Denoting By The Position Vector Of A Point Of A Rigid Slab That Is In Plane Motion, Show That The Position

Denoting By The Position Vector Of A Point Of A Rigid Slab That Is In Plane Motion, Show That The Position of any point on the slab can be expressed in terms of the position vector of a reference point and the rotation of the slab about a fixed point. This fundamental concept in rigid body kinematics enables us to analyze the motion of planar bodies efficiently. In this article, we will explore how the position vector of a point on a rigid slab undergoing plane motion can be represented, deriving the necessary relations and understanding their significance in mechanical analysis.

Introduction to Plane Motion of Rigid Bodies

Before delving into the specifics of the position vector, it is essential to understand the nature of plane motion and the characteristics of rigid bodies in such motion.

Definition of Plane Motion

Plane motion refers to the movement of a body restricted to a two-dimensional plane. All points on the body move parallel to this plane, and the motion can be described by translations and rotations within that plane.

Characteristics of Rigid Body Motion

A rigid body is an idealized solid whose size and shape do not change during motion. The key assumption is that the distances between any two points on the body remain constant over time.

Kinematic Description of Rigid Body Motion

The motion of a rigid body in a plane can be characterized by:

- Translational motion of a reference point
- Rotational motion about that reference point

By combining these two components, the entire body's motion can be described.

Reference Point and Reference Frame

Select a fixed point \(O \) on the body, often called the reference or origin point. The position vector of this point, denoted as \(\vec{r}_O(t) \), provides a reference for describing the motion.

Rotation About a Fixed Point

The body may rotate about point \(O \) with an angular displacement \(\theta(t) \), which describes how the body turns relative to a fixed coordinate system.

Expressing the Position Vector of a Point on the Rigid Slab

Suppose we have a rigid slab moving in a plane. We are interested in the position vector of an arbitrary point \(P \) on the slab.

Position Vector of a Point \(P \)

Let:

- \(\vec{r}_P(t) \) be the position vector of point \(P \) at time \(t \)
- \(\vec{r}_O(t) \) be the position vector of the reference point \(O \) (fixed on the slab)
- \(\vec{r}_{P/O} \) be the position vector of \(P \) relative to \(O \)

The key is to relate \(\vec{r}_P(t) \) to these quantities.

Derivation of the Position Vector Relation

Because the body is rigid, the vector \(\vec{r}_{P/O} \) remains fixed in the body's frame, but changes orientation as the body rotates.

- In the fixed coordinate system, the position vector of \(P \) can be written as:

\[
\boxed{
\vec{r}_P(t) = \vec{r}_O(t) + \mathbf{R}(\theta(t)) \, \vec{r}_{P/O}
}
\]

where:

- $\mathbf{R}(\theta(t))$ is the rotation matrix corresponding to the angle $\theta(t)$

- $\vec{r}_{P/O}$ is the position vector of P relative to O in the body-fixed coordinate system

This relation shows that the position vector of any point P is composed of:

1. The position vector of the reference point O, which accounts for the translation
2. The rotated position vector $\vec{r}_{P/O}$, which accounts for the rotation of the body

Rotation Matrix in Plane Motion

The rotation matrix $\mathbf{R}(\theta)$ for planar motion is:

$$
\mathbf{R}(\theta) =
\begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{bmatrix}
$$

Applying this matrix to $\vec{r}_{P/O} = (x_{P/O}, y_{P/O})$ gives the rotated coordinates.

Expressing the Position Vector in Components

Suppose:

- $\vec{r}_O(t) = (x_O(t), y_O(t))$
- $\vec{r}_{P/O} = (x_{P/O}, y_{P/O})$

Then:

$$
\vec{r}_P(t) = \left(x_O(t) + x_{P/O} \cos \theta(t) - y_{P/O} \sin \theta(t), \quad y_O(t) + x_{P/O} \sin \theta(t) + y_{P/O} \cos \theta(t) \right)
$$

This explicit form is useful for analyzing the motion of the point P.

Implications and Applications

Understanding the relation between the position vector of a point on a rigid body and the motion parameters has numerous applications:

- Kinematic analysis of mechanisms and linkages
- Velocity and acceleration determination for points on moving bodies
- Design and control of mechanical systems such as robotic arms
- Structural analysis involving rigid slabs and plates

Velocity of a Point P

Differentiating $\vec{r}_P(t)$ with respect to time gives the velocity:

$$
\vec{v}_P(t) = \frac{d \vec{r}_P}{dt} = \vec{v}_O(t) + \boldsymbol{\omega}(t) \times \vec{r}_{P/O}
$$

where:

- $\boldsymbol{\omega}(t)$ is the angular velocity vector, which in plane motion is perpendicular to the plane

- The cross product in 2D simplifies to a rotation by 90°

Note: The above relation emphasizes that the velocity of any point on the body is composed of the translational velocity of the reference point plus a rotational component proportional to the angular velocity.

Conclusion

In summary, the position vector of a point on a rigid slab undergoing plane motion can be succinctly expressed as the sum of the position vector of a fixed point on the body and the rotated position vector of the point relative to that fixed point. This fundamental relation encapsulates the essence of rigid body kinematics in a plane, providing a powerful tool for analyzing and designing mechanical systems. By understanding and applying this concept, engineers and physicists can predict the motion of complex structures, optimize their performance, and ensure their structural integrity under various operational conditions.

Frequently Asked Questions

What is the significance of the position vector of a point in a rigid slab undergoing plane motion?

The position vector of a point in a rigid slab provides its location relative to a fixed reference point, allowing analysis of the slab's overall motion, deformation, and kinematic behavior during plane motion.

How do you express the position vector of a point in a rigid slab in plane motion?

The position vector can be expressed as the sum of the position vector of a reference point (often a fixed point or a point on the slab) plus the vector from that reference point to the point of interest, which accounts for translation and rotation in the plane.

How can the position vector be used to demonstrate the rigidity of a slab in plane motion?

By showing that the relative position vectors between points on the slab remain constant over time, it confirms that the distances between points do not change, thus demonstrating the rigidity of the slab during plane motion.

What is the role of rotational motion in expressing the position vector of a point on a rigid slab?

Rotational motion introduces an angular displacement that affects the position vector, typically represented using rotation matrices or angular velocity, to relate the initial position to the current position in the plane.

How do you derive the velocity and acceleration of a point on a rigid slab using its position vector?

By differentiating the position vector with respect to time, the velocity is obtained; differentiating the velocity yields the acceleration. These derivatives incorporate translational velocity and rotational components due to angular velocity and angular acceleration.

Show how the position vector relates to the centroid and the rotation of the rigid slab in plane motion.

The position vector of a point can be written as the position vector of the centroid plus the rotated position vector of the point relative to the centroid, where the rotation accounts for the slab's angular displacement in the plane.

What assumptions are made when showing that the position vector in plane motion demonstrates the rigidity of the slab?

It is assumed that the slab is rigid, meaning distances between points remain constant, and that the motion is planar with no deformation, allowing the use of rotation and translation to describe the position vectors accurately.

Additional Resources

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Introduction

In the realm of rigid body kinematics, understanding the motion of a rigid slab in a plane involves analyzing the position of individual points within the body relative to a fixed reference frame. The concept of the position vector plays a fundamental role in representing the spatial location of points on the body as it moves. This article endeavors to investigate the representation of the position vector of a point on a rigid slab undergoing plane motion, and to demonstrate how this vector encapsulates the entire motion of the slab.

The precise notation and derivation of the position vector are essential in fields spanning mechanical engineering, robotics, and physics. The core objective here is to establish that the position vector of any point on the rigid slab can be expressed in a form that clearly links the body's translational and rotational movements.

Fundamental Concepts and Definitions

Before delving into the derivation, it is important to clarify several foundational concepts:

- Rigid Body Motion in a Plane: Movement where the distances between all pairs of points on the body remain constant over time. It can be decomposed into translation (movement of the body’s reference point) and rotation (about a point on the body).

- Reference Frames:
- Fixed (Inertial) Frame: Denoted by axes \(OX, OY \), stationary in space.

- Body Frame (Moving Frame): Denoted by axes \(x', y' \), fixed to the rigid slab, moving with it.

- Position Vector: For a point \(P \) in the plane, the position vector \(\vec{r}_P \) locates \(P \) relative to a fixed origin \(O \).

The Position Vector of a Point on a Rigid Slab

Consider a rigid slab moving in the plane. Let:

- \(O \) be a fixed origin in the inertial frame.
- \(P \) be an arbitrary point on the slab.
- \(O' \) be the instantaneous center of translation of the slab, or alternatively, a reference point on the slab (such as the centroid or a chosen point \(P_0 \)).

The goal is to express the position vector \(\vec{r}_P \) of point \(P \) in terms of known quantities.

Derivation of the Position Vector

Step 1: Establish a Reference Point

Choose a reference point \(P_0 \) on the slab with position vector \(\vec{r}_{P_0} \). This point serves as a reference for expressing the position of other points.

The position vector of \(P_0 \) relative to the fixed origin \(O \) is:

\[
\vec{r}_{P_0} = \vec{R}(t)
\]

where \(\vec{R}(t) \) is a function of time representing the translational motion of the reference point.

Step 2: Incorporate Rotation

The slab undergoes rotation about a point \(O' \) with an angular displacement \(\theta(t) \). The rotation tensor in plane motion reduces to a rotation matrix \(\mathbf{Q}(\theta) \):

\[
\mathbf{Q}(\theta) =
\begin{bmatrix}
\cos \theta & - \sin \theta \\
\sin \theta & \cos \theta

\end{bmatrix}
\]

Any point \(P \) on the slab can be represented relative to \(P_0 \) as:

\[
\vec{r}_P = \vec{r}_{P_0} + \mathbf{Q}(\theta) \cdot \vec{r}_{P_0 P}
\]

where:

- \(\vec{r}_{P_0 P} \) is the position vector of \(P \) relative to \(P_0 \), expressed in the body frame.

Step 3: Expressing the Position Vector

Assuming \(\vec{r}_{P_0 P} \) is known in the body frame, the position vector of \(P \) in the inertial frame becomes:

\[
\boxed{
\vec{r}_P(t) = \vec{R}(t) + \mathbf{Q}(\theta(t)) \cdot \vec{r}_{P_0 P}
}
\]

This expression encapsulates the entire motion:

- \(\vec{R}(t) \): the translational component, representing the movement of the reference point.
- \(\mathbf{Q}(\theta(t)) \): the rotation matrix encoding the angular displacement \(\theta(t) \).
- \(\vec{r}_{P_0 P} \): the fixed vector from \(P_0 \) to \(P \) in the body frame.

Showing That The Position Vector Represents The Motion

The derived formula demonstrates that the position vector of any point \(P \) on the slab is explicitly composed of:

1. The translational motion \(\vec{R}(t) \): a vector that shifts the entire body in space.
2. A rotated version of the point’s position relative to \(P_0 \): accounting for the rotation of the slab.

This fundamental relationship highlights that the kinematics of a rigid body in plane motion are fully described by:

- The translation \(\vec{R}(t) \),
- The rotation \(\theta(t) \),

- The initial configuration $\vec{r}_{P_0 P}$.

Thus, the position vector $\vec{r}_P(t)$ is a comprehensive descriptor of the motion of point P.

Alternative Formulation: Using the Instantaneous Center of Rotation

In some scenarios, the motion can be characterized by an instantaneous center of rotation O''. Let $\vec{r}_{O''}$ be the position vector of this point, then:

$$
\vec{r}_P(t) = \vec{r}_{O''}(t) + \vec{r}_{P}^{\text{rel}}
$$

where:

$$
\vec{r}_{P}^{\text{rel}} = \vec{r}_{P O''} = \text{vector from } O'' \text{ to } P
$$

rotated by the angular velocity $\omega(t)$. This approach provides a different but equivalent way of demonstrating the same concept: the position vector reflects the combined effects of translation and rotation.

Practical Implications in Engineering and Physics

The significance of this derivation extends beyond theoretical interest:

- Kinematic Analysis: Enables precise calculation of the position, velocity, and acceleration of points on moving bodies.
- Structural Dynamics: Assists in analyzing how forces and moments propagate through the body.
- Robotics and Mechanisms Design: Facilitates the development of motion plans and control algorithms by understanding how individual points move.

Conclusion

The investigation confirms that the position vector of a point P on a rigid slab in plane motion can be succinctly expressed as:

$$
\boxed{
\vec{r}_P(t) = \vec{R}(t) + \mathbf{Q}(\theta(t)) \cdot \vec{r}_{P_0 P}

}
\]

This formula demonstrates that the position of any point on the slab is a superposition of the translational motion of a reference point and the rotational motion about that point. It encapsulates the fundamental nature of rigid body motion in a plane, revealing that the entire movement can be reconstructed from the motion of a single reference point and the body's angular displacement.

By rigorously deriving and analyzing this vector relationship, we reaffirm the central principle of rigid body kinematics: the motion of any point within a rigid body is fully determined once the translational and rotational movements are known. This insight not only enriches our understanding of planar kinematics but also underpins practical applications across numerous engineering disciplines.

References

- Beer, F. P., & Johnston, E. R. (2013). Vector Mechanics for Engineers: Statics and Dynamics. McGraw-Hill Education.
- Shames, I. H. (2002). Engineering Mechanics: Dynamics. Pearson.
- Meriam, J. L., & Kraige, L. G. (2012). Engineering Mechanics: Dynamics. Wiley.
- Hibbeler, R. C. (2016). Engineering Mechanics: Dynamics. Pearson.

Note: For further study, consider exploring the concepts of instantaneous centers of rotation, velocity and acceleration analysis within the same framework, and extending these ideas to three-dimensional rigid body motion.

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