Assume That U=ln(x) + Y. Solve For X* In Terms Of Px, Py, And I. Group Of Answer Choicesa. Px/Pyb. Py/Pxc.

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This question delves into the realm of consumer choice theory and utility maximization, key concepts in microeconomics. By understanding how to derive the optimal consumption bundle \(X^\) given a utility function and budget constraints, economists and students can better analyze consumer behavior and market dynamics. In this article, we will explore the steps to solve for \(X^\) when the utility function is specified as \(U = \ln(x) + y\), and how the solution relates to the prices of goods \(P_x\) and \(P_y\), as well as the consumer's income \(I\).

Understanding the Utility Function \(U = \ln(x) + y\)

Before solving for the optimal choice \(X^\), it is essential to understand the form of the utility function provided:

Nature of the Utility Function

- Additive Utility: The function \(U = \ln(x) + y\) suggests that utility depends on two goods, \(x\) and \(y\), in an additive manner.
- Concavity and Preferences: Since \(\ln(x)\) is a monotonically increasing and concave function for \(x > 0\), and \(y\) enters linearly, the utility is increasing in both goods but with diminishing marginal utility for \(x\).

Implications for Consumer Behavior

- Consumers derive utility from consuming \(x\) and \(y\), but the marginal utility of \(x\) diminishes as \(x\) increases.
- The linear term \(y\) indicates constant marginal utility for \(y\).

Setting Up the Optimization Problem

To find the optimal consumption bundle \((x^, y^)\), the consumer maximizes utility subject to their budget constraint:

Budget Constraint

$$
P_x x + P_y y = I
$$

Where:
- P_x = price of good x
- P_y = price of good y
- I = consumer's income

Formulating the Lagrangian

The problem becomes:

$$
\mathcal{L} = \ln(x) + y + \lambda (I - P_x x - P_y y)
$$

Where λ is the Lagrange multiplier representing the marginal utility of income.

Deriving the First-Order Conditions

To solve for the optimal x and y, we take partial derivatives:

Partial derivatives

$$
\frac{\partial \mathcal{L}}{\partial x} = \frac{1}{x} - \lambda P_x = 0
$$
$$
\frac{\partial \mathcal{L}}{\partial y} = 1 - \lambda P_y = 0
$$
$$
\frac{\partial \mathcal{L}}{\partial \lambda} = I - P_x x - P_y y = 0
$$

From the first two conditions, we can express λ:

$$
\lambda = \frac{1}{x P_x}
$$
$$
\lambda = \frac{1}{P_y}
$$

Solving for \(x^\) in Terms of \(P_x\), \(P_y\), and \(I\)

By equating the two expressions for \(\lambda\):

\[
\frac{1}{x P_x} = \frac{1}{P_y}
\]

Rearranged, this yields:

\[
P_y = x P_x
\]
\[
x^ = \frac{P_y}{P_x}
\]

This is a critical step: the optimal quantity of \(x\) depends on the ratio of the prices of the two goods.

Determining \(y^\)

Using the budget constraint:

\[
I = P_x x^ + P_y y^
\]
\[
y^ = \frac{I - P_x x^}{P_y}
\]

Since \(x^ = \frac{P_y}{P_x}\):

\[
y^ = \frac{I - P_x \times \frac{P_y}{P_x}}{P_y} = \frac{I - P_y}{P_y}
\]

Interpreting the Results

The key takeaway from the derivation is that the consumer's optimal choice for \(x\) given the utility function \(U = \ln(x) + y\) and the budget constraint is:

\[
\boxed{
x^ = \frac{P_y}{P_x}
}
\]

This result aligns with the intuition that, at optimality, the consumer balances the marginal utilities per dollar spent across goods:

\[
\frac{\text{Marginal Utility of } x}{P_x} = \frac{\text{Marginal Utility of } y}{P_y}
\]

Given the marginal utilities:

\[
MU_x = \frac{1}{x}
\]
\[
MU_y = 1
\]

The equality at the optimum becomes:

\[
\frac{1/x}{P_x} = \frac{1}{P_y}
\]
\[
\Rightarrow x^ = \frac{P_y}{P_x}
\]

Expressing \(X^\) in Terms of Income \(I\)

While the optimal \(x^\) is primarily determined by the ratio of prices, the consumer's choice of \(y^\) depends on income:

\[
y^ = \frac{I - P_x x^}{P_y}
\]

Substituting \(x^ = \frac{P_y}{P_x}\):

\[
y^ = \frac{I - P_x \times \frac{P_y}{P_x}}{P_y} = \frac{I - P_y}{P_y}
\]

This shows that as income increases, \(y^\) increases linearly, while \(x^\) remains constant, determined solely by the ratio of prices.

Comparing to the Group of Answer Choices

The multiple-choice options are:

- a. \(\frac{P_x}{P_y} \)
- b. \(\frac{P_y}{P_x} \)

Given our derivation, the correct expression for the optimal \(x^\) in terms of prices is:

\[
x^ = \frac{P_y}{P_x}
\]

which matches choice b.

Conclusion and Practical Implications

The analysis reveals that for the utility function \(U = \ln(x) + y\), the consumer's optimal choice for \(x\) is directly proportional to the ratio of the price of \(y\) to the price of \(x\). This insight is valuable for understanding consumer preferences and how they respond to changes in prices and income.

Key Takeaways

- The optimal quantity of \(x\) depends only on the relative prices \(P_y\) and \(P_x\).
- Income \(I\) influences the consumption of \(y\), not \(x\), in this specific utility setup.
- The choice \(x^ = \frac{P_y}{P_x}\) aligns with the principle of equating the marginal utility per dollar spent across goods.

Real-World Applications

- Pricing strategies: Understanding how consumers adjust their consumption based on relative prices.
- Policy analysis: Evaluating how changes in taxes or subsidies affect consumer choices.
- Market simulations: Modeling consumer responses to price fluctuations.

In summary, solving for \(X^\) within this utility framework clarifies the fundamental relationships between prices, income, and consumer preferences, providing a practical toolkit for microeconomic analysis.

Note: The correct answer choice, based on the derivation, is b. \(\frac{P_y}{P_x} \).

Frequently Asked Questions

Given the utility function U = ln(x) + y, how do we solve for the optimal x in terms of prices Px, Py, and income I?

To find x, set the marginal utility per dollar equal: $(\partial U/\partial x)/Px = (\partial U/\partial y)/Py$. Since $\partial U/\partial x = 1/x$ and $\partial U/\partial y = 1$, the condition becomes $(1/x)/Px = 1/Py$, leading to $x = Py / Px$.

What is the optimal consumption bundle (x, y) when utility is U = ln(x) + y with given prices Px, Py and income I?

From the first-order condition, $x = Py / Px$. The budget constraint $I = Px\ x + Py\ y$ gives $y = (I - Px\ x) / Py$.

If the utility function is U = ln(x) + y, how does the ratio of prices Px and Py influence the choice of x?

The optimal x is directly proportional to Py and inversely proportional to Px. Specifically, $x = Py / Px$, so a higher Py increases x, while a higher Px decreases x.

How do you express x solely in terms of prices Px, Py, and income I using the utility function U = ln(x) + y?

First, find $x = Py / Px$ from the utility maximization condition. Then, substitute into the budget constraint $I = Px\ x + Py\ y$ to find y. The key relationship for x remains $x = Py / Px$.

What is the significance of the ratio Px / Py in determining the optimal consumption x?

The ratio Px / Py represents the relative price of good x to good y. The optimal x is proportional to the ratio Py / Px, meaning as Px increases relative to Py, x decreases.

In the context of the utility maximization problem, which of the following options correctly represents x? a) Px / Py, b) Py / Px, c) Py / Px, d) Px / Py.

The correct answer is b) Py / Px.

Given the answer choices, which correctly expresses x in terms of Px and Py based on the utility function U = ln(x) + y?

The correct choice is b. Py / Px, because the optimal x equals the ratio of Py to Px.

Additional Resources

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Introduction

Understanding consumer behavior and demand functions is a cornerstone of microeconomic theory. When analyzing how consumers allocate their income across different goods, economists often turn to utility functions to model preferences and derive demand functions. In this context, the utility function U = ln(x) + y offers a tractable yet insightful form to explore how consumers optimize their consumption bundle given their budget constraints.

This article delves into the process of solving for the consumer's optimal choice of X (denoted as X\), expressed in terms of the prices of goods Px, Py, and the consumer's income I. By examining the utility maximization problem, deriving the demand functions, and interpreting the solutions, we aim to clarify the relationship between these variables and the implications for consumer choice theory.

Utility Function and Its Significance

The Form of the Utility Function

The utility function under consideration is:

$$U = \ln(x) + y$$

This form suggests that the consumer derives utility from two goods, X and Y, with the utility depending logarithmically on X and linearly on Y.

- The logarithmic component $\ln(x)$ indicates diminishing marginal utility with respect to X — each additional unit of X provides less added utility than the previous one.
- The linear component in Y implies a constant marginal utility from Y.

This utility structure simplifies the optimization process because the derivatives are straightforward, and the properties of the logarithmic function ensure concavity, which is vital for the existence of a unique maximum.

Implications for Consumer Preferences

The form indicates that the consumer prefers X and Y to be positive quantities (since $\ln(x)$ is only defined for $x > 0$), emphasizing the necessity of feasible, positive consumption choices. The utility function's additive nature allows for straightforward analysis of substitution and income effects, especially when subject to a budget constraint.

Setting up the Optimization Problem

The Budget Constraint

The consumer's budget constraint is:

$$ P_x \cdot x + P_y \cdot y = I $$

where:

- P_x is the price of good X,
- P_y is the price of good Y,
- I is the consumer's income.

The goal is to maximize utility $U = \ln(x) + y$ subject to this constraint.

Solving the Utility Maximization Problem

Step 1: Formulate the Lagrangian

To incorporate the budget constraint, we set up the Lagrangian function:

$$ \mathcal{L} = \ln(x) + y + \lambda (I - P_x x - P_y y) $$

where λ is the Lagrange multiplier.

Step 2: Compute First-Order Conditions (FOCs)

Differentiating $\mathcal{L}$ with respect to x, y, and λ, we derive the optimality conditions:

1. $\frac{\partial \mathcal{L}}{\partial x} = \frac{1}{x} - \lambda P_x = 0 \Rightarrow \lambda = \frac{1}{x P_x}$

2. $\frac{\partial \mathcal{L}}{\partial y} = 1 - \lambda P_y = 0 \Rightarrow \lambda = \frac{1}{P_y}$

3. $\frac{\partial \mathcal{L}}{\partial \lambda} = I - P_x x - P_y y = 0$

Step 3: Equate the Lagrange Multipliers

From conditions 1 and 2:

$$
\frac{1}{x P_x} = \frac{1}{P_y} \Rightarrow \frac{1}{x P_x} = \frac{1}{P_y}
$$

Cross-multiplied:

$$
P_y = x P_x
$$

or equivalently,

$$
x = \frac{P_y}{P_x}
$$

This critical step reveals the consumer's optimal choice for X in relation to the prices.

Step 4: Find the Optimal X\

Using the budget constraint and the optimal X, solve for Y\:

$$
I = P_x x + P_y y
$$

Substitute $x = \frac{P_y}{P_x}$:

$$
I = P_x \left(\frac{P_y}{P_x} \right) + P_y y = P_y + P_y y
$$

Rearranged:

$$
I - P_y = P_y y \Rightarrow y = \frac{I - P_y}{P_y}
$$

which simplifies to:

$$
y = \frac{I}{P_y} - 1
$$

Key Result:

The optimal quantity of X, denoted as X\, is:

\[
\boxed{
X^ = \frac{P_y}{P_x}

}
\]

This indicates that, given the utility function $U = \ln x + y$ and the budget constraint, the consumer chooses X proportional to the ratio of the prices of Y and X.

Expressing \(X^\) in Terms of P_x, P_y, and I

The derivation above suggests that the optimal X depends primarily on the price ratio, but importantly, the initial assumption that this ratio directly determines X\ needs reinterpretation.

The earlier derivation yields an X\ independent of income I, which seems counterintuitive because, in typical utility maximization, demand functions depend on income. This indicates that, with the utility form $U = \ln x + y$, X is determined solely by the price ratio, and Y adjusts accordingly to exhaust income.

To clarify, we revisit the steps to incorporate income explicitly.

Revisiting the Demand Function for X

Step 1: Express Y in terms of X

From the budget constraint:

$$
Y = \frac{I - P_x x}{P_y}
$$

The utility function in terms of X:

$$
U = \ln x + \frac{I - P_x x}{P_y}
$$

Maximize $U(x)$ with respect to x:

$$
\frac{dU}{dx} = \frac{1}{x} - \frac{P_x}{P_y} = 0
$$

Solve for x:

$$
\frac{1}{x} = \frac{P_x}{P_y} \Rightarrow x = \frac{P_y}{P_x}
$$

Again, same as before, this suggests that X\ is independent of income, which aligns with the properties of the utility function: the consumer chooses X to maximize the utility contribution of \(\ln x\), constrained by the budget, but because \(\ln x\) is unbounded as \(x \to 0\) and increasing, the demand for X is driven solely by the price ratio.

Step 2: Derive Y\

Using the budget constraint:

\[
Y^ = \frac{I - P_x X^}{P_y}
\]

which explicitly involves income \(I\).

Final Expression for X\

Based on the derivations, the demand for X as a function of prices and income is:

\[
X^ = \frac{P_y}{P_x}
\]

which does not depend on income.

However, the question asks for an expression of \(X^\) in terms of \(P_x\), \(P_y\), and \(I\).

Given the prior derivations, the demand for X is constant with respect to income, implying:

\[
X^ = \frac{P_y}{P_x}
\]

and Y adjusts to exhaust income.

Interpreting the Group of Answer Choices

The multiple-choice options provided are:

a. \(\frac{P_x}{P_y} \)

b. \(\frac{P_y}{P_x} \)

c. The options are only partial, but based on the derivation, the correct expression for X\ is:

\[
X^ = \frac{P_y}{P_x}
\]

which corresponds to option b.

Broader Context and Implications

The demand function's independence from income in this specific utility form illustrates a key point in consumer theory: the elasticity and demand responses depend heavily on the functional form of the utility and the constraints involved.

In typical utility maximization problems, demand functions are functions of prices and income; however, certain utility functions, especially those with specific properties like constant marginal utility or particular forms like the logarithmic utility, can lead to demand functions that are independent of income or other parameters.

This behavior underscores the importance of understanding the utility function's properties when predicting consumer responses to price changes.

Additional Considerations

Price Changes and Demand

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