

Dante Is Solving The System Of Equations Below. He Writes The Row Echelon Form Of The Matrix. Which Matrix

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Which Matrix is a common scenario encountered by students and professionals working with systems of linear equations. Understanding this process is fundamental in linear algebra, as it allows for efficient solutions to complex systems. In this article, we will explore the steps Dante takes to convert a matrix into its row echelon form, the significance of this form, and how to identify the original matrix from the row echelon form. Whether you're a student preparing for exams or a math enthusiast interested in matrix transformations, this guide will clarify the concept and application of row echelon forms in solving systems of equations.

Understanding the System of Equations and Matrices

What Is a System of Equations?

A system of equations consists of multiple linear equations involving the same set of variables. For example:

- $2x + 3y - z = 5$
- $-x + 4y + 2z = 6$
- $3x - y + z = 4$

Solving such systems involves finding values for the variables that satisfy all equations simultaneously.

Representing Systems with Matrices

Matrices provide a structured way to handle systems of equations. The coefficients of the variables are placed into a matrix called the coefficient matrix, and the constants form the augmented matrix:

- Coefficient matrix: a 3×3 matrix containing only the coefficients.

- Augmented matrix: includes the coefficients and the constants from each equation.

For the above system, the augmented matrix is:

```
\[
\begin{bmatrix}
2 & 3 & -1 & | & 5 \\
-1 & 4 & 2 & | & 6 \\
3 & -1 & 1 & | & 4
\end{bmatrix}
```

Dante's goal is to convert this matrix into row echelon form to simplify the process of solving for the variables.

What Is Row Echelon Form?

Definition and Characteristics

A matrix is in row echelon form if it satisfies the following conditions:

- All non-zero rows are above any rows of all zeros.
- The leading coefficient (pivot) of a non-zero row is always to the right of the leading coefficient of the row above it.
- All entries in a column below a pivot are zeros.

This form makes it straightforward to perform back-substitution to find variable values.

Why Is Row Echelon Form Important?

Transforming a matrix into row echelon form simplifies the process of solving the system. It reduces the problem to a sequence of easier steps, often leading quickly to solutions or indicating if the system has no solutions or infinitely many solutions.

Dante's Process: Converting a Matrix to Row Echelon Form

Step 1: Write the Augmented Matrix

Dante begins with the augmented matrix representing the system:

```
\[
\begin{bmatrix}
2 & 3 & -1 & | & 5 \\
-1 & 4 & 2 & | & 6 \\
3 & -1 & 1 & | & 4
\end{bmatrix}
```

Step 2: Make the First Pivot a 1

To facilitate calculations, Dante prefers to have a leading 1 in the first row, first column. He performs row operations:

- Divide the first row by 2:

```
\[
R_1 \leftarrow \frac{1}{2} R_1
```

Resulting in:

```
\[
\begin{bmatrix}
1 & 1.5 & -0.5 & | & 2.5 \\
-1 & 4 & 2 & | & 6 \\
3 & -1 & 1 & | & 4
\end{bmatrix}
```

Step 3: Zero Out Elements Below the Pivot

Next, Dante eliminates the entries below the pivot in the first column:

- For row 2:

$$\begin{aligned} & \\ R_2 & \leftarrow R_2 + R_1 \\ & \end{aligned}$$

- For row 3:

$$\begin{aligned} & \\ R_3 & \leftarrow R_3 - 3 R_1 \\ & \end{aligned}$$

Calculations:

$$\begin{aligned} & \\ R_2: & (-1) + 1 = 0, \quad 4 + 1.5 = 5.5, \quad 2 + (-0.5) = 1.5, \quad 6 + 2.5 = 8 \\ & \end{aligned}$$

$$\begin{aligned} & \\ R_3: & 3 - 3(1) = 0, \quad -1 - 3(1.5) = -1 - 4.5 = -5.5, \quad 1 - 3(-0.5) = 1 + 1.5 = 2.5, \quad 4 - 3(2.5) = 4 - 7.5 = -3.5 \\ & \end{aligned}$$

The matrix becomes:

$$\begin{aligned} & \\ & \begin{bmatrix} 1 & 1.5 & -0.5 & | & 2.5 \\ 0 & 5.5 & 1.5 & | & 8 \\ 0 & -5.5 & 2.5 & | & -3.5 \end{bmatrix} \\ & \end{aligned}$$

Step 4: Make the Second Pivot a 1

Divide row 2 by 5.5:

$$\begin{aligned} & \\ R_2 & \leftarrow \frac{1}{5.5} R_2 \\ & \end{aligned}$$

Yielding:

$$\begin{aligned} & \\ & 0 \quad 1 \quad \frac{1.5}{5.5} \quad | \quad \frac{8}{5.5} \end{aligned}$$

\]

Simplify fractions:

\[

$$\frac{1.5}{5.5} = \frac{3/2}{11/2} = \frac{3/2 \times 2/11}{11} = \frac{3}{11}$$

\]

\[

$$\frac{8}{5.5} = \frac{8}{11/2} = 8 \times \frac{2}{11} = \frac{16}{11}$$

\]

Resulting in:

\[

$$\begin{bmatrix} 1 & 1.5 & -0.5 & | & 2.5 \\ 0 & 1 & \frac{3}{11} & | & \frac{16}{11} \\ 0 & -5.5 & 2.5 & | & -3.5 \end{bmatrix}$$

\]

Step 5: Zero Out Elements Below the Second Pivot

Eliminate the entry in row 3, column 2:

$$-R_3 + 5.5 R_2:$$

Calculate:

\[

$$R_3: 0 + 5.5 \times 0 = 0$$

\]

\[

$$-5.5 + 5.5 \times 1 = 0$$

\]

\[

$$2.5 + 5.5 \times \frac{3}{11} = 2.5 + 5.5 \times \frac{3}{11} = 2.5 + \frac{5.5 \times 3}{11} = 2.5 + \frac{16.5}{11} = 2.5 + 1.5 = 4$$

\]

\[

$$-3.5 + 5.5 \times \frac{16}{11} = -3.5 + \frac{5.5 \times 16}{11} = -3.5 + \frac{88}{11} = -3.5 + 8 = 4.5$$

\]

The new row 3:

$$\left[\begin{array}{ccc|c} 0 & 0 & 4 & 4.5 \end{array} \right]$$

The matrix in row echelon form:

$$\begin{bmatrix} 1 & 1.5 & -0.5 & | & 2.5 \\ 0 & 1 & \frac{3}{11} & | & \frac{16}{11} \\ 0 & 0 & 4 & | & 4.5 \end{bmatrix}$$

This matrix is now in row echelon form, with zeros below each pivot.

Identifying the Original Matrix from Row Echelon Form

From Row Echelon Form to the Original Matrix

The row echelon form primarily shows the structure of the matrix after transformations. To trace back to the original matrix:

- Remember that row operations are reversible.
- By reversing the sequence of operations, you can recover the original matrix.
- Typically, the initial coefficient matrix is the part of the augmented matrix before any row operations are performed.

Which Matrix Corresponds to the Final Row Echelon Form?

The matrix Dante ends up with is a transformed version of the original coefficient matrix.

Frequently Asked Questions

What is the purpose of converting a matrix to row echelon form when solving a system of equations?

Converting a matrix to row echelon form simplifies the system, making it easier to perform back substitution and find the solutions efficiently.

Which matrix operation is primarily used to transform a matrix into row echelon form?

The primary operation is Gaussian elimination, which involves row operations like swapping rows, multiplying a row by a non-zero scalar, and adding multiples of one row to another.

How can you identify the row echelon form of a matrix after applying row operations?

The matrix in row echelon form has zeros below the leading entries (pivots) in each row, and each leading entry is to the right of the leading entry in the row above.

In solving the system of equations, what does the row echelon form reveal about the solution set?

It helps determine whether the system has a unique solution, infinitely many solutions, or no solution by exposing the presence of inconsistent or dependent equations.

What is the significance of identifying the matrix in row echelon form during system solving?

It provides a clear pathway to perform back substitution to find the variable values, streamlining the process of solving the system efficiently.

Additional Resources

Dante is solving the system of equations below. He writes the row echelon form of the matrix. Which matrix? This question invites a detailed exploration of the process of solving systems of equations through matrix methods, specifically focusing on the transformation into row echelon form. The problem underscores the importance of understanding how matrices are manipulated to find solutions efficiently, and analyzing what the original matrix could be based on the given steps is fundamental in linear algebra.

This article will delve into the methods Dante might be using, the significance of row echelon form, and how to identify the original matrix from this form, providing a comprehensive overview suitable for students, educators, and enthusiasts alike.

Understanding the System of Equations and Matrices

Before examining the row echelon form, it's crucial to understand the foundational concepts of systems of equations and their matrix representations.

Systems of Equations

A system of linear equations consists of multiple equations involving the same set of variables. For example:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \end{cases}$$

This can be represented in matrix form as $A\mathbf{x} = \mathbf{b}$, where:

- A is the coefficient matrix,
- $\mathbf{x}$ is the vector of variables,
- $\mathbf{b}$ is the constants vector.

Matrix Representation and Operations

The matrix representation simplifies manipulation, especially when applying methods like Gaussian elimination. The key steps involve row operations:

- Swapping rows,
- Multiplying a row by a non-zero scalar,
- Adding a multiple of one row to another.

These operations aim to convert the matrix into an echelon form, simplifying the solution process.

What is Row Echelon Form?

Row echelon form (REF) is a specific matrix form achieved after applying elementary row operations. It has the following characteristics:

- All zero rows, if any, are at the bottom.
- The leading coefficient (pivot) of a non-zero row is always to the right of the leading coefficient of the row above.
- The leading coefficients are typically non-zero, and often normalized to 1 in reduced row echelon form.

Significance of Row Echelon Form

Transforming a matrix into REF simplifies solving the system:

- It makes back-substitution straightforward.
- It reveals whether the system has a unique solution, infinitely many solutions, or none.

For example, consider a generic 3x4 augmented matrix in REF:

```
\[
\begin{bmatrix}
1 & & & | & \\
0 & 1 & & | & \\
0 & 0 & 1 & | & 
\end{bmatrix}
```

This form indicates a clear path to solving for the variables.

Reconstructing the Original Matrix from Row Echelon Form

Given that Dante has achieved the row echelon form of a matrix derived from a system of equations, a natural question arises: which original matrix could this be? Understanding this process involves analyzing the transformations and their reversibility.

Elementary Row Operations and Their Effect on the Original Matrix

Elementary row operations are invertible, which means:

- The original matrix can be recovered if the sequence of operations is known.
- The row echelon form is a product of applying these operations to the original matrix.

So, if Dante's final matrix is in row echelon form, the original matrix was obtained by applying inverse operations in reverse order.

Inferring the Original Matrix

If the row echelon form is known, and the sequence of transformations is documented, one can:

- Reverse the row operations to reconstruct the initial matrix.
- Alternatively, analyze the pattern of the echelon form to deduce possible original matrices.

Note: Without the specific row echelon matrix or the sequence of transformations, it's impossible to determine the exact original matrix. However, certain features can help infer its general structure.

Features of Matrices Leading to the Row Echelon Form

When examining a row echelon form, several features help identify characteristics of the original matrix:

- Number of pivots: Indicates the rank and possible number of solutions.
- Position of leading entries: Helps infer the original arrangement of coefficients.
- Zero rows: Signal dependent equations or redundant information.
- Pattern of zeros: The pattern can suggest particular structures, such as upper-triangular matrices.

Examples and Analysis

Let's analyze some common scenarios, providing insight into how the original matrices relate to their row echelon forms.

Example 1: Full Rank Square Matrix

Suppose Dante's row echelon form is:

$\backslash$

```

\begin{bmatrix}
1 & 2 & 3 \\
0 & 1 & 4 \\
0 & 0 & 1
\end{bmatrix}

```

This indicates the original matrix was full rank and likely invertible. The process to obtain this form involved:

- Using Gaussian elimination.
- Normalizing leading coefficients to 1.

The original matrix could have been:

```

\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}

```

with specific entries, but the form above indicates the original had non-zero leading entries in each row.

Pros:

- Easy to solve via back-substitution.
- System has a unique solution.

Cons:

- Doesn't explicitly show the original coefficients unless the elimination steps are tracked.

Example 2: Matrix with Zero Rows

Suppose the row echelon form has:

```

\begin{bmatrix}
1 & 3 & 0 & | & 2 \\
0 & 0 & 1 & | & -1
\end{bmatrix}

```

This indicates the original matrix had dependent equations leading to zero rows after elimination.

Implication:

- The original matrix likely had some rows that were linear combinations of others.
- The system may have infinitely many solutions or be inconsistent if augmented with contradictory constants.

Pros and Cons of the Row Echelon Approach

Pros:

- Simplifies complex systems into manageable steps.
- Clearly shows the rank and solvability.
- Facilitates computational algorithms in software.

Cons:

- Can be computationally intensive for large matrices.
- The process may obscure the original coefficients if steps are not documented.
- Not always straightforward to reverse engineer the original matrix without detailed operations.

Practical Applications and Significance

Understanding how to derive and interpret the row echelon form is vital in various fields:

- Engineering: Solving systems in circuit analysis.
- Computer Science: Algorithms for linear algebra computations.
- Data Science: Dimensionality reduction techniques.
- Mathematics Education: Teaching systems of equations and matrix operations.

Dante's process exemplifies fundamental linear algebra techniques that are central to these disciplines.

Conclusion

Dante's work in solving the system of equations by transforming its matrix into row echelon form is a classic example of linear algebra in action. The key takeaway is that the row echelon form serves as a bridge from complex systems to straightforward solutions. Reconstructing the original matrix from this form involves understanding the elementary row operations and their reversibility. While the exact original matrix depends on the specific steps taken during elimination, analyzing the features of the echelon form provides insights into the structure and properties of the initial matrix. This process not only facilitates solving systems efficiently but also deepens comprehension of the underlying linear relationships, making it an essential skill in mathematical problem-solving and applied sciences.

Final thoughts: Whether you're a student learning about matrices for the first time or a professional applying these techniques in real-world scenarios, mastering the process of converting matrices to row echelon form and understanding their origins is invaluable. Dante's example underscores the importance of systematic methods and the power of linear algebra in deciphering complex systems.

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