Consider The ODEdy/dx = (y/x) +x^2(a) Find Two Particular Solutions, One For Each Of The Following Initial

Consider The ODE dy/dx = (y/x) + x^2. Find Two Particular Solutions, One For Each Of The Following Initial Conditions

Understanding how to solve differential equations is fundamental in mathematics, especially in fields like physics, engineering, and applied sciences. Among the various types of differential equations, first-order linear differential equations are particularly common. In this article, we will delve into the process of solving the differential equation:

$$ \frac{dy}{dx} = \frac{y}{x} + x^2 $$

and focus on finding two particular solutions corresponding to different initial conditions. This exploration will not only enhance your problem-solving skills but also deepen your understanding of methods such as integrating factors, substitution, and solving initial value problems.

Understanding the Differential Equation $\frac{dy}{dx} = \frac{y}{x} + x^2$

Before jumping into solutions, it’s crucial to analyze the structure of the differential equation and determine the most suitable methods for solving it.

Recognizing the Type of Differential Equation

The given differential equation:

$$ \frac{dy}{dx} = \frac{y}{x} + x^2 $$

can be rewritten as:

$$ \frac{dy}{dx} - \frac{1}{x} y = x^2 $$

This form reveals that it is a first-order linear differential equation. The standard form for such equations is:

$$ \frac{dy}{dx} + P(x) y = Q(x) $$

where:

- $P(x) = -\frac{1}{x}$
- $Q(x) = x^2$

Recognizing this allows us to apply the integrating factor method to find the general solution.

Initial Conditions and Their Significance

The problem asks us to find two particular solutions, each satisfying different initial conditions. An initial condition specifies the value of y at a particular point $x = x_0$, typically written as:

$$y(x_0) = y_0$$

Having these conditions helps us determine the arbitrary constant in the general solution, thus identifying specific solutions relevant to real-world scenarios.

Solving the Differential Equation: Step-by-Step Approach

Now, let's systematically solve the differential equation using the integrating factor method, and then discuss how initial conditions influence the particular solutions.

Step 1: Write the Equation in Standard Linear Form

As previously identified:

$$\frac{dy}{dx} - \frac{1}{x} y = x^2$$

Step 2: Find the Integrating Factor (IF)

The integrating factor $\mu(x)$ is given by:

$$\mu(x) = e^{\int P(x) dx}$$

where $P(x) = -\frac{1}{x}$. Therefore:

$$\mu(x) = e^{\int -\frac{1}{x} dx} = e^{-\ln|x|} = |x|^{-1}$$

Since the equation involves x, and solutions are typically considered for $x > 0$ or \(

$x < 0$), we can simplify:

$$\mu(x) = \frac{1}{x}$$

(hoping that the domain is such that $x \neq 0$).

Step 3: Multiply Through by the Integrating Factor

Multiplying the entire differential equation by $\mu(x) = \frac{1}{x}$:

$$\frac{1}{x} \frac{dy}{dx} - \frac{1}{x^2} y = x$$

Observe that the left side simplifies to the derivative of $y \times$ the integrating factor:

$$\frac{d}{dx} \left(\frac{y}{x} \right) = x$$

This step is crucial because it transforms the differential equation into an easily integrable form.

Step 4: Integrate Both Sides

Integrate both sides with respect to x:

$$\int \frac{d}{dx} \left(\frac{y}{x} \right) dx = \int x dx$$

which simplifies to:

$$\frac{y}{x} = \frac{x^2}{2} + C$$

where C is an arbitrary constant of integration.

Step 5: Solve for y

Multiply both sides by x:

$$y = x \left(\frac{x^2}{2} + C \right) = \frac{x^3}{2} + Cx$$

This is the general solution to the differential equation.

Finding Particular Solutions with Given Initial Conditions

The general solution contains an arbitrary constant C. To find particular solutions, we need specific initial conditions, such as:

- $y(x_0) = y_0$

Suppose the two initial conditions are:

1. $y(1) = 2$
2. $y(2) = 5$

We will use these to find the particular solutions.

Particular Solution 1: $y(1) = 2$

Plugging into the general solution:

$$2 = \frac{(1)^3}{2} + C \times 1$$
$$2 = \frac{1}{2} + C$$
$$C = 2 - \frac{1}{2} = \frac{3}{2}$$

Thus, the particular solution is:

$$y = \frac{x^3}{2} + \frac{3}{2} x$$

Particular Solution 2: $y(2) = 5$

Plug into the general solution:

$$5 = \frac{(2)^3}{2} + C \times 2$$
$$5 = \frac{8}{2} + 2 C$$
$$5 = 4 + 2 C$$
$$2 C = 1$$
$$C = \frac{1}{2}$$

Correspondingly, the particular solution is:

$$y = \frac{x^3}{2} + \frac{1}{2} x$$

Interpreting and Applying the Solutions

The solutions derived above are tailored to specific initial conditions, illustrating how initial data influence the particular solution within the family of solutions.

Applications of These Solutions

- Physics: Modeling motion where initial position and velocity are known.
- Engineering: Calculating system responses given initial states.
- Economics: Forecasting trends with initial investment conditions.

Key Takeaways

- The general solution to the differential equation is $y = \frac{x^3}{2} + Cx$.
- Specific initial conditions determine the value of C.
- The process involves recognizing the type of equation, applying the integrating factor, and solving for the particular solution.

Additional Insights and Tips for Solving Similar Differential Equations

To master solving first-order linear differential equations like this one, consider the following tips:

1. **Identify the form:** Always check if the differential equation can be written in the standard form $dy/dx + P(x) y = Q(x)$.

2. **Calculate the integrating factor:** Use $\mu(x) = e^{\int P(x) dx}$, being mindful of domain restrictions.

3. **Transform the equation:** Multiply through by the integrating factor to simplify into an exact derivative.

4. **Integrate carefully:** Integrate both sides with respect to x, keeping track of the constant of integration.

5. **Apply initial conditions:** Substitute x_0 and y_0 to find the particular solution.

Conclusion

Solving the differential equation $\frac{dy}{dx} = \frac{y}{x} + x^2$ exemplifies the power of the integrating factor method for first-order linear equations. By transforming the equation into an integrable form, we derived the general solution $y = \frac{x^3}{2} + Cx$. Applying specific initial conditions allowed us to determine particular solutions, each relevant to different initial states.

Understanding this process enhances your toolkit for tackling a wide array of differential equations encountered in scientific and engineering contexts. Practice with various initial conditions and equations will deepen your mastery and confidence in solving differential equations efficiently and accurately.

Keywords: differential equation, first-order linear, integrating factor, particular solution, initial condition, solving differential equations, ODEs

Frequently Asked Questions

How do you find particular solutions to the differential equation dy/dx = (y/x) + x^2 with given initial conditions?

To find particular solutions, first solve the differential equation generally, then substitute the initial conditions to determine the specific constants, resulting in the particular solutions.

What method can be used to solve the differential equation dy/dx = (y/x) + x^2 for different initial values?

This equation can be solved using an integrating factor or substitution methods, such as setting v = y/x to transform it into a linear differential equation, then applying the initial conditions to find particular solutions.

Can you provide an example of a particular solution for dy/dx = (y/x) + x^2 with initial condition y(1)=k?

Yes, by solving the differential equation generally and then substituting x=1 and y= k, you can find a particular solution. For example, if initial condition is y(1)=2, you substitute to determine the constant, resulting in a specific solution.

What challenges might arise when finding solutions to dy/dx = (y/x) + x^2 with different initial conditions?

The main challenge is solving the nonhomogeneous linear differential equation and accurately applying initial conditions to determine the particular solutions, especially if the integral solutions involve complex functions or constants.

How does the initial condition influence the particular solutions of the differential equation dy/dx = (y/x) + x^2?

The initial condition provides the specific value needed to determine the integration constant in the general solution, thus customizing the solution to fit the initial value, resulting in a unique particular solution for each initial condition.

Additional Resources

Differential Equations: Unlocking the Secrets of \(\frac{dy}{dx} = \frac{y}{x} + x^2 \)

Introduction

When exploring the fascinating world of differential equations, one encounters a variety of challenges and techniques that reveal the underlying behaviors of complex systems. Among these, first-order ordinary differential equations (ODEs) stand out for their relative simplicity yet profound applications across physics, engineering, and mathematical modeling.

Today, we focus on a specific first-order linear ODE:

\[
\frac{dy}{dx} = \frac{y}{x} + x^2
\]

This equation elegantly combines a linear term involving \(y \) with a nonhomogeneous component \(x^2 \). Our goal? To find two particular solutions, each corresponding to different initial conditions, illustrating the depth and flexibility of solving such equations.

Understanding the Equation: Structure and Significance

The Equation at a Glance

The differential equation:

\[

\frac{dy}{dx} = \frac{y}{x} + x^2
\]

can be rewritten to better understand its structure:

\[
\frac{dy}{dx} - \frac{1}{x} y = x^2
\]

This form reveals that it is a linear first-order differential equation in the standard form:

\[
\frac{dy}{dx} + P(x) y = Q(x)
\]

where:

- \(P(x) = -\frac{1}{x} \)
- \(Q(x) = x^2 \)

The linear form is advantageous because it allows us to employ the integrating factor method, a powerful technique for solving such equations systematically.

Why is this important?

Understanding the structure helps in:

- Simplifying the problem
- Applying the correct solution method
- Interpreting the physical or geometrical significance of solutions

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Step-by-Step Solution Strategy

1. Standardizing the Equation

Express the differential equation as:

\[
\frac{dy}{dx} + P(x) y = Q(x)
\]

with:

\[
P(x) = -\frac{1}{x}
\]
\[
Q(x) = x^2
\]

Note that the domain excludes $x=0$ because the coefficient $\frac{1}{x}$ becomes undefined there.

2. Computing the Integrating Factor

The integrating factor $\mu(x)$ is given by:

$$
\mu(x) = e^{\int P(x) dx} = e^{\int -\frac{1}{x} dx} = e^{-\ln|x|} = \frac{1}{|x|}
$$

For simplicity and since we're dealing with positive x (or considering the domain accordingly), we take:

$$
\mu(x) = \frac{1}{x}
$$

3. Multiplying Through by the Integrating Factor

Multiply the entire differential equation by $\mu(x) = \frac{1}{x}$:

$$
\frac{1}{x} \frac{dy}{dx} - \frac{1}{x^2} y = x
$$

Observe that the left side is the derivative of $y \times \frac{1}{x}$:

$$
\frac{d}{dx} \left(\frac{y}{x} \right) = x
$$

This step leverages the product rule in reverse, confirming that the integrating factor simplifies the equation.

Finding the General Solution

1. Integrate Both Sides

Integrate with respect to x:

$$
\frac{y}{x} = \int x \, dx + C
$$

where C is the constant of integration.

Calculate the integral:

\[
\int x \, dx = \frac{x^2}{2}
\]

So,

\[
\frac{y}{x} = \frac{x^2}{2} + C
\]

2. Solve for \(y \)

Multiply through by \(x \):

\[
y = x \left(\frac{x^2}{2} + C \right) = \frac{x^3}{2} + C x
\]

This is the general solution to the differential equation:

\[
\boxed{
y(x) = \frac{x^3}{2} + C x
}
\]

Finding Two Particular Solutions

The general solution contains an arbitrary constant \(C \). To find particular solutions, we need initial conditions, which specify the value of \(y \) at specific \(x \)-values. Let's explore two different scenarios.

Particular Solution 1: Initial Condition \(y(1) = 2 \)

1. Plug in the initial condition:

\[
y(1) = \frac{1^3}{2} + C \times 1 = \frac{1}{2} + C = 2
\]

2. Solve for \(C \):

\[
C = 2 - \frac{1}{2} = \frac{4}{2} - \frac{1}{2} = \frac{3}{2}
\]

3. Write the particular solution:

$$
\boxed{
y(x) = \frac{x^3}{2} + \frac{3}{2} x
}
$$

Interpretation: This solution describes the specific trajectory of the system when the initial value at $x=1$ is 2. Its behavior is dominated by the cubic term $\frac{x^3}{2}$, with a linear adjustment based on the initial condition.

Particular Solution 2: Initial Condition $y(2) = 5$

1. Plug in the initial condition:

$$
y(2) = \frac{(2)^3}{2} + C \times 2 = \frac{8}{2} + 2 C = 4 + 2 C = 5
$$

2. Solve for C:

$$
2 C = 5 - 4 = 1 \Rightarrow C = \frac{1}{2}
$$

3. Write the particular solution:

$$
\boxed{
y(x) = \frac{x^3}{2} + \frac{1}{2} x
}
$$

Interpretation: Different initial conditions lead to different particular solutions, each reflecting unique starting points within the solution family. Despite the similarity in form, the solutions diverge based on the constant C.

Key Insights and Practical Applications

1. The Role of Initial Conditions

The ability to tailor solutions to specific initial conditions makes these equations invaluable in modeling real-world phenomena — from physics to biology. For example, in heat transfer, population dynamics, or electrical circuits, initial states dictate the evolution of the system, much like our particular solutions.

2. The Significance of the General Solution

The general solution acts as a "solution family," encompassing all possible solutions that satisfy the differential equation. Particular solutions are specific members chosen based on initial conditions, offering precise predictions or descriptions.

3. Mathematical Flexibility and Robustness

The method of integrating factors exemplifies the robustness of linear differential equation solutions. Its systematic nature ensures that, once the integrating factor is known, solutions can be derived efficiently for a wide class of equations.

Extending the Approach: Broader Context and Techniques

While the specific equation we examined is manageable with elementary techniques, more complex differential equations may require advanced methods such as:

- Variation of parameters
- Exact equations
- Substitutions and transformations
- Numerical methods for equations not amenable to closed-form solutions

Recognizing the linear form and employing the integrating factor remains a foundational skill, empowering practitioners to approach a range of problems with confidence.

Final Thoughts: The Power of Analytical Solutions

The process of solving $\frac{dy}{dx} = \frac{y}{x} + x^2$ underscores the elegance and utility of analytical methods in differential equations. By systematically transforming the problem, integrating, and applying initial conditions, we obtain explicit solutions that illuminate the behavior of the system.

Whether modeling physical phenomena or serving as a pedagogical example, these solutions demonstrate how mathematical rigor translates into practical insights — a true testament to the enduring power of differential equations in science and engineering.

Summary at a Glance:

Step	Description	Result
1	Convert to standard linear form	$\frac{dy}{dx} - \frac{1}{x} y = x^2$
2	Compute integrating factor	$\mu(x) = \frac{1}{x}$
3	Multiply through by $\mu(x)$	$\frac{d}{dx} \left(\frac{y}{x} \right) = x$
4	Integrate both sides	$\frac{y}{x} = \frac{x^2}{2} + C$
5	Solve for y	$y = \frac{x^3}{2} + C x$
6	Apply initial conditions	Find particular C for specific $y(x_0)$

Through this detailed exploration, we've demonstrated not only the solution process

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