with numerous practical applications across disciplines.

Remember: Always identify the longest side before applying the theorem, and verify whether the sum of the squares of the shorter sides equals the square of the longest side to confirm the right triangle property.

Frequently Asked Questions

Which set of numbers can represent the side lengths in inches of a right triangle?

Both sets A (8, 12, 15) and B (10, 24, 26) can represent right triangles because they satisfy the Pythagorean theorem.

Are the sets 8, 12, 15 and 10, 24, 26 Pythagorean triples?

Yes, both sets are Pythagorean triples, meaning each set satisfies the Pythagorean theorem and can be the side lengths of a right triangle.

How can you verify if a set of three numbers can be the sides of a right triangle?

Check if the square of the largest number is equal to the sum of the squares of the other two numbers (Pythagorean theorem).

Is the set 8, 12, 15 a Pythagorean triple?

Yes, because $8^2 + 12^2 = 64 + 144 = 208$, and $15^2 = 225$. Since $208 \neq 225$, 8, 12, 15 is not a Pythagorean triple, so it cannot represent the sides of a right triangle.

Is the set 10, 24, 26 a Pythagorean triple?

Yes, because $10^2 + 24^2 = 100 + 576 = 676$, and $26^2 = 676$. Since both are equal, it can represent the side lengths of a right triangle.

Which of the given sets are valid Pythagorean triples?

Only set B (10, 24, 26) is a valid Pythagorean triple; set A (8, 12, 15) is not.

Can the set 8, 12, 15 be the sides of a right triangle?

No, because it does not satisfy the Pythagorean theorem, so these lengths cannot form a right triangle.

What is the significance of the numbers 8, 12, 15 in geometry?

They are commonly used as the sides of a triangle with no right angle, but they do not form a right triangle since they do not satisfy the Pythagorean theorem.

Are 10, 24, 26 considered a scaled version of a Pythagorean triple?

Yes, they are a scaled version of the triple 5, 12, 13, multiplied by 2.

Which set of side lengths is an example of a Pythagorean triple in inches?

Set B (10, 24, 26) is an example of a Pythagorean triple in inches, representing a right triangle.

Additional Resources

Which Set Of Numbers Can Represent The Side Lengths In Inches Of A Right Triangle? A. 8,12,15 B. 10,24,26 C.

Understanding whether a set of numbers can serve as the side lengths of a right triangle is a fundamental question in geometry. This inquiry becomes particularly intriguing when examining specific triplets, such as 8, 12, 15 and 10, 24, 26. Are these sets compatible with the Pythagorean theorem? Can they truly form right triangles when used as side lengths? This article delves into the mathematical principles that determine the validity of these triplets, providing a comprehensive exploration suitable for both students and enthusiasts alike.

The Foundations: The Pythagorean Theorem and Right Triangles

Before analyzing the specific sets, it is essential to understand the core principle underlying right triangles—the Pythagorean theorem. Formulated by the ancient Greek mathematician Pythagoras, this theorem states:

In a right triangle, the square of the length of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the lengths of the other two sides.

Mathematically, if a and b are the legs, and c is the hypotenuse, then:

$$a^2 + b^2 = c^2$$

This relationship serves as the primary test to verify whether a given set of three numbers can be side lengths of a right triangle.

Analyzing the Sets: Step-by-Step Approach

To determine whether each set of numbers can represent the side lengths of a right triangle, the following process is employed:

1. Identify the largest number in the triplet; this will be considered as the potential hypotenuse.
2. Calculate the sum of the squares of the other two numbers.
3. Compare the sum to the square of the largest number.
4. Determine validity:
 - If the sum equals the square of the largest number, the set forms a right triangle (a Pythagorean triplet).
 - If the sum is less than the square, the set cannot form a right triangle (the triangle would be acute).
 - If the sum is greater than the square, the set cannot form a right triangle (the triangle would be obtuse).

Let's apply this method to the given options:

Set A: 8, 12, 15

Step 1: Largest number is 15.

Step 2: Calculate:

$$8^2 + 12^2 = 64 + 144 = 208$$

Step 3: Calculate 15^2 :

$$\backslash[\\ 15^2 = 225 \\ \backslash]$$

Step 4: Compare:

$$\backslash[\\ 208 \text{ (sum of squares)} \quad \text{vs.} \quad 225 \quad \text{(square of hypotenuse)} \\ \backslash]$$

Since $(208 < 225)$, the sum of the squares of the legs is less than the square of the hypotenuse.

Interpretation: The set does not satisfy the Pythagorean theorem. Therefore, 8, 12, 15 cannot be the side lengths of a right triangle.

Set B: 10, 24, 26

Step 1: Largest number is 26.

Step 2: Calculate:

$$\backslash[\\ 10^2 + 24^2 = 100 + 576 = 676 \\ \backslash]$$

Step 3: Square of 26:

$$\backslash[\\ 26^2 = 676 \\ \backslash]$$

Step 4: Compare:

$$\backslash[\\ 676 \text{ (sum)} \quad \text{vs.} \quad 676 \quad \text{(hypotenuse squared)} \\ \backslash]$$

They are equal!

Interpretation: The set satisfies the Pythagorean theorem exactly, meaning 10, 24, 26 can form a right triangle.

Set C: 15, 20, 25

(Note: Although not initially provided, this classic triplet is often used as an example. If you intended a different set, please specify. Here, we proceed with 15, 20, 25 for demonstration.)

Step 1: Largest number is 25.

Step 2: Calculate:

$$15^2 + 20^2 = 225 + 400 = 625$$

Step 3: Square of 25:

$$25^2 = 625$$

Step 4: Comparison:

$$625 \text{ (sum)} \quad \text{vs.} \quad 625 \text{ (hypotenuse squared)}$$

They are equal!

Interpretation: The triplet 15, 20, 25 is a well-known Pythagorean triplet, confirming that it too can form a right triangle.

Summary of Findings

Set	Largest Number	Sum of Squares of Other Two	Square of Largest Number	Can Form a Right Triangle?
8, 12, 15	15	208	225	No
10, 24, 26	26	676	676	Yes
15, 20, 25	25	625	625	Yes

From the analysis, Set B (10, 24, 26) and Set C (15, 20, 25) can serve as side lengths of right triangles, while Set A (8, 12, 15) cannot.

The Significance of Pythagorean Triplets in Geometry and Real-World Applications

The recognition of Pythagorean triplets is not merely an academic exercise. These triplets appear in various contexts, from construction to computer graphics, where right angles are fundamental. Understanding which sets of numbers satisfy the Pythagorean theorem enables engineers, architects, and students to identify right triangles quickly and accurately.

Practical applications include:

- Design and construction: Ensuring structures have right angles.

- Navigation and surveying: Calculating distances using right triangle principles.
- Technology: Programming algorithms that require geometric computations.
- Mathematics education: Reinforcing concepts of relationships between sides and angles.

Extending the Concept: Primitive and Non-Primitive Triplets

It's worth noting that some triplets are primitive, meaning their sides share no common divisor other than 1, while others are non-primitive and can be scaled versions of primitive triplets.

- Primitive triplet example: 3, 4, 5
- Non-primitive triplet example: 6, 8, 10 (which is just 3, 4, 5 scaled by 2)

In this context, both sets B and C are primitive triplets:

- 10, 24, 26: divisible by 2, so it's a scaled version of 5, 12, 13.
- 15, 20, 25: divisible by 5, which is a scaled version of 3, 4, 5.

Identifying primitive triplets is key in understanding fundamental right triangles, while scaled versions are equally valid for practical purposes.

Final Thoughts: Which Sets Are Valid?

In summary, among the options examined:

- Set A (8, 12, 15): Cannot form a right triangle, as it does not satisfy the Pythagorean theorem.
- Set B (10, 24, 26): Can form a right triangle, being a scaled triplet of 5, 12, 13.
- Set C (15, 20, 25): Can form a right triangle, being a scaled triplet of 3, 4, 5.

Recognizing these patterns helps in quick verification and enhances geometric intuition. Whether in academic settings or real-world applications, understanding the conditions under which three lengths can constitute a right triangle is essential.

Conclusion

The question of which set of numbers can represent the side lengths of a right triangle boils down to verifying the Pythagorean theorem. Through systematic analysis, it becomes clear that only certain triplets—like 10, 24, 26 and 15, 20, 25—meet the criteria, while others, such as 8, 12, 15, do not. This exploration underscores the importance of fundamental geometric principles and highlights the rich structure behind seemingly simple numerical sets. As you encounter various triplets in your studies or practical work, remember that checking the Pythagorean relationship is the quickest route to confirming their validity as right triangle sides.

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