Write An Equation Of The Line That Passes Through The Given Point And Is Perpendicular To The Given Line.

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Understanding how to find the equation of a line that passes through a specific point and is perpendicular to another line is a fundamental concept in coordinate geometry. This skill is essential for solving various geometric problems, analyzing slopes, and understanding the relationships between lines. Whether you are a student preparing for exams or a professional working on geometric modeling, mastering this process will enhance your mathematical toolkit.

In this article, we will explore the step-by-step process to write the equation of such a line, discuss key concepts like slope and perpendicularity, and provide examples to solidify your understanding.

Understanding the Basics: Slope and Perpendicular Lines

What Is Slope?

The slope of a line measures its steepness and is typically represented by the letter 'm'. In a Cartesian coordinate system, the slope is calculated as the ratio of the change in y to the change in x between two points on the line:

$$m = (y_2 - y_1) / (x_2 - x_1)$$

For example, if a line passes through points (1, 2) and (3, 4), its slope is:

$$m = (4 - 2) / (3 - 1) = 2 / 2 = 1$$

What Does Perpendicular Mean?

Two lines are perpendicular if they intersect at a right angle (90 degrees). An important property of perpendicular lines is that their slopes are negative reciprocals of each other.

This means:

If line 1 has slope m_1, then a line perpendicular to it has slope m_2 such that:

$$m_1 \times m_2 = -1$$

For example, if the given line has a slope of 2, the slope of the perpendicular line will be -1/2.

Step-by-Step Procedure to Write the Equation of the Perpendicular Line

Let's break down the process into clear steps:

1. Find the slope of the given line

Identify the slope of the original line either from its equation or from two known points.

- If the line is given in slope-intercept form (y = mx + b), the slope is m.
- If the line is given through two points, use the slope formula.

2. Determine the slope of the perpendicular line

Calculate the negative reciprocal of the original slope:

- If the original slope is m, then the perpendicular slope is $m_{perp} = -1/m$
- Special cases: if the original line is vertical (undefined slope), then the perpendicular line will be horizontal (slope = 0), and vice versa.

3. Use the point-slope form to write the equation

Once you have the slope of the perpendicular line and the point it passes through (say, (x0, y0)), plug these into the point-slope form:

$$y - y0 = mperp (x - x0)$$

4. Simplify to slope-intercept or standard form

Rearrange the equation as needed:

- To slope-intercept form (y = mx + b),
- Or to standard form (Ax + By = C).

Examples to Illustrate the Process

Example 1: Find the line passing through (3, 4) and perpendicular to $y = 2x + 1$

Step 1: Identify the slope of the given line.

- The given line is in slope-intercept form: $y = 2x + 1$

- Slope $m = 2$

Step 2: Find the perpendicular slope.

- $m_{perp} = -1/2$

Step 3: Use the point-slope form with point (3, 4):

```
y - 4 = -1/2 (x - 3)
```

Step 4: Simplify the equation:

```
y - 4 = -1/2 x + 3/2
```

Add 4 to both sides:

```
y = -1/2 x + 3/2 + 4
```

Express 4 as 8/2:

```
y = -1/2 x + 3/2 + 8/2 = -1/2 x + 11/2
```

Final Equation:
$y = -1/2\ x + 11/2$

This is the equation of the line passing through (3, 4) and perpendicular to $y = 2x + 1$.

Example 2: Find the line passing through (0, 0) and perpendicular to $x = 5$

Step 1: Recognize the given line $x = 5$ is a vertical line with undefined slope.

Step 2: The perpendicular line to a vertical line is a horizontal line with slope 0.

Step 3: Use the point (0, 0):

```
y - 0 = 0 (x - 0) → y = 0
```

Final Equation:
y = 0

This horizontal line passes through (0, 0) and is perpendicular to the vertical line x = 5.

Special Cases to Consider

When the Given Line Is Vertical

A vertical line has an undefined slope. The perpendicular line will be horizontal with a slope of 0. Its equation will be of the form y = y0, where (x0, y0) is the given point.

When the Given Line Is Horizontal

A horizontal line has a slope of 0. The perpendicular line will be vertical, with an undefined slope, and will be of the form x = x0.

Implications for the Equation

Always analyze the nature of the given line first to determine the correct approach for the perpendicular line's equation.

Tips for Accurate Calculation and Writing Equations

- Always identify the slope of the given line accurately.
- Remember that the perpendicular slope is the negative reciprocal.
- Use the point-slope form for convenience: $y - y_0 = m(x - x_0)$
- Check for special cases like vertical or horizontal lines.
- Simplify the equation to your preferred form for clarity.

Conclusion: Mastering the Art of Perpendicular Line Equations

Being able to write the equation of a line passing through a specific point and perpendicular to another line is a vital skill in analytical geometry. By understanding the relationship between slopes of perpendicular lines, recognizing special cases, and applying the point-slope form accurately, you can confidently solve a wide array of problems.

Practice with different examples, verify your calculations, and always consider the nature of the given line. With these strategies, you'll master the process and enhance your geometric problem-solving skills.

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- Coordinate geometry perpendicular lines tutorial
- Step-by-step guide to perpendicular line equations

Frequently Asked Questions

How do I write the equation of a line passing through a point and perpendicular to a given line?

First, find the slope of the given line, then determine the perpendicular slope (the negative reciprocal). Use the point-slope form with the given point and this perpendicular slope to write the equation.

What is the steps to find the perpendicular line to a given line passing through a specific point?

Identify the slope of the given line, find its negative reciprocal for the perpendicular slope, then use the point-slope form with the point provided to write the new line's equation.

If the given line is y = 2x + 3 and the point is (4, 1), how do I

find the perpendicular line through (4, 1)?

The slope of the given line is 2. The perpendicular slope is -1/2. Using point-slope form: $y - 1 = -1/2(x - 4)$. Simplify to get the equation.

Can I use the slope-intercept form to write the equation of the perpendicular line?

Yes, after finding the perpendicular slope and the point, you can substitute into point-slope form and then convert to slope-intercept form if desired.

What if the given line is vertical or horizontal? How do I find the perpendicular line?

If the line is horizontal (slope 0), the perpendicular line is vertical (x = constant). If vertical (undefined slope), the perpendicular line is horizontal (y = constant).

How is the negative reciprocal used to find the slope of the perpendicular line?

The negative reciprocal of a slope m is $-1/m$. This is used because perpendicular lines in a plane have slopes that are negative reciprocals.

What is the general formula to write the equation of a line passing through (x_1, y_1) with slope m?

The point-slope form: $y - y_1 = m(x - x_1)$. Use this with the perpendicular slope and the given point to write the equation.

How do I verify that my line is perpendicular to the given line?

Calculate the slopes of both lines; if their slopes are negative reciprocals, the lines are perpendicular.

Are there any special cases to consider when writing perpendicular lines through a point?

Yes, if the given line is horizontal or vertical, the perpendicular line will be vertical or horizontal, respectively. Adjust your approach accordingly.

Additional Resources

Perpendicular Line Equation: Mastering the Art of Geometric Precision

Understanding how to derive the equation of a line that passes through a specific point and is perpendicular to a given line is a fundamental skill in coordinate geometry. Whether you're a student preparing for exams, a teacher designing lesson plans, or a professional solving complex problems in engineering or architecture, mastering this process is invaluable. In this comprehensive guide, we will explore the nuances of this topic with clarity, depth, and practical insights, much like an expert review of a sophisticated product.

Introduction to the Concept of Perpendicular Lines

Perpendicular lines are a cornerstone concept in geometry. They intersect at a right angle (90 degrees), and their properties are extensively utilized in various fields, from design and construction to computer graphics. The core challenge addressed here is: Given a point and a line, how do you find the equation of the line passing through that point and perpendicular to the given line?

This problem involves multiple steps, including understanding the slope of the original line, determining the slope of the perpendicular line, and then formulating its equation. Let's dissect each component systematically.

Fundamental Concepts and Terminology

Slope of a Line

The slope (m) of a line measures its steepness and is calculated as the ratio of the change in y to the change in x between two points on the line:

$$
m = \frac{\Delta y}{\Delta x}
$$

For a line expressed in the form $y = mx + c$, the coefficient m directly represents the slope.

Perpendicular Slopes

Two lines are perpendicular if the product of their slopes is -1:

$$
m_1 \times m_2 = -1
$$

This relationship means that if the original line has a slope m, then the perpendicular line has a slope:

$$
m_{\perp} = -\frac{1}{m}
$$

Point-Slope Form of a Line

Once the slope of the perpendicular line is known, and a point through which it passes (say, (x_1, y_1)), the line's equation can be expressed as:

$$
y - y_1 = m_{\perp} (x - x_1)
$$

Step-by-Step Procedure for Deriving the Equation

The process involves several key steps:

1. Identify the Equation of the Given Line

The first step is to understand the form of the given line. It can be presented in various formats:

- Slope-Intercept Form: $y = mx + c$
- Standard Form: $Ax + By + C = 0$

Suppose the line is given as $y = m x + c$. If it's presented differently, algebraic manipulation can convert it into slope-intercept form.

2. Determine the Slope of the Given Line

Extract the slope m directly if the line is in slope-intercept form. For example, if the line is $y = 2x + 3$, then $m=2$.

If the line is in standard form, convert it:

$$
Ax + By + C = 0
$$

to slope-intercept form:

$$
By = -Ax - C \quad \Rightarrow \quad y = -\frac{A}{B}x - \frac{C}{B}
$$

Thus, the slope is $-\frac{A}{B}$.

3. Find the Slope of the Perpendicular Line

Using the relation:

\[

m_{\perp} = -\frac{1}{m}
\]
For example, if the original line has \(m=2 \), then:

\[
m_{\perp} = -\frac{1}{2}
\]

4. Use the Given Point to Write the Equation of the Perpendicular Line

Suppose the point is \((x_1, y_1) \). Plugging this and the perpendicular slope into the point-slope form:

\[
y - y_1 = m_{\perp} (x - x_1)
\]
This yields the desired line's equation.

5. Simplify to Standard or Slope-Intercept Form (Optional)

Depending on the requirement, you may want to express the equation in slope-intercept form:

\[
y = m_{\perp} x + b
\]
or standard form:

\[
Ax + By + D= 0
\]

Illustrative Example: Bringing Theory to Life

Let’s examine a practical example to see this process in action.

Given:

- Line \(y = 3x + 4 \)
- Point \((2, -1) \)

Objective:

Find the equation of the line passing through \((2, -1) \) and perpendicular to the given line.

Solution:

Step 1: Identify the slope of the given line:

$$
m = 3
$$

Step 2: Calculate the perpendicular slope:

$$
m_{\perp} = -\frac{1}{3}
$$

Step 3: Write the equation using point-slope form:

$$
y - (-1) = -\frac{1}{3}(x - 2)
$$

which simplifies to:

$$
y + 1 = -\frac{1}{3}x + \frac{2}{3}
$$

Step 4: Convert to slope-intercept form:

$$
y = -\frac{1}{3}x + \frac{2}{3} - 1
$$
$$
y = -\frac{1}{3}x - \frac{1}{3}
$$

Final Equation:

$$
\boxed{
y = -\frac{1}{3}x - \frac{1}{3}
}
$$

This line passes through $(2, -1)$ and is perpendicular to $y=3x+4$.

Handling Various Line Equations and Special Cases

While the steps above are straightforward, real-world problems often involve lines in different forms or special situations.

Case 1: Line in Standard Form

Suppose the line is $2x - 5y + 10= 0$.

- Convert to slope-intercept:

$$
-5 y = -2x - 10 \quad \Rightarrow \quad y = \frac{2}{5}x + 2
$$

- Slope $m = \frac{2}{5}$
- Perpendicular slope:

$$
m_{\perp} = -\frac{5}{2}
$$

- Equation of the perpendicular line through point (x_1, y_1):

$$
y - y_1 = -\frac{5}{2}(x - x_1)
$$

Case 2: Vertical and Horizontal Lines

- If the original line is vertical, e.g., $x= c$, its slope is undefined.
- The perpendicular line to a vertical line is horizontal, i.e., of the form $y= y_1$.
- Conversely, if the original line is horizontal, $y= y_0$, the perpendicular line is vertical, $x= x_1$.

Handling these cases requires special attention:

- For a vertical line passing through $x= c$, the perpendicular line passing through (x_1, y_1) is $y= y_1$.
- For a horizontal line $y= y_0$, the perpendicular line is $x= x_1$.

Practical Applications and Significance

Mastering the derivation of perpendicular lines extends beyond academic exercises; it has tangible applications:

- Architectural Design: Ensuring perpendicular walls or features.
- Computer Graphics: Rendering perpendicular lines or features in digital images.
- Navigation and Mapping: Calculating shortest paths or perpendicular distances.
- Engineering: Structural analysis involving orthogonal components.
- Robotics: Path planning involving perpendicular trajectories.

Common Pitfalls and Expert Tips

Achieving accuracy in deriving the perpendicular line equation requires awareness of common mistakes:

- Incorrect slope calculation: Always verify the line's form before extracting the slope.
- Forgetting to invert the slope: Remember, the perpendicular slope is the negative reciprocal.
- Ignoring special cases: Vertical and horizontal lines require different approaches.
- Sign errors: Carefully manage signs during algebraic manipulation.
- Not verifying the solution: Plug the point and the slope back into the equation to confirm correctness.

Expert Tip: Always double-check the perpendicularity by verifying that the product of the slopes equals -1.

Conclusion: Your Geometric Toolkit Enhanced

Determining the equation of a line passing through a given point and perpendicular to a given line is a fundamental skill that combines algebraic manipulation with geometric insight. By understanding the underlying principles—such as slope relationships, line forms, and point-slope equations—you can confidently tackle a broad spectrum of problems involving perpendicular lines.

Think of this process as an advanced tool in your mathematical toolkit: precise,

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