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Consider The Following. T Is The Counterclockwise Rotation Of 120 In R2, V = (8, 8). (a) Find The Standard

Understanding rotations in a two-dimensional plane (R2) is fundamental in linear algebra, computer graphics, physics, and engineering. When we discuss rotation transformations, especially counterclockwise rotations by a certain angle, we are referring to a linear transformation that rotates every point around the origin by that specified angle. In this context, the problem specifies a counterclockwise rotation of 120 degrees in R2 applied to a vector V = (8, 8). Additionally, it asks for the standard form of this transformation.

This comprehensive article explores the mathematical background of such rotations, how to derive their standard matrix forms, and applies these concepts to the specific vector V. By the end, you will understand how to represent the rotation transformation mathematically and interpret its effects on vectors in R2.

Understanding Rotation in R2

The Geometric Perspective

Rotation in the plane involves turning points around the origin by a specific angle. For counterclockwise rotation by an angle θ, each point (x, y) in the plane is mapped to a new point (x', y') according to the rotation rules.

- Counterclockwise Rotation: When an object is rotated counterclockwise by an angle θ, it moves in the positive angular direction from the initial position.
- Origin as Center: All rotations are performed around the origin (0,0) unless specified otherwise.

Mathematical Representation

The rotation transformation can be expressed as a linear transformation, represented by a matrix R(θ), which acts on vectors in R2:

$$R(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

Applying R(θ) to a vector V = (x, y) gives the rotated vector V' = R(θ) V.

Deriving the Standard Rotation Matrix for 120 Degrees

Step 1: Convert the Angle to Radians

Since trigonometric functions typically operate in radians, convert 120 degrees to radians:

$$ 120^\circ = \frac{120 \times \pi}{180} = \frac{2\pi}{3} $$

Step 2: Compute Cosine and Sine of 120 Degrees

Calculate the cosine and sine values:

$$ \cos \frac{2\pi}{3} = -\frac{1}{2} $$
$$ \sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2} $$

Step 3: Write the Rotation Matrix

Substituting these into the standard rotation matrix:

$$ R(120^\circ) = \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} $$

This matrix is the standard form representing a rotation of 120 degrees counterclockwise in R2.

Applying the Rotation to Vector V = (8, 8)

Step 1: Write the Vector as a Column Matrix

Express V as a column vector:

$$ V = \begin{bmatrix} 8 \\ 8 \end{bmatrix} $$

Step 2: Multiply the Rotation Matrix by the Vector

Perform matrix multiplication:

$$
V' = R(120^\circ) V = \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 8 \\ 8 \end{bmatrix}
$$

Compute component-wise:

- $x' = -\frac{1}{2} \times 8 + -\frac{\sqrt{3}}{2} \times 8 = -4 - 4 \sqrt{3}$
- $y' = \frac{\sqrt{3}}{2} \times 8 + -\frac{1}{2} \times 8 = 4 \sqrt{3} - 4$

Resulting Vector:

$$
V' = \left(-4 - 4 \sqrt{3}, \quad 4 \sqrt{3} - 4 \right)
$$

This is the image of V after a 120-degree counterclockwise rotation.

Interpreting the Results

The computed vector illustrates how the original point (8, 8) is transformed under the rotation:

- The x-component becomes more negative, influenced by both the original x-value and the rotation angle's sine component.
- The y-component involves a similar transformation, with the sine of 120 degrees amplifying the original x-value.

This transformation preserves magnitude and orientation relative to the origin, consistent with the properties of rotation transformations.

Additional Insights and Applications

Properties of Rotation Matrices

- Rotation matrices are orthogonal, meaning $R(θ)^T R(θ) = I$.
- They have determinants of 1, indicating they preserve area and orientation.
- Their inverses are their transposes: $R(θ)^{-1} = R(-θ)$.

Practical Applications

- Computer Graphics: Rotating objects or camera views.
- Robotics: Calculating joint rotations.
- Physics: Analyzing rotational motion.
- Engineering: Structural analysis involving rotational components.

Conclusion

In summary, the problem of applying a counterclockwise rotation of 120 degrees to a vector in R2 involves deriving the standard rotation matrix corresponding to that angle and then multiplying it by the vector. The standard form of the rotation matrix for 120 degrees is:

$$ R(120^\circ) = \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} $$

Applying this to the vector V = (8, 8) yields the transformed vector:

$$ V' = \left(-4 - 4 \sqrt{3}, \quad 4 \sqrt{3} - 4 \right) $$

Understanding these matrix transformations provides valuable insight into how rotation operations work in the plane, with broad applications across various scientific and engineering disciplines.

Frequently Asked Questions

What is the standard matrix for a counterclockwise rotation of 120 degrees in R2?

The standard matrix for a 120-degree counterclockwise rotation in R2 is
[[cos 120°, -sin 120°], [sin 120°, cos 120°]] which simplifies to
[[-0.5, -√3/2], [√3/2, -0.5]].

How do you apply a rotation matrix to a vector in R2?

To apply a rotation matrix to a vector in R2, multiply the matrix by the vector. For example, if R is the rotation matrix and V is the vector, then the new vector V' = R V.

Given V = (8, 8), how do you find its rotated image after a 120-degree rotation?

Multiply the rotation matrix for 120 degrees by the vector V = (8, 8). This yields the new coordinates:
V' = R V.

What is the result of applying a 120-degree counterclockwise rotation to the vector (8, 8)?

The rotated vector is approximately (8 -0.5 - 8 √3/2, 8 √3/2 - 8 0.5), which simplifies to roughly (-8, 4√3).

What does the phrase 'Consider The Following' imply in a mathematical problem?

It indicates that a problem or scenario is being introduced for analysis or calculation, prompting the reader to perform specific operations or reasoning based on the given data.

How is the standard matrix for rotation in R2 derived?

The standard matrix for rotation by an angle θ in R2 is derived from the unit circle definitions of sine and cosine, resulting in [[cos θ, -sin θ], [sin θ, cos θ]].

What are common applications of rotation matrices in linear algebra?

Rotation matrices are used in computer graphics, robotics, physics simulations, and any field involving coordinate transformations and spatial manipulations.

Additional Resources

Consider The Following. T Is The Counterclockwise Rotation Of 120° In R2, V = (8, 8). (a) Find The Standard

In the realm of linear algebra and vector transformations, understanding how rotation matrices operate in two-dimensional space (R2) is fundamental. Whether you're a student delving into the intricacies of coordinate transformations or a professional applying these concepts in computer graphics, robotics, or engineering, mastering the process of identifying and applying rotation matrices is crucial. Today, we explore a specific problem: given a vector V = (8, 8) and a rotation transformation T that rotates vectors counterclockwise by 120°, how do we find the standard matrix associated with T? Let's demystify this problem step-by-step, providing clarity and insight into the mathematics behind it.

Understanding Rotation in R2

Before diving into the specific problem, it's essential to understand the general principles of rotation matrices in two-dimensional space.

The Concept of Rotation

In R2, rotation refers to turning a vector around the origin by a specific angle, either clockwise or counterclockwise. When we specify that a rotation is counterclockwise by θ degrees, it means we're rotating the vector in the positive direction of the coordinate plane.

For example:
- A rotation of 90° counterclockwise transforms (x, y) into (-y, x).
- A rotation of 180° flips the vector to its negative, i.e., (x, y) becomes (-x, -y).

The Rotation Matrix

The standard matrix for a rotation by an angle θ (in degrees or radians) in R2 is derived from trigonometry. It is given by:

$$R(\theta) = \begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{bmatrix}$$

This matrix, when multiplied by a vector V, results in the vector V rotated counterclockwise by θ degrees.

Key points:
- The matrix is orthogonal, meaning its transpose is its inverse.
- It preserves lengths and angles, characteristic of isometric transformations.

Given Data and Objective

The problem provides:
- A rotation transformation T: counterclockwise by 120°.
- A vector V = (8, 8).

The question asks: Find the standard matrix for transformation T.

In essence, our goal is to determine the matrix $R(120^\circ)$ that, when applied to any vector, rotates it counterclockwise by 120°, and verify how it acts on the vector V.

Step-by-Step Solution Approach

Let's outline the method to find the standard matrix for T and its application to the vector V.

Step 1: Convert the Rotation Angle to Radians

Since trigonometric functions in mathematics typically operate in radians, convert 120° to radians:

$$
\theta = 120^\circ = \frac{120 \pi}{180} = \frac{2\pi}{3} \text{ radians}
$$

Step 2: Compute Cosine and Sine of 120°

Using known values:
- $\cos 120^\circ = -\frac{1}{2}$
- $\sin 120^\circ = \frac{\sqrt{3}}{2}$

Expressed in radians:
- $\cos \frac{2\pi}{3} = -\frac{1}{2}$
- $\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

Step 3: Write the Rotation Matrix

Plugging these into the standard rotation matrix:

$$ R(120^\circ) = \begin{bmatrix}
-\frac{1}{2} & -\frac{\sqrt{3}}{2} \\
\frac{\sqrt{3}}{2} & -\frac{1}{2}
\end{bmatrix} $$

This matrix represents the linear transformation T.

Step 4: Apply the Rotation Matrix to Vector V

To verify the transformation, multiply R(120°) by V = (8, 8):

$$
V' = R(120^\circ) \times \begin{bmatrix}8 \\ 8\end{bmatrix}
$$

Calculations:
- First component:

$$
x' = -\frac{1}{2} \times 8 + -\frac{\sqrt{3}}{2} \times 8 = -4 - 4 \sqrt{3}
$$

- Second component:

$$
y' = \frac{\sqrt{3}}{2} \times 8 + -\frac{1}{2} \times 8 = 4 \sqrt{3} - 4
$$

Thus, the rotated vector is:

$$
V' = \left(-4 - 4 \sqrt{3}, \quad 4 \sqrt{3} - 4 \right)
$$

This confirms how the vector transforms under T.

Implications and Applications of the Rotation Matrix

Understanding the standard rotation matrix has broad implications in numerous fields.

Computer Graphics and Animation

In rendering scenes, objects are often rotated to achieve realistic perspectives. The rotation matrix provides a straightforward way to perform these transformations mathematically.

Robotics and Kinematics

Robots operating in 2D spaces need to rotate their arms or sensors efficiently. Applying rotation matrices ensures precise control and movement.

Engineering and Physics

Analyzing forces, motion, and dynamics often involves rotating coordinate systems or vectors, making the rotation matrix an essential tool.

Signal Processing

Rotations can be used in phase shifts and Fourier transforms—fundamental in signal analysis.

Summary and Key Takeaways

- The standard rotation matrix in R2 for an angle θ is:

\[
R(\theta) = \begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{bmatrix}
\]

- For a counterclockwise rotation of 120°, the matrix becomes:

\[
R(120^\circ) = \begin{bmatrix}

-\frac{1}{2} & -\frac{\sqrt{3}}{2} \\
\frac{\sqrt{3}}{2} & -\frac{1}{2}
\end{bmatrix}
\]

- Applying this matrix to a vector yields its rotated position in the plane.

- The mathematical process involves converting angles to radians, calculating trigonometric functions, constructing the matrix, and performing matrix-vector multiplication.

Final Remarks

Mastering the process of deriving and applying rotation matrices is foundational in understanding and executing transformations in two-dimensional space. Whether for theoretical purposes or practical applications, the ability to manipulate vectors through these matrices enables precise control of spatial transformations. The example of rotating the vector V = (8, 8) by 120° encapsulates a core concept in linear algebra—how simple matrix operations underpin complex spatial manipulations.

By grasping these principles, you're equipped to approach a wide array of problems involving rotations, from computer graphics to mechanical engineering, with confidence and mathematical rigor.

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