Write An Equation In Slope-intercept Form Of The Line That Passes Through The Given Point And Is Parallel

Write An Equation In Slope-intercept Form Of The Line That Passes Through The Given Point And Is Parallel

When working with linear equations, one common task is to find the equation of a line that passes through a specific point and is parallel to another given line. This process involves understanding the concepts of slope, point-slope form, and the slope-intercept form of a line. The slope-intercept form is particularly useful because it clearly displays the slope and y-intercept, making it straightforward to graph and interpret. In this article, we will explore how to derive the equation of such a line step-by-step, providing clear explanations and examples to deepen your understanding.

Understanding the Basics: Slope-Intercept Form and Parallel Lines

What is the Slope-Intercept Form?

The slope-intercept form of a line's equation is expressed as:

$$y = mx + b$$

where:
- m is the slope of the line,
- b is the y-intercept, the point where the line crosses the y-axis.

This form is favored because it makes it easy to identify the slope and the y-intercept directly from the equation.

What Does It Mean for Lines to Be Parallel?

Two lines are parallel if they lie in the same plane and never intersect, no matter how far they extend. The key property of parallel lines in the plane is that they have identical slopes but different y-intercepts.

Mathematically:
- If line 1 has slope m_1,
- Then line 2, parallel to line 1, must have the same slope $m_2 = m_1$,
- But $b_2 \neq b_1$ (they have different y-intercepts).

Steps to Write the Equation of a Parallel Line Through a Given Point

To find the equation of a line in slope-intercept form that passes through a given point and is parallel to another line, follow these steps:

Step 1: Identify the Slope of the Given Line

- If the problem provides an equation of the existing line, rewrite it in slope-intercept form $y = mx + b$ to find its slope m.
- If only the slope is given directly, note that as the slope of the parallel line.

Step 2: Use the Slope for the New Line

- Since parallel lines have equal slopes, the new line will have the same slope m.

Step 3: Use the Given Point and Slope to Find the Y-Intercept

- Apply the point-slope form of a line:

$$y - y_1 = m(x - x_1)$$

where:
- (x_1, y_1) is the given point,
- m is the slope.

- Substitute the known point and slope into this equation.

Step 4: Convert to Slope-Intercept Form

- Simplify the point-slope form to solve for y, obtaining the equation in $y = mx + b$ form.

Practical Example: Step-by-Step Solution

Suppose we are asked:

"Write an equation in slope-intercept form of the line that passes through the point (3, 4) and is parallel to the line $y = 2x + 1$."

Let's break down the solution:

Step 1: Identify the slope of the given line

- The given line's equation is $y = 2x + 1$.
- The slope m of this line is 2.

Step 2: Use this slope for the new line

- The new line must be parallel, so it has the same slope $m = 2$.

Step 3: Use the point (3, 4) and the slope to find the line equation

- Plug into the point-slope form:

$$y - 4 = 2(x - 3)$$

Step 4: Convert to slope-intercept form

- Expand the right side:

$$y - 4 = 2x - 6$$

- Add 4 to both sides:

$$y = 2x - 6 + 4$$

$$y = 2x - 2$$

Final Answer:

$$y = 2x - 2$$

This is the equation in slope-intercept form of the line passing through (3, 4) and parallel to $y = 2x + 1$.

Additional Tips and Common Mistakes

Tips for Correctly Writing the Equation

- Always identify the slope first from the given line.
- Use the point-slope form to incorporate the specific point.
- Simplify the equation carefully to reach slope-intercept form.
- Double-check your slope to ensure the lines are indeed parallel.

Common Mistakes to Avoid

- Using the wrong slope: Remember, parallel lines must have identical slopes.
- Mixing up the coordinates: Ensure the point used is correct.
- Forgetting to simplify the equation fully into $y = mx + b$.
- Confusing the slope-intercept form with other forms like standard form.

Practice Problems for Mastery

To reinforce your understanding, try solving these problems:

1. Find the equation of the line passing through (1, 2) and parallel to $y = -3x + 4$.

2. Write the equation in slope-intercept form of the line passing through (0, -1) and parallel to $y = \frac{1}{2}x - 5$.

3. Determine the equation of the line passing through (5, 3) and parallel to $y = 4x + 2$.

Solutions:

1. Slope $m = -3$, point (1, 2):

$$y - 2 = -3(x - 1) \Rightarrow y - 2 = -3x + 3 \Rightarrow y = -3x + 5$$

2. Slope $m = \frac{1}{2}$, point (0, -1):

$$y - (-1) = \frac{1}{2}(x - 0) \Rightarrow y + 1 = \frac{1}{2}x \Rightarrow y = \frac{1}{2}x - 1$$

3. Slope $m = 4$, point (5, 3):

$$y - 3 = 4(x - 5) \Rightarrow y - 3 = 4x - 20 \Rightarrow y = 4x - 17$$

Conclusion

Writing the equation of a line in slope-intercept form that passes through a given point and is parallel to another line is a fundamental skill in algebra and coordinate geometry. The process hinges on understanding the properties of parallel lines, correctly identifying slopes, and applying the point-slope form to incorporate the specific point. With practice, this method becomes intuitive, enabling you to solve a wide array of problems involving linear equations efficiently. Whether for academic purposes or practical applications, mastering this technique provides a solid foundation for further exploration of linear functions and their behaviors.

Frequently Asked Questions

How do I write the equation of a line in slope-intercept form that passes through a given point and is parallel to a specific line?

First, find the slope of the given line. Since parallel lines have the same slope, use that slope and the given point to write the equation in slope-intercept form (y = mx + b) by solving for b.

What is the slope of a line parallel to y = 2x + 5 that passes through (3, 4)?

The slope of the parallel line is 2, same as the original line. To find its equation passing through (3, 4): y = 2x + b, then 4 = 2(3) + b, so b = -2. The equation is y = 2x - 2.

If a line passes through (1, -3) and is parallel to y = -4x + 1, what is its equation in slope-intercept form?

Since the original line has slope -4, the new line also has slope -4. Using point (1, -3): -3 = -4(1) + b, so b = 1. The equation is y = -4x + 1.

How can I determine the slope of the line if only a point and a parallel line's equation are given?

Identify the slope from the given line's equation (in slope-intercept form). Since the lines are parallel, use that same slope for the new line.

What steps are involved in writing the slope-intercept form of a parallel line passing through a point?

Steps include: 1) Find the slope of the given line. 2) Use the point-slope formula with the point and slope. 3) Solve for y to write in slope-intercept form.

Can a line pass through a point and be parallel to a vertical line? How do I write its equation?

Vertical lines have undefined slope, so a line passing through a point and parallel to a vertical line is also vertical, with an equation x = constant, not in slope-intercept form.

What if the given point is the same as a point on the original line? Will the new line be identical?

Yes, if the point lies on the original line, then the new line passing through that point and parallel to the original will be the same line.

How do I verify that my line is parallel to the given line after writing the equation?

Check that both lines have the same slope. If the slopes are equal, the lines are parallel.

Is it possible for a line to pass through a point and be parallel to a horizontal line? How do I write its equation?

Yes, if the original line is horizontal (slope 0), the line passing through the point and parallel to it will also be horizontal, with the equation y = constant, where 'constant' is the y-coordinate of the point.

What common mistakes should I avoid when writing the equation of a parallel line through a given point?

Common mistakes include using the wrong slope, forgetting to solve for the y-intercept, or mixing up the point's coordinates. Always identify the correct slope and carefully substitute the point into the equation.

Additional Resources

Write An Equation In Slope-intercept Form Of The Line That Passes Through The Given Point And Is Parallel

Understanding how to write the equation of a line that passes through a specific point and is parallel to a given line is an essential skill in algebra and analytic geometry. This process not only enhances problem-solving abilities but also deepens comprehension of the geometric concepts underlying linear equations. When tackling such problems, the slope-intercept form of a line—expressed as y = mx + b—serves as a foundational tool, allowing students and practitioners to easily interpret and manipulate line equations.

In this comprehensive review, we will explore the step-by-step methodology for writing the equation of a line in slope-intercept form that passes through a specific point and maintains parallelism to another line. We will delve into the underlying principles, illustrate with examples, discuss common pitfalls, and highlight best practices to master this fundamental aspect of coordinate geometry.

Understanding the Slope-Intercept Form

Definition and Components

The slope-intercept form of a line is given by:

$$ y = mx + b $$

Where:
- m represents the slope of the line.
- b is the y-intercept, the point where the line crosses the y-axis.

This form is particularly advantageous because:
- It directly reveals the slope and y-intercept.
- It simplifies graphing and analyzing the line.
- It provides a straightforward way to formulate equations once the slope and a point are known.

Relevance in Parallel Line Problems

Parallel lines share the same slope but differ in their y-intercepts. When tasked with writing an equation for a line passing through a specific point and parallel to another, the key is to:

- Find the slope of the given line.
- Use that slope for the new line.
- Determine the y-intercept using the given point.

Step-by-Step Process for Writing the Equation

Step 1: Identify the Slope of the Given Line

The first step is to find the slope of the line to which the new line must be parallel. If the original line's equation is given in slope-intercept form, the slope is directly available as the coefficient of x (m). If the line is given in standard form or another form, algebraic manipulation may be necessary to extract the slope.

Example:

Given line: $3x - 2y = 6$

Rearranged to slope-intercept form:

$$ -2y = -3x + 6 $$
$$ y = \frac{3}{2}x - 3 $$

The slope (m) is $\frac{3}{2}$.

Features:
- Straightforward extraction when the line is in slope-intercept form.
- May require rearrangement if in standard form.

Pros:
- Clear identification of the slope.
- Facilitates the process of parallelism.

Cons:
- Additional algebra needed if the original form isn't slope-intercept.

Step 2: Confirm Parallelism (Same Slope)

Since parallel lines have identical slopes, the new line's slope (m_{new}) equals the original slope ($m_{original}$).

Example:
If the original line has slope $\frac{3}{2}$, then:

$$m_{\text{new}} = \frac{3}{2}$$

Note:
- Be cautious with negative slopes and fractions.
- Confirm the slope before proceeding.

Step 3: Use the Point-Slope Formula

The point-slope form of a line is:

$$y - y_1 = m(x - x_1)$$

Where:
- (x_1, y_1) is the given point through which the line passes.
- m is the slope (found in Step 1).

This form is versatile for plugging in known values directly.

Example:

Given point: (4, 1)
Slope: $\frac{3}{2}$

Equation:

$$y - 1 = \frac{3}{2}(x - 4)$$

Step 4: Convert to Slope-Intercept Form

Transform the point-slope form into slope-intercept form:

$$y = mx + b$$

Continuing the Example:

$$y - 1 = \frac{3}{2}x - 6$$
$$y = \frac{3}{2}x - 6 + 1$$
$$y = \frac{3}{2}x - 5$$

Thus, the line passing through (4, 1) and parallel to the original line has the equation:

$$ y = \frac{3}{2}x - 5 $$

Features:
- Straightforward algebraic manipulation.
- Clear visualization of the y-intercept.

Common Challenges and Tips

Handling Different Line Forms

- Standard form ($Ax + By = C$): Rearrange to slope-intercept form to find the slope.
- Point-slope form: Directly usable; only conversion to slope-intercept is needed.
- Vertical lines: Have undefined slope; not parallel to any line with finite slope.

Tip: Always verify the form of the given line before proceeding.

Dealing with Fractions and Negative Slopes

- When working with fractional slopes, maintain consistent notation and simplify where possible.
- Be mindful of signs: a negative slope indicates the line decreases as x increases.

Tip: Use parentheses to avoid errors during algebraic manipulation.

Common Mistakes to Avoid

- Confusing the slope of the original line with the slope of the new line.
- Forgetting to use the point in the point-slope formula.
- Incorrectly converting from standard or other

forms to slope-intercept form.
- Overlooking the importance of the point's coordinates.

Practical Applications and Examples

Example 1: Basic Scenario

Find the equation of the line passing through the point (2, 3) and parallel to the line y = -4x + 1.

Solution:

- Slope of given line: $m = -4$
- New line passes through (2, 3):

$$y - 3 = -4(x - 2)$$

$$y - 3 = -4x + 8$$

$$y = -4x + 8 + 3$$

$$y = -4x + 11$$

Example 2: Line in Standard Form

Find the line passing through (-1, 4) and parallel to 2x + 3y = 6.

Solution:

- Convert to slope-intercept:

$$ 3y = -2x + 6 $$
$$ y = -\frac{2}{3}x + 2 $$

- Slope: $m = -\frac{2}{3}$

- Equation using point (-1, 4):

$$ y - 4 = -\frac{2}{3}(x + 1) $$

$$ y - 4 = -\frac{2}{3}x - \frac{2}{3} $$

$$ y = -\frac{2}{3}x - \frac{2}{3} + 4 $$

$$ y = -\frac{2}{3}x + \frac{10}{3} $$

Conclusion and Final Thoughts

Mastering the process of writing an equation in slope-intercept form of a line passing through a given point and parallel to a specified line is an essential skill that combines algebraic manipulation with geometric understanding. The core principle hinges on recognizing that parallel lines share the same slope, and leveraging point-slope and slope-intercept forms simplifies the derivation process.

Features of this method include:
- Clear, step-by-step approach.
- Flexibility across different line representations.
- Ease of visualization and graphing.

Pros:
- Facilitates quick formulation of lines in various contexts.
- Enhances understanding of the relationship between algebraic equations and geometric lines.

Cons:
- Susceptible to algebraic mistakes if steps are rushed.
- Requires familiarity with manipulating different line forms.

In practice, consistent practice with diverse examples builds confidence and proficiency. Remember to verify the slope, carefully perform algebraic steps, and always double-check your final equation for accuracy. By mastering this process, students and professionals can confidently handle a wide array of linear equations and geometric problems, reinforcing their foundational understanding of coordinate geometry.

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