Describe The Sampling Distribution Of P^. Assume The Size Of The Population Is 30,000 . N=400,p=0.2 Choose

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Understanding the sampling distribution of $\hat{p}$ (the sample proportion) is fundamental in statistics, especially when dealing with large populations. In this article, we will explore how to describe the sampling distribution of $\hat{p}$ when the population size is 30,000, the sample size is 400, and the population proportion p is 0.2. This comprehensive overview will guide you through the key concepts, calculations, and implications for statistical inference, ensuring you grasp the essentials for applying these principles in real-world scenarios.

Introduction to Sampling Distributions of Proportions

Before diving into specific calculations, it's important to understand what a sampling distribution of a sample proportion entails.

What Is a Sampling Distribution?

A sampling distribution describes the probability distribution of a given statistic—here, the sample proportion $\hat{p}$—over many repeated samples drawn from the same population. It tells us how $\hat{p}$ varies from sample to sample, providing the foundation for estimating population parameters and conducting hypothesis tests.

The Significance of the Sample Proportion $\hat{p}$

- $\hat{p}$ is the proportion of successes in a sample.
- It is a point estimate of the true population proportion p.
- The sampling distribution of $\hat{p}$ allows us to assess the variability and reliability of this estimate.

Key Parameters and Assumptions

When describing the sampling distribution of $\hat{p}$, certain parameters and assumptions are essential.

Parameters Given

- Population size $N = 30,000$
- Sample size $n = 400$
- Population proportion $p = 0.2$

Assumptions for Approximate Normality

- The sample size is sufficiently large (here, $n = 400$)
- The sample is drawn randomly from the population
- The success-failure condition is met: Both np and $n(1-p)$ are at least 10

Calculations:

- $np = 400 \times 0.2 = 80$
- $n(1 - p) = 400 \times 0.8 = 320$

Since both are greater than 10, the normal approximation is appropriate.

Finite Population Correction Factor

Because the population size is finite and the sample size is a significant fraction of the population, we need to consider the finite population correction (FPC).

What Is the FPC?

The FPC accounts for the reduced variability when sampling without replacement from a finite population:

$$
\text{FPC} = \sqrt{\frac{N - n}{N - 1}}
$$

Given:

$$
N = 30,000,\quad n = 400
$$

Calculating:

$$
\text{FPC} = \sqrt{\frac{30,000 - 400}{30,000 - 1}} = \sqrt{\frac{29,600}{29,999}} \approx \sqrt{0.9867} \approx 0.9933
$$

This correction slightly reduces the standard error, making the sampling distribution narrower.

Describing the Sampling Distribution of $\hat{p}$

Now, let's articulate the key features of the sampling distribution.

1. Shape of the Distribution

- Due to the Central Limit Theorem, for sufficiently large n, the sampling distribution of $\hat{p}$ is approximately normal.
- The normal approximation is valid here because both np and $n(1-p)$ exceed 10.

2. Mean of the Distribution

The mean of the sampling distribution of $\hat{p}$ equals the population proportion:

$$
\boxed{\text{Mean} = p = 0.2}
$$

This indicates that, on average, the sample proportion $\hat{p}$ centers around the true proportion p.

3. Standard Error (SE) of $\hat{p}$

The standard error measures the variability of $\hat{p}$ across samples:

$$
SE_{\hat{p}} = \sqrt{\frac{p(1 - p)}{n}} \times \text{FPC}
$$

Calculating:

$$
SE_{\hat{p}} = \sqrt{\frac{0.2 \times 0.8}{400}} \times 0.9933
$$

$$
SE_{\hat{p}} = \sqrt{\frac{0.16}{400}} \times 0.9933 = \sqrt{0.0004} \times 0.9933 = 0.02 \times 0.9933 \approx 0.01987
$$

Thus,

$$
\boxed{SE_{\hat{p}} \approx 0.01987}
$$

This small standard error indicates that the sample proportion will typically be close to the true proportion of 0.2.

4. Distribution Summary

Feature	Description
Shape	Approximately normal (by CLT)
Mean	$p = 0.2$
Standard Error	≈ 0.01987
Population Size	30,000 (large enough for approximation)

Constructing Confidence Intervals Using the Sampling Distribution

One of the primary applications of understanding the sampling distribution of $\hat{p}$ is constructing confidence intervals to estimate the population proportion p.

Example: 95% Confidence Interval

Using the normal approximation:

$$
\hat{p} \pm Z_{\alpha/2} \times SE_{\hat{p}}
$$

Where $Z_{0.025} \approx 1.96$.

Suppose a sample yields $\hat{p} = 0.22$. The confidence interval is:

$$
0.22 \pm 1.96 \times 0.01987 \approx 0.22 \pm 0.0389
$$

Thus, the 95% confidence interval:

$$
(0.1811,\ 0.2589)
$$

This means we are 95% confident that the true population proportion p lies within this interval.

Implications for Statistical Inference

Understanding the sampling distribution of $\hat{p}$ allows researchers to:

- Estimate Population Parameters: Use sample data to infer p with specified confidence levels.
- Perform Hypothesis Tests: Test claims about p (e.g., $p=0.2$) by calculating the probability of observing a sample proportion as extreme as the one obtained.
- Assess Variability: Recognize how sample size and population size influence the precision of estimates.

Summary of Key Points

- The sampling distribution of $\hat{p}$ is approximately normal when the sample size is large and success-failure conditions are met.
- Its mean equals the true population proportion $p=0.2$.
- The standard error, accounting for the finite population correction, is approximately 0.01987.

- The shape is symmetric and centered around \(p\), enabling the construction of confidence intervals.
- The finite population correction factor slightly reduces variability, which is particularly relevant when sampling a significant fraction of the population.

Final Remarks

In practical applications, understanding the sampling distribution of \(\hat{p}\) provides vital insights into the reliability and accuracy of estimates derived from sample data. With a large population of 30,000 and a substantial sample size of 400, the normal approximation offers a reliable method for inference about the population proportion. Recognizing the role of the finite population correction ensures precise calculations, especially when the sampling fraction is not negligible.

By mastering these concepts, statisticians and data analysts can confidently interpret sample data, make informed decisions, and communicate findings effectively. Whether conducting surveys, opinion polls, or quality control assessments, a solid grasp of the sampling distribution of \(\hat{p}\) is essential for rigorous statistical analysis.

Keywords for SEO Optimization:

Sampling distribution of \(\hat{p}\), population proportion, finite population correction, standard error, confidence interval, normal approximation, statistical inference, large population, sample size, success-failure condition.

Frequently Asked Questions

What is the sampling distribution of $\hat{p}$ when sampling 400 from a population of 30,000 with p=0.2?

The sampling distribution of $\hat{p}$ will be approximately normal due to the large sample size (n=400), with a mean equal to the population proportion p=0.2 and a standard error calculated as √[p(1-p)/n] = √[0.20.8/400] ≈ 0.020.

How do you compute the standard error of $\hat{p}$ in this scenario?

The standard error (SE) of $\hat{p}$ is computed as √[p(1-p)/n] = √[0.20.8/400] ≈ 0.020, indicating the variability expected in the sample proportion.

Is the sampling distribution of $\hat{p}$ approximately normal here, and why?

Yes, because the sample size is large enough to satisfy the conditions for normal approximation, with np = 4000.2=80 and n(1-p)=320 both greater than 10.

What is the probability that the sample proportion $\hat{p}$ exceeds 0.25?

Using the normal approximation, z = (0.25 - 0.2)/0.020 = 2.5. The probability P($\hat{p}$ > 0.25) is approximately 1 - Φ(2.5) ≈ 1 - 0.9938 = 0.0062.

How does the finite population size (30,000) affect the sampling distribution of p̂ ?

Since the sample size (400) is less than 5% of the population (30,000), the finite population correction factor is negligible, and the standard error remains approximately 0.020 without adjustment.

What is the expected value of p̂ in this sampling scenario?

The expected value of p̂ is equal to the population proportion p=0.2, meaning on average, the sample proportion will be around 0.2.

Additional Resources

Describe The Sampling Distribution Of P̂ . Assume The Size Of The Population Is 30,000. N=400, p=0.2.

Introduction

Understanding the sampling distribution of the sample proportion, denoted as p̂ (pronounced "p-hat"), is fundamental in statistical inference, especially when estimating population parameters based on sample data. This concept provides insight into how the proportion observed in a sample varies from the true population proportion, and it forms the backbone of hypothesis testing and confidence interval estimation for proportions.

In this article, we delve into the characteristics of the sampling distribution of p̂ , assuming a finite population of 30,000, a sample size of 400, and a true population proportion p of 0.2. We will explore the theoretical underpinnings, practical implications, and the assumptions necessary for valid inference, providing a comprehensive understanding for students, researchers, and practitioners alike.

Fundamentals of the Sampling Distribution of p̂

Definition of p̂

The sample proportion p̂ is calculated as:

$$
\hat{p} = \frac{x}{n}
$$

where:

- x = number of successes in the sample,
- n = sample size.

In our case, with $n=400$, $p=0.2$, the variable x follows a binomial distribution with parameters $n=400$ and $p=0.2$.

Purpose of Studying the Sampling Distribution

The sampling distribution describes the probability distribution of $\hat{p}$ over many repeated samples drawn from the population. It enables statisticians to:

- Quantify the variability of $\hat{p}$ around the true p,
- Construct confidence intervals for p,
- Conduct hypothesis tests regarding the population proportion.

Theoretical Characteristics of the Sampling Distribution of P̂

Distribution Type

When sampling from a population, the distribution of $\hat{p}$ depends on the underlying binomial distribution of successes x. For sufficiently large samples, the distribution of $\hat{p}$ approximates a normal distribution due to the Central Limit Theorem.

Key conditions for normal approximation:

- $np \geq 10$,
- $n(1-p) \geq 10$.

Given our parameters:

- $np = 400 \times 0.2 = 80$,
- $n(1-p) = 400 \times 0.8 = 320$,

both exceed 10, satisfying the normal approximation criteria.

Mean of the Sampling Distribution

The expected value (mean) of the sampling distribution of $\hat{p}$ is:

$$
\text{Mean} = p = 0.2
$$

This indicates that, on average, the sample proportion will equal the true population proportion over many repeated samples.

Variance and Standard Deviation

The variability of $\hat{p}$ is captured by its variance:

$$
\text{Var}(\hat{p}) = \frac{p(1 - p)}{n} \times \left(\frac{N - n}{N - 1}\right)
$$

where:

- N = population size,
- n = sample size.

The term $\frac{N - n}{N - 1}$ accounts for the finite population correction (FPC), which adjusts variance when sampling without replacement from a finite population.

Calculating the variance:

$$
\text{Var}(\hat{p}) = \frac{0.2 \times 0.8}{400} \times \left(\frac{30,000 - 400}{30,000 - 1}\right)
$$

$$
= \frac{0.16}{400} \times \frac{29,600}{29,999}
$$

$$
= 0.0004 \times 0.9867 \approx 0.0003947
$$

The standard deviation (standard error):

$$
\text{SE} = \sqrt{0.0003947} \approx 0.01986
$$

This standard error quantifies the typical deviation of the sample proportion from the true p.

Finite Population Correction (FPC)

When sampling without replacement from a finite population, the variability of $\hat{p}$ is reduced compared to sampling with replacement. The FPC factor:

$$
\text{FPC} = \sqrt{\frac{N - n}{N - 1}}
$$

adjusts the standard error accordingly.

In our case:

$$
\text{FPC} = \sqrt{\frac{30,000 - 400}{30,000 - 1}} \approx \sqrt{\frac{29,600}{29,999}} \approx 0.9867
$$

This implies a slight reduction (about 1.33%) in the variability of $\hat{p}$ due to finite population sampling, emphasizing the importance of considering population size when the sampling fraction is non-negligible.

Practical Implications of the Sampling Distribution

Estimating Population Proportion

The sampling distribution's characteristics inform the construction of confidence intervals for p. For example, a 95% confidence interval:

$$
\hat{p} \pm Z_{0.975} \times \text{SE}
$$

where $Z_{0.975} = 1.96$, assumes the normal approximation holds.

Given the parameters:

$$
\text{Margin of Error} = 1.96 \times 0.01986 \approx 0.0389
$$

Thus, if a sample yields $\hat{p} = 0.2$, the 95% confidence interval for p is approximately:

$$
0.2 \pm 0.0389 \Rightarrow (0.1611, 0.2389)
$$

This interval suggests with 95% confidence that the true population proportion lies within this range.

Sampling Distribution in Hypothesis Testing

Suppose we want to test $H_0: p = 0.2$ against an alternative hypothesis such as $H_A: p \neq 0.2$. The sampling distribution of $\hat{p}$ under H_0 is used to compute the test statistic:

$$
z = \frac{\hat{p} - p_0}{\text{SE}_0}
$$

where SE_0 is calculated assuming $p = p_0 = 0.2$. The resulting z-score helps determine whether to reject H_0.

Limitations and Assumptions

While the normal approximation simplifies analysis, certain assumptions underpin its validity:

- Sample Size: The approximation relies on sufficiently large np and $n(1-p)$, which are satisfied here.
- Random Sampling: The sample must be randomly selected to ensure representativeness.
- Finite Population Correction: Necessary when the sampling fraction $\frac{n}{N}$ is significant. In this case, $\frac{400}{30,000} \approx 1.33\%$, which is generally considered low enough that the correction is optional but still recommended for accuracy.
- Independence: The successes in the sample must be independent, which holds under simple random sampling.

Conclusion

The sampling distribution of the sample proportion \(\hat{p}\) is a cornerstone concept in inferential statistics, enabling estimation and hypothesis testing about the true population proportion. With a population size of 30,000, a sample size of 400, and a true proportion of 0.2, the distribution of \(\hat{p}\) can be approximated by a normal distribution with a mean of 0.2 and a standard error of approximately 0.01986, after accounting for the finite population correction.

This distribution's properties facilitate the creation of confidence intervals and the conduction of hypothesis tests, providing researchers with powerful tools to infer about the population based on sample data. Recognizing the assumptions and limitations associated with this approximation ensures the validity and reliability of conclusions drawn from statistical analyses.

As statistical methods continue to evolve, understanding the behavior of the sampling distribution remains vital for accurate data interpretation, effective decision-making, and advancing knowledge across disciplines.

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